The Non-Relativistic Models of the Relativistic Bell’s Paradox

Vadim N. Matveev¹, Oleg V. Matveev

Abstract
Here the so-called Bell’s accelerating rockets paradox is examined. The non-relativistic models of Bell's effect are presented, where likewise the theory of special relativity the proper distance between two rockets following one another is increased them being accelerated on identical programmes. It becomes clear that the proper distance increase is determined by Einstein’s simultaneity of the moments of the start of the programmes execution on the rockets. It is also shown that Einstein’s relative simultaneity does not ensure reversibility of the proper distance between the rockets upon their joint return to their initial state. The reversibility is only achieved by the introduction of the preferred reference frame (not necessarily absolute!) and of the universal time in all inertial reference frames.

Keywords: Bell’s relativistic paradox, Einstein’s simultaneity, Einstein’s clock synchronization, preferred reference frame

¹matwad@mail.ru
1. Introduction: The Break of the String and the Essence of the Relativistic Bell’s Paradox

The relativistic Bell’s paradox or, in other words, the accelerating rockets paradox refers to the solution of the following problem.

In a certain inertial reference frame $K$, two identical rockets at rest, with absolutely identical engines are considered. A thin non-stretch, delicate stiff string connects the tail of one rocket to the nose of the other rocket. At a certain moment in time $t=0$ the engines are simultaneously ignited on identical programmes, and the rockets start accelerating, following one another along a straight line on which the string connecting the rockets lies. Due to space homogeneity, the rockets at each moment in time $t$ moving with identical acceleration $g(t)$ and identical velocity $v(t)$, travel in strict synchronism along the straight line, due to mutual synchronism of travel, staying an invariable distance apart in the frame $K$. The question arises as to what will happen to the string when the rockets continue to accelerate indefinitely long.

According to the solution given by Bell, the string will break, which is often presented as a paradoxical effect. Bell’s solution has been disputed by a number of physicists and, according to Wikipedia, it has even been brought forward for informal conference at the CERN theory division. At the same time, there is no paradox related to the string break whatsoever, and the discussion at CERN only once again confirms that there is often no understanding of the basic effects of the theory of special relativity even among high-ranking physicists. It is only this lack of understanding that can explain an appearance of a whole series of discussion papers [1-5] on Bell’s paradox in the American Journal of Physics. The source of misunderstandings when discussing the effects related to Lorentz contraction is an attempt to regard it as having purely kinematic nature [6]. The Soviet physicist Skobeltsyn in his book “The Twin Paradox in Relativity” gave a sufficiently complete explanation of the effect related to the break of a string connecting synchronously accelerating rockets. Written in 1959, the book was published in 1966 [7].

Within the framework of the theory of special relativity, the Bell's paradox has a simple explanation.

The Lorentz contraction formula relates the longitudinal length $L$ of an object moving with a velocity $v$ to proper length $L_0$ of this same object in the following way:

$$L = L_0 \sqrt{1 - (v/c)^2}$$

where $c$ is the speed of light. According to the Lorentz contraction formula, proper length at all times exceeds the length of a moving object.

At all times! If after acceleration of an object its proper length stays invariable, then the length of an object moving after acceleration decreases. However, if in the process of and after acceleration of an object the length constancy of a moving object is forced, then its proper length increases.

Let us in what follows assume that an object has accelerated to a velocity $v$, whereupon the acceleration stops, and the object becomes inertial. For the sake of convenience, let us assume that a velocity $v$ is such that $\sqrt{1 - (v/c)^2}$ equals $\frac{1}{2}$. Then, if for instance an elastic rod with proper length $L_0$, being carefully drawn at one of its ends, is gradually accelerated along its length to a velocity $v$, and in so doing having secured
the integrity of its proper length \( L_0 \), then upon acceleration to a velocity \( v \) the moving rod becomes twice shorter, i.e. its length \( L \) becomes equal to \( \frac{1}{2}L_0 \).

Again, if within the reference frame \( K \) an elastic rod has been accelerated to a velocity \( v \), retaining its ends an invariable (within this reference frame) distance apart (synchronously accelerating them) and preventing the rod from contracting, then the length \( L \) of the rod moving within the frame \( K \) at a velocity \( v \) stays invariable and numerically equal to the value \( L_0 \). The proper length of the rod \( L_0' \), i.e. the length of the rod within the inertial system \( K' \), where it is at rest after the acceleration has been discontinued, becomes equal to \( 2L_0 \), i.e. the relation \( L_0'=2L_0 \) becomes valid, now as \( L=L_0 \), then \( L_0'=2L_0 \). Thus, an elastic rod with the proper length \( L_0 \) prior to its acceleration will stretch twice to its proper length \( L_0' \) after the acceleration is discontinued. It is quite clear that if the rod is not elastic but fragile to rupture even at the slightest stretching, it will break, and in our case, it will happen before the rod accelerates to a velocity \( v \).

