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Review of the Microscopic Approach to the Higgs Mechanism and to Quark and Lepton Masses and Mixings

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Abstract

This review summarizes the results of a series of recent papers[1, 2, 3, 4, 5], where a microscopic model underlying the physics of elementary particles has been proposed. The model relies on the existence of an internal isospin space, in which an independent physical dynamics takes place. This idea is critically re-considered in the present work. As becomes evident in the course of discussion, the model not only describes electroweak phenomena but also modifies our understanding of other physical topics, like the big bang cosmology and the nature of the strong interactions.

1 Particle Physics and Cosmology in the Tetron Model

The Standard Model of elementary particles is very successful on the phenomenological level. The outcome of (almost) any particle physics experiment can be predicted accurately within this model, and where not, by some straightforward extension. For example, one may introduce right handed neutrinos to account for tiny neutrino masses[6].

Nevertheless, it is widely believed that the SM is only an effective low-energy theory valid below a certain energy scale, which is supposed to be larger than 1 TeV. This view is based on the fact that the SM has many unknown parameters with hitherto unexplained hierarchies. Furthermore, there is one rather mysterious component, the so-called Higgs field, needed for the spontaneous symmetry breaking (SSB) to take place in the model.

In recent papers a microscopic model has been developed[1, 2, 3, 4, 5], whose central assumption is the existence of a 3-dimensional *internal tetrahedral structure* attributed to each point of Minkowski space, in which an independent physical dynamics takes place.

Under this assumption spacetime originally is 6+1 dimensional, and at the time when the tetrahedrons are formed, it fibers into internal space and Minkowski space as $R^3 \times R^{3,1}$.

The sites $i = 1, 2, 3, 4$ of the tetrahedron are populated by 6+1 dimensional spinor fields ψ, called tetrons. The tetron on site i will be denoted ψ_i.

The fundamental spinor representation in R^{6+1} is of dimension 8. It decomposes as[38]

$$8 \rightarrow (1, 2, 2) + (2, 1, 2) = ((1, 2) + (2, 1), 2) \tag{1}$$

under the fibration $SO(6, 1) \rightarrow SO(3, 1) \times SO(3)$.[1]

[1]Representations of $SO(3, 1) \times SO(3)$ are denoted by a set of 3 numbers (a, b, c), where (a, b) are representations of the Lorentz group and c is the dimension of a $SO(3)$-representation. For example, c=2 corresponds to a non-relativistic Pauli spinor in internal space, whose 2 spin orientations are

Eq. (1) means that each tetron is an isospin doublet $\psi = (U, D)$ of 3+1 dimensional Dirac fermions U and D. One may write it as a 2-index object ψ^a_α, where $\alpha = 1, 2, 3, 4$ is the Dirac index and $a = 1, 2$ the internal index. The internal spin will be called isospin.

Using the triplet $\vec{\tau}$ of internal Pauli matrices an isospin (pseudo)vector

$$\vec{Q} = \psi^\dagger \vec{\tau} \psi \tag{2}$$

may be defined for any tetron ψ. It fixes a direction in the internal space and, up to an overall constant, can be interpreted as the internal angular momentum vector of the tetron ψ.

Since the tetrons are Dirac fermions on Minkowski space, $\vec{Q}$ can be written in terms of creation and annihilation operators of a tetron and an antitetron as

$$\vec{Q} = \psi^\dagger \vec{\tau} \psi = a^\dagger \vec{\tau} a - b^\dagger \vec{\tau} b \tag{3}$$

For the calculation of the quark and lepton masses the chiral iso-vectors

$$\vec{S} := \vec{Q}_L = \frac{1}{2} \psi^\dagger (1 - \gamma_5) \vec{\tau} \psi \qquad\qquad \vec{T} := \vec{Q}_R = \frac{1}{2} \psi^\dagger (1 + \gamma_5) \vec{\tau} \psi \tag{4}$$

turn out to be of particular importance. For simplicity of notation they are called $\vec{S}$ and $\vec{T}$ in the following. Obviously, they fulfill $\vec{Q} = \vec{S} + \vec{T}$.

