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Black Holes without Singularity?

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Abstract

We propose a model scheme of microscopic black holes. We assume that at the center of the hole there is a spin $\frac{1}{2}$ core field. The core is proposed to replace the singularity of the hole. Possible frameworks for non-singular models are discussed briefly.

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1 Introduction and Summary

The motivation behind the model described here is to find a way to go beyond the Standard Model (BSM), including gravity [1]. Gravity would mean energies of the Planck scale, which is far beyond any accelerator experiment. This work is hoped to be a small step forward in exploring the role on gravity in particle physics while any complete theory of quantum gravity is still in an early developmental phase, and certainly beyond the scope of this note.

In particular we pay attention to the nature of microscopic quantum black holes at zero temperature. We make a gedanken experiment of what might happen when exploring a microscopic black hole deep inside with a probe. In [2] we made two assumptions

(i) when probed with a very high energy $E \gg E_{Planck}$ point particle a microscopic black hole is seen as a fermion core field in Kerr, and ultimately Minkowski, metric. The point-like core particle of the hole may have a high mass, something like the GUT unification energy or Planck mass. The core field is called here gravon.

(ii) the core may be the stable (or decaying) remnant of the hole, and the black hole singularity is replaced by the core field.

The core is considered to illustrate the idea of singularity free black hole, but it may also be a possible remnant with certain interactions of its own. In this note we try to discuss theoretical models available in the literature to get support for the above assumptions.

With the Planck scale having its the conventional value 10^{19} GeV testing is hard. Gamma-ray signals from the sky may be a promising way. A gamma-ray, or particle, with energy half the Planck mass would be a clear signal of the models of this type.

There are models which may bring the relevant energy scale down within reach of the LHC. Provided the Planck scale is down at TeV scale black holes with mass in the TeV region may be formed. The basic idea discussed here does not depend on the value of the Planck scale.

In sections below we discuss briefly some possible scenarios of microscopic black holes, renormalization group improved gravity, bouncing universes, their roles in cosmology and testing the model. The nature of the present note is a short survey of limited scope.

2 Einstein-Dirac Cosmology

Though not directly relevant to the present work ref. [3] gives another indication of singularity avoidance in Friedmann-Robertson-Walker (FRW) cosmology. The authors study Einstein-Dirac (ED) equations

$$R^i_j - \frac{1}{2}R\delta^i_j = 8\pi\kappa T^i_j \tag{1}$$

$$(\mathcal{D} - m)\Psi = 0 \tag{2}$$

where T^i_j is the energy-momentum tensor of the Dirac particles, κ is the gravitational constant, $\mathcal{D}$ is the Dirac operator and Ψ the wave function. For metric the closed Friedmann-Robertson-Walker is chosen

$$ds^2 = dt^2 - R^2(t)d\sigma^2 \tag{3}$$

where R is the scale function and $d\sigma^2$ is the line element on the unit 3-sphere

$$d\sigma^2 = \frac{dr^2}{1-r^2} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2 \tag{4}$$

where r, θ and ϕ are the standard polar coordinates. The Dirac operator in this metric is written as

$$\mathcal{D} = i\gamma^0 \left(\partial_t + \frac{3\dot{R}(t)}{2R(t)}\right) + \frac{1}{R(t)}\begin{pmatrix} 0 & \mathcal{D}_{S^3} \\ -\mathcal{D}_{S^3} & 0 \end{pmatrix}, \tag{5}$$

where γ^0 is the standard Dirac matrix, and D_{S^3} is the Dirac operator on the unit 3-sphere. The operator D_{S^3} has discrete eigenvalues $\lambda = \pm\frac{3}{2}, \pm\frac{5}{2}, \ldots$, corresponding to quantization of momenta of the particles. The Dirac equation is separate with the ansatz

$$\Psi_\lambda = R(t)^{-\frac{3}{2}} \left[\frac{8\pi\kappa}{3}\left(\lambda^2 - \frac{1}{4}\right)\right]^{-\frac{1}{2}} \begin{pmatrix} \alpha(t)\, \psi_\lambda(r,\vartheta,\varphi) \\ \beta(t)\, \psi_\lambda(r,\vartheta,\varphi) \end{pmatrix}, \tag{6}$$

where α and β are complex functions. For a homogenous system the components of the energy-momentum tensor simplify and the time component is

$$8\pi\kappa T^t_t = \left[m\Big(|\alpha|^2 - |\beta|^2\Big) - \frac{2\lambda}{R}\,\mathrm{Re}(\alpha\overline{\beta})\right]. \tag{7}$$