The cause for the rod to break is twofold.

Within the reference frame \( K \) the rod shall break due to Lorentz contraction, which is restrained by synchronous motion of the rod ends. Yet from the point of view of observers within the co-moving inertial reference frames or observers in the accelerating reference frame related to the rod, the break occurs due to stretch of the rod.

In the case of two rockets, the distance \( L \) between them within the reference frame \( K \) during synchronous acceleration in this system remains numerically equal to the proper distance \( L_0 \) separating the rockets at their start. At the same time, the distance between the accelerating rockers as registered by inside observers increases, i.e. according to the observers the rockets are moving away from one another. The engines having stopped, the proper distance \( L_0' \) between the rockets in the inertial reference frame \( K' \) where the rockets are at rest becomes equal to \( 2L_0 \), which is the same, to \( 2L_0 \) – it is only due to the motion synchrony and retention of the distance between the rockets in the reference frame \( K \) that the distance \( L \) in it remains numerically equal to the distance \( L_0 \). It is apparent that if the rockets are connected by a thin fragile string, the latter will break.

The string between the rockets is present in Bell’s problem for better clarity and intrigue. There is nothing strange that the string breaks when the rockets are moving apart in their proper reference frame. The essence of Bell’s paradox is not in the break of the string, but in the increase of proper distance between the rockets, resulting in the break. What is the mechanism of the increase of proper distance between the rockets at their acceleration? It is namely on this issue and not on the break of the string that we are going to focus our attention in what follows.

2. The Conditions of Reversibility of the Proper Distance Between the Rockets Upon Return to Their Initial State

Before looking for an answer to the question on the mechanism of increase of the proper distance between the rockets upon their acceleration on identical programmes, let us consider a somewhat extended two-stage modification of Bell’s thought experiment. In this modification, one may indeed perceive paradoxicality as to the behaviour of the rockets.

Let us assume that the rockets, having moved apart during the first stage of the thought experiment and finding themselves at rest in the reference frame \( K \), after the
engines have stopped turn 180 degrees, and then this same rockets acceleration experiment is repeated under identical initial conditions. To do this, some time elapsing after the turn of the rockets, the engine are started on the same programme used for the first stage. Then the rockets at the second stage of the experiment are accelerated to a velocity $v$ within the reference frame $K'$ (they are slowed down accordingly to zero velocity within the reference frame $K$). It is clear that following this each rocket will return to a state of rest within the initial reference frame $K$. To verify this, it is enough from the inertial reference frame $K$ to consider acceleration of each rocket separately in the forward direction, and then their slowdown (exactly the same acceleration, but in the opposite direction). The question consists in the following: what proper distance will separate the rockets after their stop within the reference frame $K$?

To answer this question, let us go back to the first stage of the rockets being acceleraled within the initial reference frame $K$ (in the forward direction). Let us note that after the stop of both engines the clocks mounted on the rockets will become misaligned within the inertial reference frame $K'$ [7]. In order to meet the above-mentioned initial conditions and to “correctly” replicate the experiment in the backward direction having simultaneously started the engines, the clocks have to be resynchronised, i.e. to be synchronised anew using Einstein’s method, meeting the condition of the equality of the speed of light in opposite directions. As regards Bell, it is not the clocks that are synchronised, but the moments of the start of the engines. To this purpose, a light signal emitted from the point equally remote from the rockets at rest is used; that, however, does not affect the heart of the problem because such a start is equivalent to the one on the clock moving synchronously in the Einstein’s sense.

During the second stage of the experiment on the return of the rockets to the state of rest within the system $K$, the proper distance between the rockets will not contract to the initial proper distance $L_0$, but it will again increase two-fold, giving $L_1=2L'$ or $L_1=2L_0$ within the reference frame $K$. This is quite clear because the inertial systems $K$ and $K'$ are equal and the acceleration results within them should be identical. Considering that $L_0=2L_0$, the proper distance $L_1$ between the rockets will be equal to $4L_0$, i.e. four times more than the initial proper distance $L_0$.

And what will happen if we do not resynchronise the clocks?