In fig. 1 the *local* ground state of the model is drawn, a configuration with 4 tetrons on the 4 sites of a tetrahedron, their isospin vectors $\vec{Q}$ pointing in radial directions away from the origin. These internal vectors fulfill the commutation relations of a system of decoupled internal angular momenta. In other words, they play the role of angular momentum observables corresponding to rotations of the internal R^3 space.

While the coordinate symmetry is S_4, the arrangement of isospin vectors in fig. 1 respects the Shubnikov symmetry[9, 11, 12]

$$G_4 := A_4 + S(S_4 - A_4) \tag{5}$$

where $A_4(S_4)$ is the (full) tetrahedral symmetry group and S the internal time reversal operation that changes the direction of internal spin vectors. This is equivalent

identified with the SU(2) flavors U and D. It should be noted that (1,2,2) and (2,1,2) are complex conjugate with respect to each other, so one is the antiparticle representation of the other.

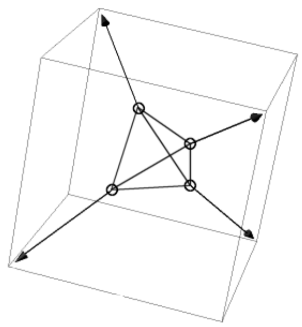

Figure 1: The local ground state of the model, living in a 3-dimensional isospin space. Shown are the tetron locations (open circles) and the 4 isospin vectors $\vec{Q}$. Note there are actually 2 tetrahedrons in this figure, one with respect to the internal coordinates (tetron locations) and the other one with respect to isospin vectors. The alignment of isospin vectors with respect to neighboring tetrahedrons (fig. 2) forms the basis for the electroweak phase transition, while the coordinate alignment of neighboring tetrahedrons is relevant for crystal formation at much higher energies, cf. the discussion after (16).

to saying that S interchanges the role of the internal spinors in the following way

$$S : (U, D) \to (-D^*, U^*) \tag{6}$$

As shown later in (2.4.14), one may actually use charge conjugation instead of the concept of an internal time operation to define the Shubnikov group G_4.

Note the arrangement fig. 1 does not respect S or internal parity R, but only the product SR. Furthermore it is chiral, the configuration with opposite internal chirality being given when the isospin vectors would point inwards instead of outwards. As will be shown in (2.1.18), this internal chirality is dynamically related to the $V-A$ nature of the q/l interactions.

As for the *global* ground state the set of all tetrahedrons forms a flat 3-dimensional crystal structure within the original R^6, similar to what is shown in fig. 2. This structure will be called the hyper-crystal or, within the Lorentz covariant cosmological framework to be developed later, the discrete micro-elastic spacetime. It is our

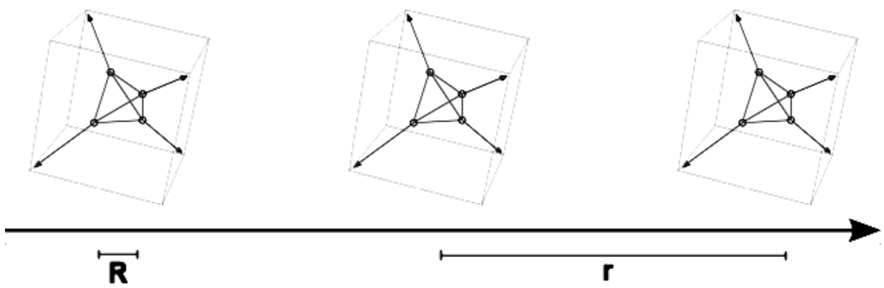

Figure 2: The global ground state of the model after SSB consists of an aligned system of chiral tetrahedrons over physical space (the latter represented by the long arrow). R is the internal magnitude of one tetrahedron and r the distance between two of them. Before the SSB the isospin vectors in each tetrahedron are oriented randomly (not shown) and there is a corresponding *local* $SU(2)$ symmetry. The figure is a bit misleading, not only because the tetrahedrons do not have an extension into physical space, but also the relative magnitudes are not correctly drawn. While r and R are tiny (of the order of the Planckh length), the tetrahedrons formed by the isospin vectors are much larger, of the order of the Fermi scale (in inverse energy units).

world, the space in which all physical processes take place.