Substituting ψ and T^j_i into the Einstein-Dirac equation one gets

$$i\frac{d}{dt}\begin{pmatrix}\alpha \\ \beta\end{pmatrix} = \begin{pmatrix} m & -\lambda/R \\ -\lambda/R & -m \end{pmatrix}\begin{pmatrix}\alpha \\ \beta\end{pmatrix} \tag{8}$$

$$\dot{R}^2 + 1 = \frac{m}{R}\left(|\alpha|^2 - |\beta|^2\right) - \frac{\lambda}{R^2}\left(\overline{\beta}\alpha + \overline{\alpha}\beta\right). \tag{9}$$

With the ansatz all single particle wave functions have the same time dependence thus they form a coherent macroscopic quantum state. The fermionic many-particle state is a spin condensate.

The ED equations further reduce to ordinary differential equations involving the scale function $R(t)$ and the complex functions $\alpha(t)$ and $\beta(t)$. In the limits $\lambda = 0$ and m $= 0$ the equations reduce to the Friedmann equations for dust and radiation universes, respectively. For large R the universe behaves classically as in the dust case. But near the singularities big bang and big crunch quantum effects change the situation. Under certain conditions $\dot{R}$ can become zero and change sign even for small values of R. Now the formation of a big bang or big crunch is prevented. This effect is called the bouncing scale function.

3 Asymptotically Free Quantum Gravity

Super-renormalizable and asymptotically free theories of gravity have been discussed in the literature. Asymptotic freedom removes the singularity. Secondly, asymptotic freedom due to higher derivative form factor causes an effective negative pressure. Repulsive gravity at high density produces a bounce of a black hole. Black holes in fact never form. A distant observer sees a long lifetime for the trapped surface and interprets it as a black hole. The bounce is not given by Heisenberg uncertainty but follows from the dynamics of the system.

In [4] the following non-polynomial extension of the quadratic gravitational action of [5] has been considered

$$\mathcal{S} = \int d^4x \frac{2\sqrt{|g|}}{\kappa^2} \left[R - G_{\mu\nu} \frac{V(-\Box/\Lambda^2)^{-1} - 1}{\Box} R^{\mu\nu} \right], \tag{10}$$

where $\kappa^2 = 32\pi G_N$ and Λ is the Lorentz invariant energy scale. Its value is of the order of Planck mass. The form factor, an entire function V contains the non-polynomial property of the theory. V cannot have poles in the complex plane to ensure unitarity and it must have at least logarithmic behavior in the UV to give super-renormalizability at the quantum level. The theory reduces to general relativity in the low energy limit since all the corrections to the Einstein-Hilbert action are suppressed by the factor Λ^{-1}.

The form factor is related to the propagator and to the effective potential of the theory. An example of a form factor is

$$V(z)^{-1} = exp(z^n) \tag{11}$$

where $z = -\Box/\Lambda^2$ and n is a positive integer. String theory suggests n $= 1$. These theories have only the graviton pole. There are no ghosts or tachyons. The UV is dominated by the bare action, counterterms are negligible. Further details of these theories are discussed in [4].

According to [6] the behavior of running coupling constant in the coupled system of higher derivative gravity and gauge fields is renormalizable to all order loops. The leading contribution to the gauge coupling beta function comes entirely from quantum gravity effects and it vanishes to all order loops.

4 Asymptotic Safety

Asymptotic safety was proposed by Weinberg [7] in 1976 as a condition of renormalizability. It is based on a nontrivial, or non-Gaussian, fixed point (NGFP) of the underlying renormalization group (RG) flow for gravity. It is nonperturbative in character and it guaranties finite results for measureable quantities. The method for investigation of this scenario is functional renormalization group equation (FRGE) for gravity. The FRGE defines a Wilsonian RG flow on a theory space which consists of all diffeomorphism invariant functionals of the metric $g_{\mu\nu}$ of the type occuring in the action of general relatvity. From this construction emerges a theory called Quantum Einstein Gravity (QEG). QEG is not a quantization of classical general relativity, but it is consistent and predictive theory within the framework of quantum field theory.