If the clocks are not resynchronised, and the engines are started under different initial conditions, i.e. according to the “incorrectly” going within the reference frame $K'$ non-resynchronised clocks, then the rockets upon their return to a state of rest within the reference frame $K$ will stay the initial distance $L_0$ apart. This can be verified having examined the acceleration of a pair of rockets, their stop and slowdown from the reference frame $K$. In this system, all actions performed by the rockets with missing resynchronization of the clocks will become synchronous, and the distance $L$ between them will prove invariable at both stages of the experiment. The proper distance between the rockets, which increased during the first stage, will decrease two-fold during the second stage (from the value $L_0'$ to the value $L_0$).

We can make the this experiment more sophisticated if we demand return of the rockets not simply to a state of rest within the initial reference frame $K$, but to the points within the system $K$, from which the rockets started at the beginning of the experiment.

The behaviour of each of the rockets in this case is clear.
If one of the rockets – Rocket \( A \) – from point \( a \) within the reference frame \( K \) is accelerated within this system to a velocity \( v \), and then, elapsing some time after the stop of the engine, turn 180 degrees, and, having started the engine again, slow it down within this same reference frame to zero velocity, then the rocket will return to its initial state of rest within the reference frame \( K \), even though it will find itself in point \( c \), remote from point \( a \). Now, if the rocket is subjected to exactly the same actions that were performed with it when moving it from point \( a \) to point \( c \) on an identical programme, but in the opposite direction, then, fuel consumption neglected, the rocket will return from point \( c \) to point \( a \). It is understood that it will happen only if the hands of the rocket clock according to which the programme of forward and backward motion is performed have not been at some moment arbitrarily put to a different time. Moving the clock hands during inertial motion of the rocket within the system \( K \) would lead to change in the travel time at different stages of inertial motion and also to the fact that the rockets would not have returned to point \( a \), but instead, it would have arrived at a different point – point \( d \).

Rocket \( B \) would behave in just the same way, and, should its flight be performed independent of the flight of Rocket \( A \), then, having departed from point \( b \) and having completed the programmes of accelerations, braking and inertial flight without touching the clock, it would have returned to point \( b \). Thus, a pair of rockets, each of them behaving independently, with the inside observers not intervening in the movement of the clocks, would return to their initial points. But a pair of rockets, if the start of the engines is performed synchronously within inertial systems, in which they for some time find themselves at rest, will not return to their starting points, because in order to ensure the synchronism of the clocks one has to intervene in their natural movement. At best, one of the rockets, for example, Rocket \( B \), may return to the initial point \( b \), whereas the other one will find itself not in point \( a \), but in point \( d \), which is remote from point \( b \). It is easy to understand that if the hands of the clock on rocket \( B \) are not touched, and the synchronism of the clock within each of the inertial reference frames (where the rockets find themselves at rest) is ensured by moving the hands of the clock on Rocket \( A \), it is Rocket \( B \) that will return to point \( b \), whereas Rocket \( A \) will not return to point \( a \), but will find itself in point \( d \). If we do not apply resynchronisation of the clocks altogether, then both rockets will return to their initial points and will find themselves at the initial distance apart.

So, we have a rather strange situation, in which the rockets to do not return to their initial position provided the experiment is performed under the “correct” synchronisation, and do return there if the moments of the start of the engines are synchronised “incorrectly”. Under the “correct” performance of the multi-stage experiment and with multiple accelerations of a pair of rockets in the forward and backward directions, the proper distance between the rockets constantly increases. At the same time, we call an experiment “correct” in which our subjective deductions and a forced movement of the clock hands are needed, and “incorrect” in which it is performed without our artificial manipulations with the clocks. Thus, the principal condition of the reversibility of the proper distance between the rockets upon their return to the initial state of rest within the initial reference system \( K \) is absence of clock resynchronization in the course of the experiment.
Why under “correct” synchronisation does separation of rockets occur? What is the reason for separation? Let us try to answer these questions from different aspects, having examined Bell's effect within the framework of the ether-less theory of special relativity, Lorenz ether theory (LET) and auxiliary models reproducing this effect. We will appeal to the Lorentz ether theory, firstly, owing to the obviousness of this theory of processes that occur at the acceleration of two rockets, and secondly, because of the revival of physicists’ interest to the universal medium, the existence of which is questioned again with the discovery of the Higgs boson and the emergence of the Higgs field on the physics arena as a variety of the universal medium.