Contrary to what is drawn, the tetrahedrons extend into internal space alone, not into physical space. In other words, physical space is *defined* to be the 3-dimensional subspace of R^6 orthogonal to the 3 dimensions spanned by the aligned tetrahedrons.

Note the parallel alignment of isospins of neighboring tetrahedrons in fig. 2. Before the formation of this structure the internal spins U and D, which are the building blocks of the isospin vectors, can freely rotate and thus there is an internal spin $SU(2)_L$ symmetry group. In ordinary magnetism this group is usually called Heisenberg's SU(2).

It is given the index L here, because of the dynamical relation between internal and external chirality to be explained in (2.1.18). Furthermore, $SU(2)_L$ is local in the sense that the isospin vectors can be rotated independently over each point of Minkowski space. It gets broken to G_4 when the chiral arrangement fig. 1 is formed.

Note, the electromagnetic $U(1)$ symmetry has not been introduced at this point. Together with the tetron model interpretation of the electroweak mixing angle, this deficit will be rectified in (2.1.11) and (2.1.8).

One wants to interpret the 3 generations of quarks and leptons as isospin wave excitations on the internal isospin structure. These excitations will be called mignons. They behave as quasi-particles while they travel through Minkowski space and can be grouped according to representations of G_4.

G_4 is a finite symmetry group which remains intact to the lowest energies. As shown in [9] it has only 1- and 3-dimensional representations. To generate all possible excitations describing the quarks and leptons one has to consider the vibrations of $\vec{Q}_L$ and $\vec{Q}_R$ for each of the 4 tetrons separately, cf. (2.3.1).

The isospin vibrational excitations are described by deviations δ from the ground state fig. 1, i.e.

$$\vec{S}_i = \langle \vec{S}_i \rangle + \delta \vec{S}_i \qquad \vec{T}_i = \langle \vec{T}_i \rangle + \delta \vec{T}_i \tag{7}$$

or, more precisely, by certain linear combinations of them – the eigenmodes of the isospin Hamiltonian to be discussed later in (12) and (13).

In order to construct a left-handed mignon, all tetrons involved have to be left-handed as well, and the ground state fig. 1 is formed by the $\langle \vec{S}_i \rangle$, while $\langle \vec{T}_i \rangle = 0$. For right-handed mignon it is the other way round. For the calculation of the q/l masses, both left- and right-handed isospin are non-vanishing in the vacuum, i.e. $\langle \vec{S}_i \rangle = \langle \vec{T}_i \rangle \neq 0$. This is because mass terms $\bar{q}_R q_L + c.c.$ involve right-handed as well as left-handed contributions.

The resulting 24 mignon states can be arranged in six singlet and six triplet representations $A_{\uparrow,\downarrow}$ and $T_{\uparrow,\downarrow}$ of G_4 to yield precisely the multiplet structure of the 24 fermion states of the 3 generations, not less and not more:

$$\begin{aligned} A_\uparrow(\nu_e) + A_\uparrow(\nu_\mu) + A_\uparrow(\nu_\tau) \quad &+ \quad T_\uparrow(u) + T_\uparrow(c) + T_\uparrow(t) + \\ A_\downarrow(e) + A_\downarrow(\mu) + A_\downarrow(\tau) \quad &+ \quad T_\downarrow(d) + T_\downarrow(s) + T_\downarrow(b) \end{aligned} \tag{8}$$

The SM quantum numbers can be recovered from this spectrum in the following way:

–the $\uparrow$ representations can be obtained from the $\downarrow$ ones by the transformation

$\delta\vec{S} \leftrightarrow \delta\vec{T}$ for any of the tetrons, i.e. by interchanging left and right. As shown in (2.4.14) this is precisely what is needed for an isospin transition on the level of mignons.
–singlet and triplet Shubnikov states have a different U(1) charge. The corresponding symmetry can be interpreted as gauged fermion oder $B-L$ number. Details will be given in (2.1.8) and (2.2.3). The mixture with the photon and the appearance of the Weinberg angle will be discussed in (2.1.11).
–the 3 states within each triplet T in (8) are always degenerate, because G_4 remains unbroken. The relation between those triplets and the QCD color triplets will be discussed in (2.3.22).

