Abstract

In 2015, Liu et al. proposed a study relationship between RSA public key cryptosystem and Goldbach’s conjecture properties. They discussed the relationship between RSA and Goldbach conjecture, twin prime and Goldbach conjecture. In this paper the author will extend to introduce the relationship among Goldbach conjecture, twin prime and Fibonacci number. Based on their contribution, the author completely lists all combinations of twin prime in Goldbach conjecture.

Keywords
Goldbach conjecture; Twin prime; Fibonacci number;

1 INTRODUCTION

Whether the Goldbach conjecture or the twin prime issue, those are unsolved problems in Number Theory. It is well known, Chan [1] has major breakthrough on the Goldbach’s conjecture by his “1 + 2” formal proof in 1973. Zhang [2] has a very good significant work on the twin prime recently. There are someone also gave good research contributions in [3]-[10]. Liu, Chang, Wu and Ye [11] proposed a study of relationship between RSA public key cryptosystem and Goldbach’s conjecture properties. They connected the RSA and Goldbach conjecture relationship, and also linked the Goldbach conjecture and twin prime. In their article, Liu [11] et al. list two situations which there probable exists twin prime in Goldbach partition combinations such as proposition 1 and 2. In this paper the author will point out 8 of all situations that occur twin prime conditions in Goldbach partition.

2 THE RELATIONSHIP BETWEEN OF GOLDBACH’S CONJECTURE AND THE TWIN PRIME

In this section, the author describes a relationship of Goldbach’s conjecture and twin prime. Our article is extending work on the basis of Liu [11] et al.’s research contribution. In Liu et al.’s article, they proposed 4 theorems, 6 propositions and 1 lemma. However, in their work, there is still insufficient. The author continues his work and increases 6 situations twin prime in Goldbach partition. This part is discussed in section 2.3.

2.1 Related work
To Goldbach partition number, Brickman [12] estimated the value too large on the number of error range. Ye and Liu’s [13] estimation is too vague, it is not clear and accurate. Based on this discussion, the author gives an exact estimating which the estimation range more close to the true value. Constant [14] and Liu [11] et al. connected the relationship between the RSA cryptosystem and the Goldbach conjecture. Ye and Liu [13], and some literatures [3], [8], [15] introduced the Goldbach conjecture and twin prime relationship. In this section the author will describe the relationship between Goldbach conjecture and the Fibonacci number in section 3. A relationship among Goldbach conjecture, twin prime, RSA and the Fibonacci number as shown in Figure 1. Notations are described in the following.

Notations:

\( GC(x) \): denote the number of Goldbach partition.

\( GC \): denote an even number for the Goldbach Conjecture (GC) number.

\( GC \equiv 2 \mid 4 \): \( GC \) is congruent to two modulo four, we usually write \( GC \equiv 2 \pmod{4} \). But for convenience, we use \( GC \equiv 2 \mid 4 \) instead here.

A variety of situations that may arise the twin primes in Goldbach conjecture, the all possible combination shown in Table 1.
The twin prime probable appears in the Goldbach conjecture. The expression of a given even number as a sum of two primes is called a 'Goldbach partition' of that number.

Goldbach conjecture, where the Goldbach function

\[ GC_i \equiv 0 \mod 4 \]

\[ GC_{\frac{x}{2}} \equiv 0 \mod 4 \]

\[ GC_{\frac{x}{2}} \equiv 0 \mod 4 \]

\[ GC_{\frac{x}{2}} \equiv 0 \mod 4 \]

\[ GC_{\frac{x}{2}} \equiv 0 \mod 4 \]

\[ GC_{\frac{x}{2}} \equiv 0 \mod 4 \]

\[ GC_{\frac{x}{2}} \equiv 0 \mod 4 \]

2.2 The Goldbach partition

The expression of a given even number as a sum of two primes is called a ‘Goldbach partition’ of that number. For example: The integer 138 can be expressed in 8 ways. We say the GC number can be described in the form as

\[ GC = P_i + P_j \rightarrow (P_i - 2n) + (P_j + 2n) \]