3. The Relativist Concept in Bell’s Problem Solution and Modifications Thereof

In fact, everything is clear as regards the theory of special relativity. As a rule, in this theory the question “why” proves irrelevant. Within this theory everything takes place in a definite mode, because it takes place exactly so. The question why the rockets after their acceleration to a great velocity on identical programmes turn out to move away from each other within their proper reference frame is answered referring to Lorentz formula and to the clocks mistiming during acceleration. One would think that the reciprocal separation of the rockets could be explained by a metric reason, which is that the rockets and the onboard meter rulers accelerated to a velocity \( v \) within the reference frame \( K \) become shorter. This contraction, with a distance \( L \) between the rockets retained, must increase the ratio of the distance \( L \) to the lengths of the rocket bodies or the contracted meter rulers. It is clear that the shortening of the bodies and of the contents of the rockets, including the rulers and with the distance between the rockets remaining invariable, must be expressed by the increase in the numerical value of the distance. Indeed, it is exactly what is happening within the reference frame \( K \). However, with regard to the reference frame \( K' \) such an explanation within special relativity is unacceptable, because a state of rest is assigned to each of the proper reference frames, and observers of different reference frames do not recognize their proper motion and do not accept the fact of their proper contraction within the reference frame relative to which they are in motion themselves. If we concede the motion of a proper reference frame, for example, the system \( K' \), then we have also to concede the possibility of the clock synchronization within its proper reference frame on the basis of inequality of the time of the motion of the signal between fixed points in the space of this system in its forward and backward motion.

An increase in the proper distance between the rockets in a two-stage modification of Bell’s problem, which with Einstein’s synchronization of the moments of the start of the engines takes places both at the first and at the second stage, is accounted for by the equality of all reference frames, the isotropy of space and the relativity of such notions as acceleration and deceleration. The latter circumstance equalizes the results of the first and the second stages of Bell’s modified experiment.

4. Solutions to Bell's Problem Within the Framework of the Ether Concept
At first sight Lorentz ether theory is not compatible with the solution to Bell’s problem as given by Bell and Skobeltsyn, because in ether at rest and in the reference frame moving relative to it a pair of rockets must behave differently. Indeed, Lorentz theory does not only allow to mentally reproduce the behaviour of two rockets in ether according to a scenario described by Skobeltsyn [7] within the framework of Einstein's ether-less theory, but makes it possible to explain the reason for such behaviour within the framework of the ether concept.

Let us assume that the initial reference system $K$ considered above finds itself at rest in relation to Lorentz fixed ether. At a certain moment in time the engines of the rockets are simultaneously started, and they start acceleration in the ether. The simultaneity may be achieved either by means of a clock, pre-synchronized by Einstein's method, or by means of emitting a light signal to the rockets from an equidistant point. As the light speed in the ether at rest is the same in all directions, then the emitted light signal simultaneously reaches the rockets at rest in the ether.

During acceleration of a pair of identically pre-programmed rockets, the inside observers according to the formula of Lorentz contraction must register separation of the rockets. What is the cause of this separation? Indeed, in the ether the distance between the rockets owing to the synchrony of their motion stays invariable.

As part of Lorentz worldview, separation of the rockets registered by inside observers can be easily accounted for by actual contraction of the rockets and their contents moving through the ether.

As the rockets and all their contents travelling in the ether are actually contracting in the forward direction, the gap between the moving rockets measured with measurement rods found inside the rockets is perceived as increased (due to shortening of the rods). For example, if ten-meter long rockets found in the ether one kilometre apart, i.e. at a distance of one hundred rocket bodies, have simultaneously accelerated and gained such a speed $v$ that they have contracted twice, then the distance between the rockets that has not changed in the ether has become equal to two hundred rocket bodies. The observers stationed inside the rockets will perceive this imperceptible for them actual contraction of the rockets with all their contents as a seeming perceptible double increase of the distance between the rockets equal to two kilometres.

And what would happen if the rockets uniformly moving in the ether after stopping their engines make a 180-degree turn and, elapsing some time, (on universal ether time) having simultaneously started the engine in the direction opposite to the motion of the rockets begin to synchronously decelerate to a complete stop in the ether?

As part of the absolute universal ether time, the actual simultaneity is absolute, so resynchronization of the rocket clocks is not required. It is clear that the lengths of the rockets and measurement rods when braking in the ether will start increasing as their velocities decrease in relation to the ether, and, having come to a state of rest in relation to the ether, the rockets will obtain their initial lengths. The distance between the rockets as fixed by observers will thus decrease to the initial one. It will occur thanks to actual increase in lengths of bodies and measurement rods to their initial size.

Moreover, what about a repeated separation of rockets, which is observed in the theory of special relativity after the rockets return to the initial reference frame? Is it possible to observe it in the ether?