Actually, to obtain the quark and lepton spectrum (8) a discrete structure is compelling only in internal space, not in physical space. Looking at fig. 2, one could try to come along with a continuous model of Minkowski space, i.e. with $r \rightarrow 0$. However, it is tempting to assume $r \neq 0$, i.e. that there is a lattice underlying spacetime, with spacings so small that Lorentz symmetry is effectively maintained for all available energies.

This item will be discussed in more detail after (16) and in section 2.5, where it will be shown that this lattice must be i) elastic and ii) a Planck lattice, otherwise it would contradict i) cosmological observations and ii) Einstein's principle of equivalence[15, 16]. Due to quantum fluctuations it may be a foam[7] or a spin network[8] – although in the tetron model there is no a priori necessity to quantize gravity, cf. section 2.5.

Can the aligned structure fig. 2 be understood heuristically? The answer is yes, if one assumes that the arrangement of isospin vectors follows similar rules than that of spin vectors in a magnetic environment. What matters are value and sign of (internal) exchange integrals J of tetron wave functions as a function of the distance between 2 tetrons, because these integrals will appear as couplings in the Heisenberg isospin Hamiltonian (12).

The behavior of isospins in fig. 2 can then be understood via the so-called Bethe-Slater curve shown in fig. 3. If the tetrons are part of one tetrahedron, their distance is small $\sim R$ and according to the figure J is negative. This corresponds to antiferromagnetic behavior and leads to the formation of the frustrated structure fig. 1

with symmetry $A_4 + S(S_4 - A_4)$, because the spin vectors try to avoid each other as far as possible.

In contrast, if the internal spin vectors belong to different tetrahedrons, the distance of the corresponding tetrons is somewhat larger, of order r, and J is positive. This corresponds to ferromagnetic behavior.

Due to the tetrahedral 'star' structure fig. 1 it is appropriate to change the notion of isospin. Usually in an (anti)ferromagnetic environment, the spin vectors align into the + or - orientation of the z(=magnetization) direction, and the corresponding Pauli spinors are given by $U = (1,0)$ and $D = (0,1)$. In the present case of a frustrated system the situation is different. The anti-ferromagnetic structure is defined by isospin vectors either pointing outward or inward in the radial direction. Correspondingly, the isospinors U and D are to be understood as 'radial' spinors[22]

$$\begin{aligned} U_\star = \sqrt{\frac{1}{3}} Y_1^0 U - \sqrt{\frac{2}{3}} Y_1^1 D &= \cos\frac{\vartheta}{2}\, U + \sin\frac{\vartheta}{2} e^{i\frac{\varphi}{2}}\, D \\ D_\star = \sqrt{\frac{2}{3}} Y_1^{-1} U - \sqrt{\frac{1}{3}} Y_1^0 D &= \sin\frac{\vartheta}{2} e^{-i\frac{\varphi}{2}}\, D - \cos\frac{\vartheta}{2}\, U \end{aligned} \tag{9}$$

where Y_l^m denote the sperical harmonics and ϑ and φ are the angles of the radial vector w.r.t. some cartesian coordinate system. These new spinors are radial in the sense that they reproduce the unit vector in polar coordinates

$$\vec{e}_r = U_\star^\dagger \vec{\tau} U_\star = -D_\star^\dagger \vec{\tau} D_\star \tag{10}$$

Furthermore they are normalized in such a way that

$$U_\star^\dagger U_\star + D_\star^\dagger D_\star = U^\dagger U + D^\dagger D \tag{11}$$

Note that this presentation is equivalent to the 'universal' z-axis approach[55, 56] used in the actual mass calculations below. Although according to (10) $D_\star$ has as much to do with U as it has with D, I will leave out the star index in the following for reasons of simplicity and understand that always $U_\star$ and $D_\star$ are meant. I will include the star only in cases this is needed for clarity, e.g. in (2.1.9).

