The Laws of Motion and a New Perspective on Inertia

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Abstract

Newton's Laws of Motion (NLM) represent one of the most significant achievements in the history of science. That said, they are not without their problems. Firstly, NLM treat gravitational motion as if it is caused by a force, which as Einstein discovered, is not true. Secondly, NLM define force in terms of the acceleration a mass undergoes. However, different observers are free to employ different coordinate systems which can result in the measurement of different accelerations. This in turn can result in different observers concluding that different aggregate forces are in action when it must be true that only one aggregate force is in action. The good news is that by incorporating the fact that gravitational acceleration is not caused by a force into a slightly modified version of NLM we can eliminate these problems, and in the process derive a new perspective on the concept of inertia.

Gravitational Acceleration Is Not Caused By a Force

As the story goes, Einstein was wondering what it would be like inside an elevator car if the cable raising and lowering the car broke. He imagined that as the car fell towards the Earth he and any other objects in the car would simply hover weightlessly in their current positions relative to the car unless acted upon by some force. If a force was imparted on an object the object would accelerate until the force was discontinued after which the object would continue on in a straight line relative to the other objects in the car. At some point he realized that this would be much the same as if the elevator car was not falling towards the Earth but was instead floating in some remote region of space far from any other significant masses. In fact, the more he thought about it the more he realized that the physical circumstances in these two scenarios are identical in every respect. That is, if there were no windows or doors to look through, or more precisely if there were no way to obtain any information from outside the confines of the elevator car, then there is no experiment whatsoever that can be performed that would allow him to determine any difference between the two scenarios. The great man then made the intellectual leap that since there is clearly no force in operation in the scenario in which the elevator car is in the remote region of space, and since the two scenarios are physically identical, then there can be no force in action in the elevator car if it is in free fall towards the Earth. That is, the acceleration caused by gravity is not cause by a force.

So what causes gravitational acceleration? Einstein went on to theorize that gravitational acceleration is caused by the non-uniformity or curvature of space. This was validated a few years after it was first proposed, but that is not the focus of this paper. All we need to know to continue the current train of thought is to acknowledge and accept whole heartedly that gravitational acceleration is not caused by a force and not bother ourselves with what the actual cause of the acceleration is. With this acknowledgment, we can plainly see that NLM, which depend on the assumption that any acceleration of a mass must be due to a force, are inadequate. However, as mentioned above, by making some fairly minor modifications to NLM, we can eliminate the inadequacy.

NLM Define Force In Terms Of Acceleration

NLM define force in terms of the acceleration the force causes a mass to experience. To see why this is problematic, consider a test object in a remote region of space. An observer, O1, is also in the same region of space. Observer O1 is a curios fellow and likes to make observations of the happenings around him and to do so he uses a coordinate system oriented such that he is at the origin – a perfectly reasonable thing to do. Imagine that O1 sees the test object and determines that the object is not accelerating relative to the coordinate system he uses. If we provide the observer with NLM and ask him whether a force is acting on the object, he will conclude that since the object is not accelerating in the coordinate system he uses there is no force acting on the object.

Now consider another observer, O2. Unlike O1, this observer lives in a rocket ship and spends his days flying through the galaxy this way and that, speeding up and slowing down as he pleases. As such, he is continuously subjected to force induced accelerations. O2 needs to be able to avoid running into anything and so he must be aware of his surroundings. To do so he employs a coordinate system with his rocket ship at the origin - another perfectly reasonable and valid choice. If we give NLM to O2 as he approaches our test object while the rocket engines are turned on he will note that the object is accelerating relative to the coordinate system he uses and will therefore conclude that a force is in fact acting on the object.