Yes, indeed.
One should not forget that in Lorentz ether theory besides an absolute real time there is a local fictitious time. This fictitious local time ensures artificial equality of speeds of light in opposite directions.

Let us assume that the observers in the rockets refuse to use uniform ether simultaneity and either resort to resynchronization of the clocks by Einstein method, or, which is the same, synchronize the start of engines by a light signal emitted from the point equidistant from the rockets.

In this case we will observe the following...

The simultaneous start of engines on a local time turns not simultaneous on an absolute ether time. The engine on the rear rocket in the direction of motion – let it be rocket B – starts earlier on real ether time, so deceleration of the rocket begins earlier. The fact is that a light signal moving in the ether arrives at the rear rocket B moving in the ether towards the signal earlier than at the front rocket A, which is moving away from it. The front rocket A begins deceleration later on ether time. Owing to a difference in rocket engines start times the velocity of the rear rocket in ether at all moments in ether time will be less than the velocity of the front rocket. Such a difference in the speeds of rockets leads to lagging of the rear rocket and to actual separation of rockets decelerating in the ether. It is easy to show (see the appendix) that after the stop of the engines the rockets return to a state of rest in the ether and find themselves separated by an actual distance $L_1$, which is four times more than the actual distance $L$ separating the rockets at the first stage of the experiment. In the process of deceleration in the ether the lengths of the rockets and their contents increase. One would think it would lead to the decrease in distance between the rockets, as perceived by observers. Indeed, at the time of return of the rockets to a state of rest in the ether this seeming distance between the rockets, provided they stay the same actual distance apart, would have contracted twice. However, by this moment the rockets have moved apart and the actual distance between them has increased four times, i.e. the actual quadruple separation of rockets is twice bigger than the seeming double reduction of the distance between them. For this reason at the second stage of the experiment the observers of rockets register not a decrease in the distance between the rockets upon their return to a state of rest in the ether, but a double increase in this distance. The main conclusion of the ether theory related to behaviour of accelerating rockets is that when using Einstein simultaneity in the ether at rest and within a reference frame moving in the ether, the results of acceleration of a pair of rockets are perceived by observers as identical.

5. The Circular Model of Bell’s Effect

An entertaining version of Bell’s effect may be exampled by a circular model, which due to lack of full equality of the rotary and inertial motion, may be considered as non-relativistic[8].

Let us consider two identical rockets resting on a circle of big diameter in the inertial system $K$. The distance $L_0$ between the rockets is much less than the diameter of the circle and is practically equal to the length of the arch connecting the rockets. The length of each rocket is equal to $l_0$. Let us place a pulse light source in the centre of the circle. At a given moment in time the source emits a light pulse, which upon reaching the rockets, starts their engines for some time. If the programmes that guide the engines of
the rockets are absolutely identical, then the rockets are synchronously accelerated along the circle, at each point in time possessing identical angular acceleration \( \varepsilon(t) \) and angular velocity \( \omega(t) \) and remaining at the identical distance of \( L = L_0 \) within the inertial system \( K \), rigidly bound to the circle. Let us imagine that after the stop of the engines the linear speed \( v \) of the circling rockets is such that the lengths \( l \) of the bodies of the rockets in motion owing to Lorentz contraction become twice less than their initial length at rest \( l_0 \). As \( l = \frac{1}{2}l_0 \), and the distance \( L \) stays invariable and equal to \( L_0 \), then the ratio \( L/l \) becomes equal to \( 2L_0/l_0 \). As the lengths of the rocket bodies and their contents are perceived by observers as invariable, they register the seeming double increase in distance between the rockets to the value \( L'_0 \), numerically equal to \( 2L_0 \). If the rockets are connected by a thin fragile string, then the latter, which is tending to contract, will break during acceleration. The string becomes actually broken because of Lorentz contraction, which is constrained by synchronously moving rockets, though the inside observers seem to attribute the string break to an increase in the distance between the rockets.

Let us note that an identical result would be received if the rocket engines were started not by a pulse from the central source, but by a light pulse from the source located in the centre of the arch connecting the rockets. Having passed identical distances, the light from the source would simultaneously reach the rockets, and the engines would be simultaneously started in an Einsteianian sense.

Now, if the circling spaceships are turned 180 degrees and on an impulse from the central light emitter the engines are simultaneously started, then after synchronous deceleration on the same programmes that guided acceleration, the rockets will find themselves at rest. We think that in the course of acceleration and deceleration fuel consumption can be neglected.