On the microscopic level the SM Higgs mechanism corresponds to a pairing of tetrons and antitetrons in neighboring tetrahedrons. This pairing leads to the ferromagnetic alignment of isospin vectors in neighboring tetrahedrons depicted in fig. 2 and is

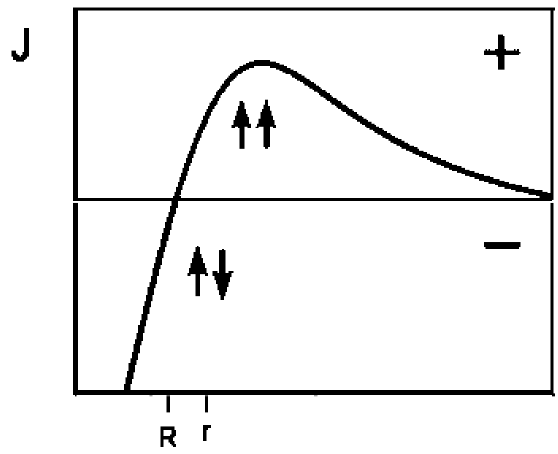

Figure 3: Bethe-Slater curve: the exchange integral coupling J as a function of the distance between 2 tetrons.

described by a non-vanishing vacuum expectation value $\langle \bar{U}_\star U_\star \rangle \neq 0$. Further details will be given in (2.1.9) and (2.3.11).

In [2] the masses of the excitations (8) have been calculated, and the observed hierarchy in the quark and lepton spectrum as well as the hierarchy in the CKM and non-hierarchy in the PMNS matrix elements has been reproduced. As described above, mignon masses can be identified with the eigenfrequencies of the vibrations of the isospin vectors $\vec{S}$ and $\vec{T}$. These eigenfrequencies get contributions both from inner- and from inter-tetrahedral interactions.

Firstly, the *inner*-tetrahedral interactions are responsible for the frustrated tetrahedral configuration fig. 1, i.e. for the structure of the local vacuum. They are small distance contributions and relatively simple to treat because they can be described by an internal Heisenberg Hamiltonian for one tetrahedron alone, with corresponding internal spin vector excitations. The most general form of this Hamiltonian is

$$H_H = -J_{SS} \sum_{i \neq j=1}^{4} \vec{S}_i \vec{S}_j - J_{TT} \sum_{i \neq j=1}^{4} \vec{T}_i \vec{T}_j - J_{ST} \sum_{i,j=1}^{4} [\vec{S}_i \vec{T}_j + \vec{T}_i \vec{S}_j] - K_{ST} \sum_{i=1}^{4} \vec{S}_i \vec{T}_i \quad (12)$$

where the couplings J are internal exchange energy densities characteristic for the internal Heisenberg interactions. By introducing K_{ST}, I have allowed that the coupling $\vec{S}_i\vec{T}_j$ is different within a site ($i = j$) than outside of it ($i \neq j$).

Using (12) and (7) one is lead to e.o.m. for $\delta\vec{S}_i$ and $\delta\vec{T}_i$ which can be solved in a similar way as the e.o.m. for magnons in solid state physics. On this basis the contributions from (12) to the eigenfrequencies of the 24 eigenmodes were calculated in [2].

Secondly, the *inter*-tetrahedral interactions are based on the parallel (='ferromagnetic') alignment of isospins between different tetrahedrons fig. 2. Their leading effect turns out to be a contribution of order $O(\Lambda_F)$ solely to the top quark mass[2]. Physically speaking, this interaction handicaps the specific eigenmode describing the top quark, because this mode disturbs the SSB alignment in the strongest possible way.