At this point it becomes obvious that there is a problem. We gave both observers the same set of laws to use, they both made reasonable, legitimate use of the laws yet one claims a force is acting on the test object while the other concludes there is no force acting on the object. We are not the first to see this problem. It has been well known for some time and a standard explanation for the confusion is confidently provided in text books and the scientific literature in general. The explanation is that NLM are only valid under certain circumstances. They are only valid in "inertial reference frames (IRFs)". But there are problems with this explanation as well – not the least of which is the fact that there are no IRFs in nature. To see why there are no IRFs in nature, consider the fact that no matter where one goes in the universe, if he were to put two objects in close proximity to one another and not restrict their movement, the objects will accelerate towards one another in the absence of any force causing said acceleration – in direct violation of NLM. So, we see that there are no IRFs in nature, and that NLM are never valid. One may be inclined to say that in the limit as the volume of space under consideration goes to zero the behavior of the objects approaches NLM, but this is not necessary. As noted above, it turns out that modifying NLM to incorporate the fact that gravitational acceleration is not caused by a force makes the laws valid in all frames of reference – inertial or otherwise – and, in fact, the entire concept of reference frames is shown to be unnecessary. We can rely exclusively on the well know and well defined concept of coordinate systems, and will see that the result is that the modified laws of motion produce the same results in all coordinate systems – regardless of whether those coordinates system are accelerating relative to one another or not.

First Principles

In order to understand how we should modify NLM, let's begin from first principles. Consider an observer standing on the surface of the Earth holding an object at location A. The observer releases the object which accelerates downwards without any force causing the downward acceleration. Next, we hold the object at location B, directly above location A, and release it. The object again accelerates in the same direction, but at a diminished magnitude because the distance between the object and the Earth has increased. We can continue to raise the object to higher and higher locations, and to combine the raises with displacements to the left and to the right and forward and backwards. Eventually when we release the object it may accelerate towards the Moon, to the Sun, or to some other galaxy. But in each case the object will accelerate away from its original position and no force will cause that acceleration. We need a name for this acceleration - let's refer to it as the Ambient Acceleration (a_A). Now, it is a testament to the genius of Isaac Newton that, while he formulated his Universal Law of Gravitation (ULG) to describe the *force* responsible for the gravitational acceleration of objects (even though we now know there is no such force) the portion of the ULG that describes the *acceleration* caused by gravity is perfectly valid. That is, Einstein showed that since gravitational acceleration is not caused by a force, we now know that Newton's ULG,

$$F = G\frac{M\,m}{r^2} \qquad (1)$$

where *F* is the force of gravity, *G* is the gravitational constant, *M* is the mass of the Earth, *m* is the mass of a test object, and *r* is the distance between the two masses is invalid. However, for large M and small test mass *m*, the acceleration derived from the ULG,

$$a = G\frac{M}{r^2} \qquad (2)$$

in which *a* represents the acceleration of the test mass, is perfectly valid. The point here is that we can use this portion of the ULG to calculate the a_A that an object will experience. And as we now begin to see that an object will accelerate according to the a_A without a force acting on it, we begin to realize that *a force will be required and will be present whenever our measurements show that an object is not accelerating according to the* a_A. A force - a real, measurable force - will be present whenever an object is not accelerating according to the a_A. And it becomes evident that it is the *difference* between the calculated a_a and the measured acceleration that determines the force acting on the object. Mathematically, the original second law of motion

$$F = ma \qquad (3)$$

is altered to

$$F = m(a_M - a_A) \qquad (4)$$

where a_M is the measured acceleration and a_A is the ambient acceleration. Some examples of the use of Equation (4) follow. (The coordinate systems to be used in these first examples will remain undefined for now and will be addressed in the "Some Additional Benefits" section below.)

Consider an object in free fall above the Earth. It is commonly said that the object will be accelerating downward at the magnitude *g*. If we calculate the value for a_A and measure the acceleration a_M we will find them to be equal. Thus Equation (4) shows that *F* = 0, which agrees with the premise that objects that are gravitationally accelerating are doing so without any force causing said acceleration.