The method of Reuter [8] uses the effective average action Γ_k, which is background independent. The RG scale dependence is governed by the FRGE of Wetterich [9]

$$k\partial_k\Gamma_k[\Phi,\bar{\Phi}] = \frac{1}{2}\mathrm{Str}\left[\left(\frac{\delta^2\Gamma_k}{\delta\Phi^A\delta\Phi^B}+\mathcal{R}_k\right)^{-1} k\partial_k\mathcal{R}_k\right]. \qquad (12)$$

where Φ^A is the collection of all dynamical fields and $\bar{\Phi}^A$ denotes their background counterparts. $\mathcal{R}_k$ is an infrared cutoff which vanishes for $p^2 \gg k^2$ and provides a k-dependent mass term for fluctuations with momenta $p^2 \ll k^2$. Solutions of the FRGE give families of effective field theories $\Gamma_k[g_{\mu\nu}], 0 \leq k < \infty$, labeled by the coarse graining scale k. The solution Γ_k interpolates between the microscopic action at $k \to \infty$ and the effective action $\Gamma_{k\to 0}$.

Suppose there is a set of basic functionals $\mathcal{P}_\alpha[\cdot]$. Any functional can be written as a linear combination of the $\mathcal{P}_\alpha$'s. The the solutions Γ_k of the FRGE have expansions of the form

$$A[\Phi,\bar{\Phi}] = \sum_{\alpha=1}^{\infty} \bar{u}_\alpha\, P_\alpha[\Phi,\bar{\Phi}]\,. \qquad (13)$$

The basis $\mathcal{P}_\alpha[\cdot]$ may include local field monomials and non-local invariants. We use the generalized couplings $\bar{u}_\alpha$ as local coordinates. Or better, we use a subset of couplings, so called essential couplings which cannot be absorbed by a field reparametrization. Though the method is non-perturbative truncations have to be made to the expansions of solutions.

Expandin Γ_k as above and inserting into FRGE we obtain a system of infinitely many coupled differential equations for the $\bar{u}_\alpha$'s

$$k\partial_k \, \bar{u}_\alpha(k) = \overline{\beta}_\alpha(\bar{u}_1, \bar{u}_2, \cdots; k) \;, \quad \alpha = 1, 2, \cdots \,. \tag{14}$$

which can be solved using analytical or numerical methods.

A simple ansatz for action is the Einstein-Hilbert action where Newton's constant $\mathcal{G}_k$ and the cosmological constant Λ_k depend on the RG scale k. Let $g_\mu \nu$ and $\bar{g}_{\mu\nu}$ denote the dynamical and background metric, respectively. The effective action then satisfies in arbitrary spacetime dimension d

$$\Gamma_k[g, \bar{g}, \xi, \bar{\xi}] = \frac{1}{16\pi G_k} \int d^dx \sqrt{g} \Big(-\mathcal{R}(g) + 2\Lambda_k \Big) + \Gamma_k^{gf}[g, \bar{g}] + \Gamma_k^{gh}[g, \bar{g}, \xi, \bar{\xi}] \tag{15}$$

where $R(g)$ is the scalar curvature from metric $g_{\mu\nu}$, Γ_k^{gf} denotes the gauge fixing action and Γ_k^{gh} the ghost action with the ghost fields ξ and $\bar{\xi}$.

The corresponding β-functions describing the evolution of the dimensionless Newton constant $g_k = k^{d-2} G_k$ and dimensionless cosmological constant $\lambda_k = k^{-2}\Lambda_k$, were derived the first time by Reuter in [8] for any value of the spacetime dimensionality. The most important result is the existence of a non-Gaussian fixed point suitable for asymptotic safety. It is UV-attractive both in g- and λ-directions (roughly $\lambda \approx .35$ and $g \approx .4$).

In the study of [10] it was shown that for $r \to 0$ the RG improved black hole metric approaches that of de Sitter space. This means that the quantum corrected spacetime is completely regular, free from any curvature singularity unlike the Schwarzschild black hole. The improved regularity comes because the 1/r-behavior of $f_{class} = 1 - 2G_0 M/r$ is tamed by very rapidly vanishing of the Newton constant at small distances.