Mathematically, the effect can be described by adding terms to the inner-tetrahedral Heisenberg interaction with a normal ferromagnetic plus a Dzyaloshinskii-Moriya (DM) component[24]. The sum of the 2 components will yield a quasi-democratic mass matrix which in leading order only contributes a term of order Λ_F to the top-quark mass and nothing to the masses of the other quarks and leptons.

More in detail, the Hamiltonian for the SSB interactions of neighboring tetrahedrons can be derived from the W-mass term of the SM Lagrangian. By considering a $SU(2)_L$ gauge transformation, which removes the longitudinal components of the W-bosons from the Higgs part of the SM Lagrangian, one obtains

$$H_{SSB} = \frac{\mu^2}{4\Lambda_F} \sum_{i,j=1}^{4} [\vec{S}_i\vec{S}'_j \; + \; i\,(\vec{S}_i \times \vec{D}_{ij})\;\vec{S}'_j] \tag{13}$$

to be added to the Heisenberg Hamiltonian (12). μ is the mass parameter of the SM Higgs potential. Only terms involving the left handed isospin vectors $\vec{S}$ appear, in accordance with the $V - A$ structure of the weak interactions. $\vec{S}'_j$ denotes the left handed isospin vector of an adjacent tetrahedron. An elaborate connection between (12) and (13) on one side and the SM Lagrangian terms on the other will be developed in (2.1.14).

Eq. (13) contains a ferromagnetic interaction plus the additional DM term which is due to the non-abelian nature of the W-bosons. The overall normalization of the

DM-term is dictated by $SU(2)_L$ gauge invariance, while the relative values of the DM-couplings $\vec{D}_{ij}$ are fixed by the internal $A_4 + S(S_4 - A_4)$ symmetry[2].

Quite in general, a DM component stands for a tendency to form a rotational structure (instead of the ordinary ferromagnetic alignment of neighboring tetrahedrons depicted in fig. 2) simply because the DM-term tends to rotate the spin vectors instead of aligning them. In the present case it appears as a consequence of the non-abelian structure of SU(2). Therefore the DM-term can be interpreted quite naturally, namely by the fact that the $SU(2)_L$ gauge fields induce a curvature of the fiber bundle formed by the system of all tetrahedrons, and the DM-term simply takes care of this curvature effect to effectively maintain the aligned structure.

This argument is supported by the fact that the gauge transformation mentioned above leads to the so-called 'unitary gauge'. As shown in (2.3.14) this is exactly the gauge where one encounters the aligned isospin structure among different tetrahedrons depicted fig. 2.

Using (12) and (13) one can derive the e.o.m. for the isospin vectors. With the usual ansatz $\sim \exp(i\omega t)$ one obtains two 24×24 eigenvalue problems, which turn out to be separately diagonalizable[2]. The eigenvalues ω correspond to the quark and lepton masses.

The quantitative results can be found in [2]. In that paper it was explicitly verified that the corresponding 24 eigenstates can be arranged into 6 singlets and 6 triplets as predicted by the Shubnikov symmetry analysis (8), i.e. as 6 lepton and 6 quark flavors. Each triplet (quark flavor) consists of 3 states with degenerate eigenvalues, because the Shubnikov symmetry $A_4 + S(S_4 - A_4)$ is unbroken at low energies.

The dominant contribution from (13) gives the top quark a mass of the order of the Fermi scale while leaving the other quark and lepton masses unchanged. b, c, s and τ get their masses mainly from (12). In contrast, there are no contributions from (12) and (13) to u, d, e and neutrino masses. These 10 excitations remain massless on this level. To obtain their masses one has to include additional small torsional interactions[2].

The masses of the neutrinos are particularly suppressed because the 3 neutrino modes correspond to the vibrations of the 3 components of the total internal angular momentum vector

$$\vec{\Sigma} := \sum_{i=1}^{4}(\vec{S}_i + \vec{T}_i) = \sum_{i=1}^{4}\vec{Q}_i \tag{14}$$

Whenever this quantity is conserved

$$d\vec{\Sigma}/dt = 0 \tag{15}$$

i.e. independent of time, the neutrino masses will strictly vanish. In fact, the interactions considered so far, i.e. (12) and (13), fulfill (15).