where \( P_i \) and \( P_j \) are both primes. Let \( R(n) \) be the number of representations of the Goldbach partition where \( \Pi_2 \) is the twin prime constant [16], say \( R(n) \sim 2 \Pi_2 \left( \prod_{p \leq N} \right) \frac{P_{n+1} - P_{n-1}}{P_{n+1} - P_{n-1}} \frac{dx}{(\ln x)^2} \). Ye and Liu [13] also gave the estimation formula \( GC(x) = 2C \prod_{p \geq 3} \left( \frac{p-1}{p+1} \right)^2 \left( \frac{p-1}{p+1} \right)^2 \frac{dx}{(\ln x)^2} \). In 2008, Bruckman [12] proposed a proof of the strong Goldbach conjecture, where the Goldbach function

\[ \theta(2N) \equiv \sum_{k=1}^{2n-3} \delta(k)(2N - k) \]

is at least equal to one. By comparison of coefficients, they result

\[ 1 \leq \theta(2k + 6) \leq k + 1, \ k = 0, 1, 2, \ldots \]
When the $k$ approaches infinity, the error range then follows larger width. For example:

\[
\begin{align*}
\theta(32) & \leq 14, \ k = 13. \\
\theta(80) & \leq 38, \ k = 37. \\
\theta(138) & \leq 67, \ k = 66. \\
\theta(101200) & \leq 50598, \ k = 50597.
\end{align*}
\]

The author obtained results from large number of experimental data. He draws the curve from data, and calculates the formula according from two curves. He found interesting situation which $GC$ is congruent to zero modulo six, or congruent to non-zero modulo six. Randomly chooses an even number $GC$, where $GC < 6$, if $GC \equiv 0 \pmod{6}$, he then finds $GC'(x) \equiv 1.75 \cdot GC \pmod{6}$. Otherwise, he finds other $GC'(x) \equiv 1.8 \cdot GC \pmod{6}$. The expression shown in Equation (4).

\[
GC \mapsto \begin{cases} 
\equiv 0 \pmod{6}, & GC'(x) = \frac{1.75 \cdot GC}{9.7259 \cdot GC + 33.9} \\
\not\equiv 0 \pmod{6}, & GC'(x) = \frac{1.8 \cdot GC}{9.7259 \cdot GC + 33.9}.
\end{cases} 
\]

(4)

The author compares his estimation with Bruckman’s method based on the true value of Goldbach partition. The results indicated that our method is better than his method according from Table 1.

### 2.3 The twin prime

To facilitate description, the author prefers to use corollary alternative proposition. Our Corollary 1 and 2 are original from Liu [11] et al.’s Proposition 1 and 2, the author expands 6 corollaries based on their work.

**Corollary 1.** If $P_i + P_j \equiv 0 \pmod{4} \equiv 0 \pmod{6} \equiv 4 \pmod{8}$, and $\frac{P_i + P_j}{2} \equiv 2 \pmod{4} \equiv 0 \pmod{6} \equiv 2 \pmod{8}$ or $\frac{P_i + P_j}{2} \equiv 2 \pmod{4} \equiv 0 \pmod{6} \equiv 6 \pmod{8}$, there may exist a twin prime where the $(\frac{P_i + P_j}{2} - 1, \frac{P_i + P_j}{2} + 1)$ is $(4n + 1) + (4n + 3)$ form.

**Proof:** As known from assumption, $\frac{P_i + P_j}{2}$ is an even number, we have

\[
\begin{cases} 
\frac{P_i + P_j}{2} - 1 \text{ is an odd number.} \\
\frac{P_i + P_j}{2} + 1 \text{ is an odd number too.}
\end{cases}
\]
Table 2
The comparison of Goldbach partition $GC(x)$, $GC'(x)$ and $\theta(2k + 6) \leq k + 1$

<table>
<thead>
<tr>
<th>Item</th>
<th>Positive Integer</th>
<th>$GC(x)$</th>
<th>Our method $GC'(x)$</th>
<th>$k$</th>
<th>$k + 1$</th>
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</table>

Note that $\frac{P_i + P_j}{2} \equiv 2 \pmod{4} \equiv 0 \pmod{6} \equiv 6 \pmod{8}$, we see the $\frac{P_i + P_j}{2}$ is $4n + 2$ form. Naturally, the $\frac{P_i + P_j}{2} - 1$ is $4n + 1$ form, and $\frac{P_i + P_j}{2} + 1$ is $4n + 3$ form. Otherwise, it is a contradiction. Since $\frac{P_i + P_j}{2} \equiv 2 \pmod{4} \equiv 0 \pmod{6} \equiv 2 \pmod{8}$, we know $(\frac{P_i + P_j}{2} - 1, \frac{P_i + P_j}{2} + 1)$ is $(4n + 1) + (4n + 3)$ form.