A very heavy black hole obeys the classical relation $T_{BH} \sim 1/M$. The mass of the hole is reduced by the radiation the temperature increases. This tendency is opposed by quantum effects. Once the mass is as small as $M_{cr} \sim M_{Planck}$ the temperature reaches its maximum value $T_{BH}(M_{cr})$ [10]. For even smaller masses it drops very rapidly and vanishes at or below the M_{Planck}. The microscopic black hole could have a remnant which does not Hawking radiate any more.

5 Planck Stars

In [11] the quantum gravitational effects come from quantum cosmology. In loop cosmology the scale factor $a(t)$ of the universe is modified by quantum gravitational effects

$$\Big(\frac{\dot{a}}{a}\Big)^2 = \frac{8\pi G}{3} \rho \Big(1 - \frac{\rho}{\rho_P} \Big) \tag{16}$$

where G is the Newton constant, ρ the energy density of matter and

$$\rho_P \sim m_P/l_P^3 \sim c^5/(\hbar G^2) \tag{17}$$

where $\hbar$ is the reduced Planck constant. Quantum gravity corrections appear when $\rho \sim \rho_c$. This may happen before $l \sim l_P$. A collapsing universe may bounce back into expansion. This repulsion is due to Heisenberg uncertainty relations. In a matter dominated universe the volume of the universe at the bounce is

$$V \sim m/m_P l_P^3 \tag{18}$$

where m is the total mass of the universe. This volume of the universe is estimated to be about 75 orders of magnitude larger than the Planck volume.

Quantum analysis of a collapsing star leads to similar results. The gravitational collapse of a star does not lead to singularity but to a new phase of the star where the large gravitational attraction is balanced by large quantum pressure. The authors call stars in this phase Planck stars, and they estimate that a stellar mass black hole could have a radius of the order of 10^{-10} cm. This is very small of course compared to the original star but still more than 20 orders of magnitude larger than the Planck length.

The lifetime of a Planck star is very long for a distant observer since it is determined to the Hawking evaporation time of the black hole. But if measured on the surface of the star it is very short, the time light takes to cross the star.

A primordial black hole with mass about 10^{12} kg has a lifetime of the order of the age of the universe $t_H \sim 14 \times 10^9$ years. So they would be at the end of their lifetime now and be detectable at present. The size of this kind of object is

$$r = \sqrt[3]{t_H/(348\pi t_P)}\, l_P \sim 10^{-14} cm \tag{19}$$

The size of the black hole is the only scale in the process and it therefore fixes the energy scale of the emitted particles in the last stage. Assuming that all fundamental particles emitted with about the same energy taken at

$$E_{burst} = \frac{hc}{2r} \sim 3.9 GeV \tag{20}$$

From detectional viewpoint it is natural to measure emitted gamma-rays. Only directly emitted gamma-rays (estimated to be about 3 per cent) are at the energy E_{burst}. Most gamma-rays come from decays of hadrons, mainly from neutral pions. The authors [11] have made a Pythia analysis of secondary gamma-rays emitted by a Planck star at the end of its life. The mean energy is of the order $0.03 \times E_{burst}$, which is in the tens of MeV range. The multiplicity is quite high at about 10 photons per $q\bar{q}$. A major hindrance comes from the maximum distance at which bursts can be detected. For measuring say 10 photons using a 1 m^2 detector surface the estimated distance of burst origin is only about 200 light years.

6 Cosmology

In the quantum schemes of gravitation the relativity theory initial singularity is most likely smoothed into a very high but finite density and temperature objects. After inflationary phase the standard model particles are formed together with occasional black holes. This does not lead to major deviations from the standard cosmological model. The abundance of primordial black holes may contribute to the distribution of dark matter of the universe. Quantitative differences to the standard model should be looked for from gamma-ray spectra. The formation of stellar size black holes proceeds as in general relativity theory.

7 Experimental Tests

As mentioned, high accuracy measurements of gamma-ray signals from the sky [12] is at present the most promising key to observe new physics. With the Planck scale at about 10^{19} GeV, all particles coupling to gravity, any particle with energy half the Planck energy is a clear signal. A remnant is expected to have two and many particle decay channels, of which the few particle channels have rather clear signals. Information and global charge conservation are imteresting questions.