In the course of deceleration an actual elongation of the bodies of rockets occurs, perceived by inside observers as a reduction of distance between the rockets. After the stop of the engines and return of the rockets to a state of rest, the inside observers discover that the distance between the rockets becomes equal to the initial value \( L_0 \).

Now let us imagine that in order to start the engines in the deceleration mode the inside observers use not a pulse from the central source, but a pulse from the source at the time of emission located in the centre of the arch connecting the circling rockets. With a large diameter of the circle and practical indistinguishability of the arch from a straight-line segment, the inside observers can during a limited period consider the reference frame \( K' \) connected with the rockets and dimensions-constrained as quasi-inertial. Starting the engines from a light source located between the rockets, the observers believe that this light, omnidirectionally propagating at an identical velocity within their quasi-inertial reference frame, simultaneously reaches the rockets and simultaneously starts the engines. However, in an inertial reference system \( K \) the rockets are moving, and the light omnidirectionally propagating in it at an identical velocity reaches the rear rocket before the front one. A delay in the start of the front rocket engine will result in a larger than \( L_0 \) value of the distance between the rockets after completion of operation of the decelerating engines. As well as in the previous model, this larger value will become equal to \( 4L_0 \) (see the appendix).

Thus, after acceleration of rockets to a speed \( v \) the inside observers will find the double seeming increase in distance between the rockets caused by a double decrease in the longitudinal sizes of bodies of rockets and their contents. After braking on a signal
from the source located between the rockets the inside observers will once again record
double increase in distance between the rockets. This double increase after braking
consists of actual quadruple increase in the distance between the rockets, caused by a
start delay of the front rocket engine and the seeming decrease in the distance caused by a
double lengthening of the bodies.

The reference system \( K \) is assigned within a circular model in a sense that, unlike
quasi-inertial systems connected with the rockets, it is inertial indeed, and unlike other
inertial systems, it is only within this one that all rockets circle at identical speeds.

6. The Simulation of Bell’s Paradox in Aqueous Medium

In recent years we have presented and discussed the kinematic model of the theory of
special relativity [9-10] at seminars and the international conferences. Taking barges
moving in still water and high-speed boats as an example, not only all known relativistic
kinematic phenomena and paradoxes including that of Bell’s are simulated, but within
this model the reason for these phenomena is also given.

A rigid (solid) body, which can also be represented by an imaginary spaceship or
rocket, within this model is simulated as a group of barges found on a surface of a flat-
bottom reservoir. Each barge is equipped with a pendulum clock. A high-speed shuttle
performs the role of the pendulum: it is taking the shortest path between the barge and the
bottom at a speed \( V \). Here, the speed \( V \) is the usual “earthly” speed equal, for example, to
100 km/h. The frequency of the pendulum of the barge at rest on a water surface is equal
to \( V/2h \), where \( h \) is the depth of this flat-bottom reservoir. If the barge is floating at a
speed \( v \), where \( v < V \), then the vertical component of speed \( V \) (the speed of floating up and
sinking) of the shuttle is equal to \( V \sqrt{1 - v/V} \), while the pendulum frequency is equal to
\( V \sqrt{1 - v/V} / 2h \). Thus, the clock on the moving barge is \( 1/\sqrt{1 - (v/V)^2} \) times slower than
the one on the barge at rest. By combining barges in groups that are at rest and those in
motion, it is possible to fully simulate the kinematic phenomena of the theory of special
relativity.

The constancy of distance between the barges in the group simulating a solid body
is maintained on a “pseudolocation” principle. The role of a “pseudolocation” signal in
realizing this method is performed by high-speed boats running to and fro between the
barges at a speed \( V \). The principle of a “pseudolocation” is as follows.

A high-speed boat regularly starts from each of the barges to the next barge, and
upon reaching it, starts its way back. The instruments on the barges by means of
simulated clocks measure the travel time of the boat to the next barge and back; if needed
they move the next barge nearer or farther so as to retain this time and the invariance of the
“pseudolocation” distance “seemingly” sensed by the instruments on the barges.

The rigidity of a simulated body is understood as the constancy of the
“pseudolocation” distance between the barges. If under external actions the
“pseudolocation” distance between the barges can be changed, then such a group can’t be
viewed as rigid.