**Corollary 2.** If $P_i + P_j \equiv 0 \pmod{4} \equiv 0 \pmod{6} \equiv 0 \pmod{8}$, and $\frac{P_i + P_j}{2} \equiv 0 \pmod{4} \equiv 0 \pmod{6} \equiv 0 \pmod{8}$ or $\frac{P_i + P_j}{2} \equiv 0 \pmod{4} \equiv 0 \pmod{6} \equiv 4 \mmod{8}$, there may exist a twin prime where $(\frac{P_i + P_j}{2} - 1, \frac{P_i + P_j}{2} + 1)$ is $(4n + 3) + (4n + 1)$ form.

**Proof:** As known, the $\frac{P_i + P_j}{2}$ is an even number. Since $\frac{P_i + P_j}{2} \equiv 0 \pmod{4}$, we see the $\frac{P_i + P_j}{2}$ is $4n$ form. Hence $\frac{P_i + P_j}{2} - 1$ is $4n + 3$ form. Therefore $\frac{P_i + P_j}{2} + 1$ is $4n + 1$ form.

Now, as $\frac{P_i + P_j}{2} \equiv 0 \pmod{4} \equiv 0 \pmod{6} \equiv 0 \pmod{8}$, the $\frac{P_i + P_j}{2}$ is $4n$ form too. Thus, the $\frac{P_i + P_j}{2} + 1$ is $4n + 1$ form. This inference is consistent with the above statement.

**Corollary 3.** If $P_i + P_j \equiv 0 \pmod{4} \equiv 4 \pmod{6} \equiv 4 \pmod{8}$, and $\frac{P_i + P_j}{2} \equiv 2 \pmod{4} \equiv 2 \pmod{6} \equiv 2 \pmod{8}$ or $\frac{P_i + P_j}{2} \equiv 2 \pmod{4} \equiv 2 \pmod{6} \equiv 6 \pmod{8}$, there may exist a twin prime where $(\frac{P_i + P_j}{2} - 1, \frac{P_i + P_j}{2} + 1)$ is $(4n + 3) + (4n + 1)$ form.

**Proof:** As known, the $\frac{P_i + P_j}{2}$ is an even number. Since $\frac{P_i + P_j}{2} \equiv 0 \pmod{4} \equiv 0 \pmod{6} \equiv 0 \pmod{8}$, we see the $\frac{P_i + P_j}{2}$ is $4n$ form. Hence $\frac{P_i + P_j}{2} - 1$ is $4n + 3$ form. Therefore $\frac{P_i + P_j}{2} + 1$ is $4n + 1$ form.

Now, as $\frac{P_i + P_j}{2} \equiv 0 \pmod{4} \equiv 0 \pmod{6} \equiv 0 \pmod{8}$, the $\frac{P_i + P_j}{2}$ is $4n$ form too. Thus, the $\frac{P_i + P_j}{2} + 1$ is $4n + 1$ form. This inference is consistent with the above statement.

**Corollary 4.** If $P_i + P_j \equiv 0 \pmod{4} \equiv 4 \pmod{6} \equiv 0 \pmod{8}$, and $\frac{P_i + P_j}{2} \equiv 0 \pmod{4} \equiv 2 \pmod{6} \equiv 0 \pmod{8}$ or $\frac{P_i + P_j}{2} \equiv 0 \pmod{4} \equiv 2 \pmod{6} \equiv 4 \pmod{8}$, there may exist a twin prime where $(\frac{P_i + P_j}{2} - 1, \frac{P_i + P_j}{2} + 1)$ is $(4n + 3) + (4n + 1)$ form.

**Proof:** As known, the $\frac{P_i + P_j}{2}$ is an even number. Since $\frac{P_i + P_j}{2} \equiv 0 \pmod{4} \equiv 0 \pmod{6} \equiv 0 \pmod{8}$, we see the $\frac{P_i + P_j}{2}$ is $4n$ form. Hence $\frac{P_i + P_j}{2} - 1$ is $4n + 3$ form. Therefore $\frac{P_i + P_j}{2} + 1$ is $4n + 1$ form.
Now, as $\frac{P_i + P_{i+1}}{2} \equiv 0 \pmod{4}$ for any $i$, the $P_i$ is 4n form too. 
Thus, the $\frac{P_i + P_{i+1}}{2} + 1$ is 4n + 1 form. This inference is consistent with the above statement.