Now, consider an object at rest on the surface of the Earth. At this location the ambient acceleration will be calculated to be *g* in the downward direction, or –*g*, while the measured acceleration will zero. Plugging these values into Equation (4)

$$F = m\big(0 - (-g)\big) = mg \qquad (5)$$

we see that there is a force pushing up on the object preventing it from accelerating downward at the ambient acceleration at the location. This is the force we feel on the bottom of our feet while standing.

Inertia

As far as the first law of motion goes, we now see that it can be re-formulated to "An object will accelerate at the ambient acceleration unless acted upon by a force". This law is sometimes referred to as the Law of Inertia in that it was the source of the idea that inertia is the property of matter that resists any acceleration. However, in light of the modified formulation inertia is now seen as the property of matter that resists deviations for the ambient acceleration.

Some Additional Benefits

The modified laws of motion are now seen to be consistent with the fact that gravitational acceleration is not caused by a force. In addition to this benefit, they are now also immune to the arbitrary results produced when we allow arbitrary coordinate systems to be used with them. That is, the new laws of motion produce the same results for all coordinate systems. Even coordinate systems that are accelerating relative to each other will produce the same results, as will be shown in the following example.

Consider an object in coordinate system A. In coordinate system A, imagine that we have measured the acceleration of the object and found it to be 12 units (meters per second squared) along the x axis, and based on the configuration of local masses the calculated value of the ambient acceleration is 7 units also along the x axis[1]. Plugging these values into Equation (4) gives

$$F = m(12 - 7) = 5m \qquad (6)$$

Now imagine that another coordinate system B has axes that are parallel to those of coordinate system A, and the B is accelerating in the x direction at a magnitude of 3 units. Clearly, the original formulation of the second law would produce different results when applied in the different coordinate systems A and B. However, with the new form of the laws of motion, in coordinate system B the measured acceleration will be 12 – 3 = 9 and the ambient acceleration will be 7 – 3 = 4, and we have

[1] The example given here keeps all accelerations along the x axis for simplicity sake. It is clear that accelerations in any directions could have been used and the conclusions arrived at would be the same.

$$F = m(9-4) = 5m \qquad (7)$$

which demonstrates that all coordinate systems, even coordinate systems that are accelerating relative to each other, will produce that same results when the modified laws of motion are used.

There are different ways to look at this fortunate result. One way is to note that the legacy laws of motion were based on an acceleration, and that acceleration was essentially an arbitrary value if we allow the choice of coordinate systems to be completely unrestricted. In contrast, the modified laws of motion define force in terms of the *difference between two accelerations*, which is not arbitrary at all. In general, if two acceleration vectors are not equal in one coordinate system, they are not equal in any coordinate system. The difference between two such accelerations cannot be altered under any coordinate transformation. While the individual accelerations will vary as different coordinate systems are employed, the difference between the two accelerations will remain constant.

Another way to look at these results is to note that any effect caused by using different coordinate systems, let's call that effect α, will be applied equally to both accelerations in Equation (4), resulting in

$$F = m((a_M + \alpha) - (a_A + \alpha)) = m(a_M - a_A + \alpha - \alpha) = m(a_M - a_A) \qquad (7)$$

showing that the same result is obtained for all α.

This relates to one of the fundamental concepts underlying the theory of relativity which can be expressed as "The laws of nature do not depend on the state of motion of the observer, so our equations that describe the behavior of nature should not produce results that vary depending on the state of motion of the observer." By integrating the fact that gravitational motion is not caused by force into the laws of motion, we now have laws that, unlike the legacy laws of motion, adhere to this notion.

Another benefit of the modified laws of motion is that, unlike the legacy laws of motion, there is no need for any caveat such as that they are valid only in certain circumstances, such as in inertial references frames. The modified laws of motion are valid in all circumstances and thus the concept of "reference frames" in general can be dispensed with entirely and we can rely on the well defined concept of the coordinate system.