There are models which may bring the relevant energy scale down within reach of the LHC. These include models of ref. [13]. Provided the Planck scale is brought down to TeV scale black holes with mass in the TeV region may be produced by gravitational interaction. The basic idea discussed here does not depend on the value of the Planck scale.

Cross sections and decay channels have been extensively calculated in [14]. Detailed analysis indicates best few body decay channels, in particular $e\mu$ pair, for black hole production.

8 Conclusions

The present note contains some tentative thoughts how to go beyond the standard model towards a model of Planck scale phenomena, assuming the standard model is valid up to that scale. At the Planck scale black holes are the key object to study. Unfortunately model calculation results concerning Planck mass region black holes are still uncertain.

Examples were given above which indicate that bouncing black holes and universes may be relevant in quantum gravity. The classical singularity is probably removed, or at least softened, by quantum gravitational effects. Asymptotic freedom of gravity is not supported unambiguously. In [6] non-abelian gauge theory coupling constants go to zero at Planck scale. The length scale where quantum gravity effects may occur is surprisingly large. The details of the theoretical models discussed in Sections 2-5 vary somewhat but the softening trends seems to be on rather sound basis. Our assumptions (i) and (ii) of section 1 could not be proven but are endorsed by some the above models. The possible remnant of black hole decay is interesting and it could be studied as a heavy decaying particle.

A realistic model of quantum gravity should start from the microscopic entities operating at the quantum scale, like the Planck scale. Then the methods of the new model theory, be it quantum field theory or something else, can be introduced to calculate the properties of the model like the UV behavior of the interaction. In principle the building blocks are known but (i) the nature of black holes changes from thermal to non-thermal when the black hole mass comes down to Planck scale, (ii) scalar particles may play a role not visible today, and (iii) the theory most likely has connection to the details of big bang.

References

[1] R. Raitio, Physica Scripta **22**, 197-198 (1980).
M. Lehto and R. Raitio, Physica Scripta **25**, 239-240 (1982).

[2] R. Raitio, Deep Inelastic Gedanken Scattering off Black Holes, viXra:1401.0006.

[3] F. Finster and C. Hainzl, Found. Phys. 40, 116-124 (2010), arXiv:0809.1693 [gr-qc].
F. Finster and C. Hainzl, J. Math. Phys. 52 (2011) 042501, arXiv:1101.1872 [math-ph].

[4] C. Bambi, D. Malafarina and L. Modesto, Eur.Phys.J. C74, 2767 (2014), arXiv:1306.1668v2 [gr-qc].

[5] K. Stelle, Phys. Rev. D16, 953 (1977).

[6] G. Narain and R. Anishetty, JHEP 1310 (2013) 203, arXiv:1309.0473 [hep-th].

[7] Weinberg, Steven (1976). "Critical Phenomena for Field Theorists". Erice School for Subnuclear Physics.

[8] M. Reuter, Phys. Rev. D57 (1998) 971-985, arXiv:9605030 [hep-th]

[9] C. Wetterich, Phys. Lett. B301 (1993) 90.

[10] A. Bonnano and M. Reuter, Phys.Rev. D62 (2000) 043008, arXiv:0002196 [hep-th].

[11] C. Rovelli and F. Vidotto, Int.J.Mod.Phys. D23 (2014) 12, 1442026, arXiv:1401.6562v4 [gr-qc].
A. Barrau and C. Rovelli, Phys.Lett. B739 (2014) 405, arXiv:1404.5821v2 [gr-qc].

[12] A. Barrau and J. Grain, Quantum gravity in the sky, arXiv:1206.1192 [gr-qc].

[13] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B 429, 263 (1998) arXiv:9803315 [hep-ph].
L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999) arXiv:9905221[hep-ph].
X. Calmet, S. Hsu and D. Reeb, Phys. Rev. D77, 125015 (2008) arXiv:0803.1836 [hep-th].

[14] X. Calmet, W. Gong, S. Hsu, Phys.Lett. B668 (2008) 20-23, [arXiv:0806.4605 [hep-ph].
A. Belyaev and X. Calmet, Quantum Black Holes and their Lepton Signatures at the LHC with CalCHEP, arXiv:1412.2661 [hep-ph].