**Corollary 5.** If $P_i + P_j \equiv 2 \pmod{4}$ or $P_i + P_j \equiv 4 \pmod{4}$, and $\frac{P_i + P_{j+1}}{2} \equiv 1 \pmod{4}$ then $\frac{P_i + P_{j+2}}{2} \equiv 3 \pmod{4}$ or $\frac{P_i + P_{j+2}}{2} \equiv 1 \pmod{4}$, and $\frac{P_i + P_{j+3}}{2} \equiv 3 \pmod{4}$ or $\frac{P_i + P_{j+3}}{2} \equiv 1 \pmod{4}$, then there may exist a twin prime where $\frac{P_i + P_{j+2}}{2} - 1, \frac{P_i + P_{j+3}}{2} + 1$ is $(4n + 3) + (4n + 1)$ form.

**Proof:** As known, the $\frac{P_i + P_{j+1}}{2} \equiv 1 \pmod{4}$, the $\frac{P_i + P_{j+3}}{2} \equiv 3 \pmod{4}$ on either side of the center point $4n + 2$. Thus, the $(\frac{P_i + P_{j+2}}{2} + 2)$ is $4n + 3$ form. If it is a contradiction.

**Corollary 6.** If $P_i + P_j \equiv 2 \pmod{4}$ or $P_i + P_j \equiv 4 \pmod{4}$, and $\frac{P_i + P_{j+1}}{2} \equiv 3 \pmod{4}$ or $\frac{P_i + P_{j+1}}{2} \equiv 1 \pmod{4}$, and $\frac{P_i + P_{j+3}}{2} \equiv 3 \pmod{4}$ or $\frac{P_i + P_{j+3}}{2} \equiv 1 \pmod{4}$, then there may exist a twin prime where $\frac{P_i + P_{j+2}}{2} - 1, \frac{P_i + P_{j+3}}{2} + 1$ is $(4n + 3) + (4n + 1)$ form.

**Proof:** This proof is same with Corollary 5, we omit the proof here.

**Corollary 7.** If $P_i + P_j \equiv 2 \pmod{4}$ or $P_i + P_j \equiv 4 \pmod{4}$, and $\frac{P_i + P_{j+1}}{2} \equiv 3 \pmod{4}$ or $\frac{P_i + P_{j+1}}{2} \equiv 1 \pmod{4}$, and $\frac{P_i + P_{j+3}}{2} \equiv 3 \pmod{4}$ or $\frac{P_i + P_{j+3}}{2} \equiv 1 \pmod{4}$, then there may exist a twin prime where $\frac{P_i + P_{j+2}}{2} - 1, \frac{P_i + P_{j+3}}{2} + 1$ is $(4n + 3) + (4n + 1)$ form.

**Proof:** This proof is same with Corollary 5, we also omit the proof here.

**Corollary 8.** If $P_i + P_j \equiv 2 \pmod{4}$ or $P_i + P_j \equiv 4 \pmod{4}$, and $\frac{P_i + P_{j+1}}{2} \equiv 3 \pmod{4}$ or $\frac{P_i + P_{j+1}}{2} \equiv 1 \pmod{4}$, and $\frac{P_i + P_{j+3}}{2} \equiv 3 \pmod{4}$ or $\frac{P_i + P_{j+3}}{2} \equiv 1 \pmod{4}$, then there may exist a twin prime where $\frac{P_i + P_{j+2}}{2} - 1, \frac{P_i + P_{j+3}}{2} + 1$ is $(4n + 3) + (4n + 1)$ form.

**Proof:** This proof is same with Corollary 5, we omit the proof here too.

**Exception:**
There are 4 exceptions of even number between $[2,1000]$ to the rule in Table 1.

$402 \mapsto \begin{cases} GC = 402 \equiv 2 \pmod{4} & \equiv 0 \pmod{6} \equiv 2 \pmod{8}, \\
GC = 201 \equiv 1 \pmod{4} & \equiv 3 \pmod{6} \equiv 1 \pmod{8}. \end{cases}$

According from Table 1, the 402 matches item 5, however, there is no one twin prime in 17 prime pairs of Goldbach partition.

$516 \mapsto \begin{cases} 516 \equiv 0 \pmod{4} & \equiv 0 \pmod{6} \equiv 4 \pmod{8}, \\
258 \equiv 2 \pmod{4} & \equiv 0 \pmod{6} \equiv 2 \pmod{8}. \end{cases}$

There are 23 prime pairs in Goldbach partition, but no one matches in the rule of item 1.