With acceleration of a group of barges to a speed of \( v \) and with constancy of the
“pseudolocation” distance, the actual longitudinal dimensions of a group of barges in
motion contract for the following reason.
In order to cover the distance $l$ between the barges one needs the time $\Delta t_1$, equal to $l/(V - v)$ for the forward motion of the group of barges, and the time $\Delta t_2$, equal to $l/(V + v)$ for its backward motion. The overall time $\Delta t_1 + \Delta t_2$ of the travel there and back between the barges makes $2l/(V^2 - v^2)$ or $2l/(1 - v^2/V^2)$. If $v = 0$, then this time is equal to $2l/V$, if $v \neq 0$, then the time $\Delta t_1 + \Delta t_2$ is $1/(1 - v^2/V^2)$ times more than $2l/V$. As the clocks on the barges in motion go $1/\sqrt{1 - (v/V)^2}$ times slower than on those at rest, then the instruments through pseudolocation monitoring the distance between the barges and retaining it constant, reduce the actual longitudinal distance between the barges not $1/(1 - v^2/V^2)$ times, but $1/\sqrt{1 - (v/V)^2}$ times.

Let us simulate Bell’s paradox with a string in the aqueous medium. For this purpose, we will consider two groups of barges at rest, simulating rockets. The groups find themselves some distance $L_0$ apart, and they are linked by a thin chain of barges, simulating a string. Let the groups and the string on the clock or on a signal from external observers simultaneously start and accelerate on identical programmes along the line on which they find themselves. In this case, after the start the actual longitudinal distances between the barges within each of the two groups moving one after another as well as in the chain of barges connecting these groups will start to decrease. If we simulate an unbreakable string, then, by definition, the pseudolocation distance between the barges of the chain is retained, meaning that the chain of barges, which is being reduced in motion becomes actually shorter. The action of the chain (string) on the groups leads to some desynchronisation of acceleration of the groups of barges and to their restraint by the chain of barges. If the breaking chain of barges is simulated, then at least in one link of the chain the forced increase in “pseudolocation” distance leads to its break. Following that, in two remainders of the chain the retention of “pseudo-location” distance between the barges can freely occur, and the “fragments” of the string as well as the groups of barges start contracting. Owing to synchronism of the motion of the groups of barges after the break of the chain, the actual distance $L$ between the groups will remain invariant, and each group will contract in the direction of motion. If after termination of acceleration the instruments of the groups will measure the distance $L_0$ between the groups in units of length of their groups, they will find out that the groups have moved apart, and the distance between them has increased.

Let us note that the same result would be received if the groups started simultaneously on the clocks of the groups of barges which had been previously synchronized by means of a high-speed boat moving between them at an already known identical speed (in relation to water and the groups at rest in water) in all directions.

If elapsing some time the moving groups of barges begin to perform the back action and simultaneously on the clock or on a signal from external observers start deceleration, then the inverse process will proceed in such a way that the groups after the completion of breaking and the stop will be expanded again and will return to an initial state. Owing to expansion of the groups, the instruments on the barges will record distance reduction between the groups in units of length of their groups up to the initial (starting) value. That will occur at an invariant actual distance between the groups.

However, if the groups of barges in motion start deceleration not on an external signal, but on a clock pre-synchronized by means of a high-speed boat, the result may be twofold.
If at synchronization of the clock one considers the fact that in relation to the groups the high-speed boat moves at a speed of \( V - v \) in the direction of the forward motion of the groups and at a speed of \( V + v \) in the direction of their backward motion, then the prior distance between the groups will remain, and the pseudolocation distance between them will decrease to the initial one. However, if at synchronization we make a false assumption on the equality of the speed of the boat relative to the barges in their forward and backward motion, then the distance between the groups will increase. It occurs because of a time delay in the start of deceleration of the front rocket in a row of those moving one after another. As well as in the cases described above and clarified in the appendix, such a delay will lead to the fact that groups of barges after their stop will find themselves at an actual distance \( 1/(1-v^2/V^2) \) times larger than they were before the start of deceleration. Such an increase in distance between the groups exceeds the reduction of pseudolocation distance between them \( 1/\sqrt{1-(v/V)^2} \) times, which is why the instruments on the groups of barges will register an increase in location distance between the groups after the stop of deceleration not \( 1/(1-v^2/V^2) \) times, but \( 1/\sqrt{1-(v/V)^2} \) times.

7. Conclusion

The behaviour of accelerating rockets in ether and circular models treated above as well as its simulation in aqueous medium physically differs from their behaviour on the theory of special relativity. In all the three models, there is a preferred reference system \( K \) at rest, and there are inertial or, as is the case of the circular model, pseudo-inertial systems moving relative to the preferred reference system \( K \). The rockets of these models moving at a speed \( v \) within the dedicated reference frame \( K \), possessing actual length \( L_0\sqrt{1-(v/c)^2} \), become actually shorter at a further increase in speed within the reference frame \( K \), and they are actually expanded to the maximum value of rest the speed decreasing. During the further run of the engines the rockets having slowed down to zero velocity start gaining speed again within the reference frame \( K \) at rest, though in an opposite direction, which accounts for the shortening of the rockets again. When applying uniform simultaneity of an preferred reference frame in all reference systems, an increase in distance between the rockets, seemingly perceived by observers, is coherent with the actual shortening of the rockets.