$786 \mapsto \begin{cases} 786 \equiv 2 \pmod{4} & \equiv 0 \pmod{6} \equiv 2 \pmod{8}, \\
393 \equiv 1 \pmod{4} & \equiv 3 \pmod{6} \equiv 1 \pmod{8}. \end{cases}$

There are 30 prime pairs in Goldbach partition, but no one matches in the rule of item 5.

$906 \mapsto \begin{cases} 906 \equiv 2 \pmod{4} & \equiv 0 \pmod{6} \equiv 2 \pmod{8}, \\
453 \equiv 1 \pmod{4} & \equiv 3 \pmod{6} \equiv 5 \pmod{8}. \end{cases}$

There are 34 prime pairs in Goldbach partition, but no one matches in the rule of item 5.

### 3 The relationship of the Goldbach’s conjecture and the Fibonacci Number

This section will introduce about Fibonacci number [17], [18] and it’s relationship with Goldbach’s conjecture. To each positive number $n$ is the sum of the previous two integers, namely

$$F_{n} = F_{n-1} + F_{n-2}. \quad (9)$$

By Equation (9), we know the Fibonacci sequence $\{0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \ldots, \infty\}$. Wall [19] had good result in his article “Fibonacci Series Modulo m”, he created a table in the appendix listing values for the function $k(n)$. This function is defined as the period of the Fibonacci numbers $\pmod{n}$ before any repeats occur. For instance, $k(7) = 16$ since

$$F_{n} \text{ mod } 7 = \{0, 1, 1, 2, 3, 5, 1, 6, 0, 6, 6, 5, 4, 2, 6, 1\}, \quad (10)$$
Table 1.  

<table>
<thead>
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<th>n</th>
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<th>2</th>
<th>3</th>
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<td>2</td>
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<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
</tr>
</tbody>
</table>

\( F_{10} \equiv X \pmod{7} \)

\( F_{11} \equiv X \pmod{7} \)

\( F_{12} \equiv X \pmod{7} \)

\( F_{13} \equiv X \pmod{7} \)

\( F_{14} \equiv X \pmod{7} \)

\[ F_6 = F_5 + F_4 \Rightarrow 3 + 5 = 8. \] (11)

The Equation (11) is only one special case while Goldbach’s conjecture in Fibonacci sequence nowadays. Since \( F_{n \equiv 0} \pmod{3} \) has never been an prime that itself even, we can say the \( F_{n \equiv 1} \pmod{3} \) or \( F_{n \equiv 2} \pmod{3} \) probable be a prime. There is one literature about Fibonacci prime in [18], but marginally different then what is discussed in this article.

Open problems:
1. Can we find the second example which Goldbach’s conjecture in Fibonacci sequence? It is so interesting.
2. To Fibonacci prime, we find interesting phenomenon in our research. If \( n \equiv 3 \pmod{4} \) and \( F_n \equiv 1 \pmod{4} \) where \( n > 5 \), the \( F_n \) probable be a prime, say

\[ \begin{align*}
\begin{cases}
F_{n \equiv 3} \pmod{4} \\
F_{n \equiv 1} \pmod{4}
\end{cases}
\end{align*} \]

(12)

3. If \( n \equiv 1 \pmod{4} \) and \( F_n \equiv 1 \pmod{4} \) where \( n > 5 \), the \( F_n \) probable be also a prime, namely

\[ \begin{align*}
\begin{cases}
F_{n \equiv 1} \pmod{4} \\
F_{n \equiv 1} \pmod{4}
\end{cases}
\end{align*} \]

(13)

We get following relationship as:
Goldbach’s conjecture \( \supseteq (\text{odd} + \text{odd} = \text{even}) \subseteq \text{Fibonacci sequence.} \)

4 Conclusions

The author cleverly assumes the Goldbach conjecture as the center, he then discusses the relationship among Goldbach conjecture, twin prime, RSA cryptosystem and Fibonacci number. 1) He analyzes the characteristics of twin prime in Goldbach conjecture and then point out all of situations of combination. 2) He also proposes an
estimation method to Goldbach partition which the result is better than Bruckman’s estimating. 3) Finally, the author explores the relationship between Goldbach conjecture and Fibonacci number, he mentions a new one discussion about searching the Fibonacci prime in its sequence. From above, the author is still studying on these unsolved problems in the future.

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