Therefore, the seeming distance between the rockets increases provided they accelerate within the reference frame \( K \), and decreases up to the value equal to \( L_0 \) if they are slowed down to a state of rest (in system \( K \)). During the further run of the engines and the acceleration of the rockets the seeming distance between the rockets increases.

Such a specific behaviour of rockets in models with an preferred reference frame would seem essentially incompatible with their behaviour in the ether-less world. However, upon close examination of the model of the ether-less universe and the non-relativistic models one may notice that the behaviour of the rockets is determined not by our ideas of preferred reference frames, but by synchronization of the clocks. The representations regarding assignment of a reference frame are used only for justification of one or the other synchronization. Going beyond such justifications and applying an
identical synchronization in different models, it is possible to obtain identical behaviour of rockets in different models.

If in the ether-less model for purely practical purposes we conditionally introduced the preferred inertial reference system and applied universal (not absolute, but, explicitly, conditionally universal!) time and uniform scales of physical quantities in other reference frames, then such a model would mathematically describe the behaviour of the material world, as though in the ether model this preferred reference frame were rigidly fixed to the ether, while the other systems were moving in relation to it. Thus an invariance of mathematical notation of physical laws is broken, but there emerges an invariance of physical quantities in different reference frames. For example, the longitudinal length of a rod moving relative to a preferred reference frame and which has shrunk Lorentz-wise will be identical in all reference frames.

On the other hand, if in the ether model, having introduced an artificial requirement of equality of the velocity of light in opposite directions, one should refuse a preferred reference frame and equalize all reference systems, there will emerge an invariance of mathematical notation of physical laws and the imaginary relativity of physical quantities (Lorentz ether theory with Poincare-Lorentz transformations). All this could be understood having analysed the results of the simulation of kinematics related to the theory of special relativity stated in the works [8-9].

8. Appendix

Let us consider two rockets, rocket A and rocket B, moving one after another in a straight line at a velocity v, within the inertial reference frame K at rest. Under an inertial reference frame at rest we understand a reference system, which is conditionally or conceptually assigned a state of rest. The distance between the rockets moving forward, the front rocket A and the rear rocket B, is equal to L. At a certain time period from the central point located between the rockets at a distance \( \frac{L}{2} \) from each of them an omnidirectional signal is emitted, propagating in the reference frame K at a speed c (in the case of simulation in aqueous medium the role of speed c is played by speed \( V \)). As the rear rocket B is moving in the reference system K towards the signal at a speed v, the signal overtakes the rocket B after a time period \( \frac{L}{2(c+v)} \). The signal travelling to the front rocket A, which is moving away from it, needs a longer time period, equal to \( \frac{L}{2(c-v)} \). Therefore, rocket A begins deceleration later than rocket B. The difference \( \Delta t \) of the times upon which the signals reach the rockets A and B is equal to \( \frac{L}{2(c+v)} - \frac{L}{2(c-v)} \), i.e. \( vL/(c^2-v^2) \).

If the rockets started simultaneous acceleration within the reference frame K, then owing to synchronism of deceleration within this reference frame they would find themselves after the stop of the engines the identical distance L apart. But the front rocket A started deceleration a time period \( \Delta t \) later than rocket B, during this time period having travelled an extra run \( \Delta x \), equal to \( v\Delta t \) or \( v^2L/(c^2-v^2) \). For this reason the distance \( L+\Delta x \) between the starting point of rocket B and the starting point of rocket A is equal to \( L+v^2L/(c^2-v^2) \), which after transformation can be written as \( L/(1-v^2/c^2) \). Owing to full identity of the rockets and of the programmes operating their engines, the distance that each of them will travel from a braking point to a point of arrival at a state of rest will be
identical. For this reason, the rockets having started deceleration at different time moments from the points that are a distance \( L/(1-v^2/c^2) \) apart will likewise finish deceleration at different times at the points staying the identical distance apart.

\[ \sqrt{1-(v/c)^2} = \frac{1}{2}, \text{ then } 1-v^2/c^2 = \frac{1}{4} \text{ and } L/(1-v^2/c^2) = 4L. \]

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