The general solution to the eigenproblem given above does not only yield the energy eigenvalues but via the corresponding eigenvectors can also be used to accommodate the CKM and PMNS mixing matrices[2]. The mass eigenstates are the states corresponding to the energy eigenvalues, while the interaction eigenstates naturally correspond to the original vectors $\vec{S}_i$ and $\vec{T}_i$.

Within this framework one can understand[2] why the CKM elements turn out to be small, whereas the PMNS matrix elements are naturally large: the lepton eigenstates (roughly given by $\vec{S} \pm \vec{T}$) are 'far away' from $\vec{S}$ and $\vec{T}$, while the up- and down-type quark eigenstates are relatively small deformations of $\vec{S}$ and $\vec{T}$, respectively. Due to the dominant contribution from (13) the top quark triplet state has the smallest mixing matrix elements with other quarks, because it corresponds to the vibration of $\sum_i \vec{S}_i$ to an accuracy of less than 1%.

In summary, the present model describes the physical world as a huge ordered crystal of internal 'molecules', each molecule of tetrahedral form and arranged in such a way that the internal Heisenberg spin symmetry is spontaneously broken. As shown below, this approach not only provides a nice microscopic understanding of particle physics phenomena but in addition substantially supplements our understanding of the (inflationary) big bang cosmology. In effect, it gives the phase transitions in the early universe a microscopic meaning.

To comprehend this fact, it is appropriate to redevelop the full history of the early universe within the assumptions of the tetron model: before the 'big bang' there were the free tetrons ψ floating around as a Fermi gas in R^{6+1} space at extremely high pressure and temperature. While the universe was cooling down, 3 fundamental transitions occurred:

I. the formation of tetrahedral 'molecules' from tetrons at very high temperature of order Λ_R, where the scale R is roughly given by the extension of one molecule. Although this process is not a phase transition in the strict sense it has certainly released a large amount of energy which has amplified the initial temperature of the universe. Note that with 4 molecular sites each molecule 'fills' only 3 of the 6 spatial dimensions.

II. the formation of the 'hyper-crystal' from tetrahedrons takes place at somewhat lower temperatures $T \sim \Lambda_r$, where r is roughly given by the distance between 2 tetrahedrons. This alignment of all tetrahedral structures is a coordinate alignment and to be distinguished from the isospin vector alignment (item III) describing the electroweak phase transition. It puts all 3-dimensional molecular structures in parallel thus separating an internal 3-dimensional space from the rest. In other words, the crystal expands into a 3+1-dimensional subspace of R^{6+1}, while the tetrahedrons extent into what becomes the 3 internal dimensions.

Actually, in most parts of the review I will follow a slightly different scenario in which there is no step I of tetrahedral molecule formation, i.e. the crystal spontaneously forms out of free tetrons below the critical temperature Λ_r. In that case the distances $R \lesssim r$ naturally are of the same order of magnitude, both representing the length of the tetron bonds.

Since II corresponds to the process, in which our 3+1 dimensional universe was born, it may rightfully be called the big bang. As a crystallization process it is a first order phase transition associated with the sudden release of a large amount of energy. As will be explained in (2.5.3), the coordinate interactions among the tetrahedrons are of elastic type. Under this condition the release of crystallization energy naturally drives an inflationary expansion of the crystal and the corresponding metric. Therefore, within the framework of the tetron model, the big bang and the beginning of inflation are more or less identical. As argued below, the characteristic scale Λ_r can be identified to lie somewhat below the Planck scale $\lesssim \Lambda_P$.

III. the arrangement of isospins at temperatures of order Λ_F. Above those temperatures the isospin vectors fluctuate randomly with an associated internal 'Heisenberg' $SU_L(2)$ symmetry, but at Λ_F they arrange into the chiral isomagnetic structure figs. 1 and 2. At that point the so far freely rotatable internal spins get ordered and

$SU_L(2)$ is broken to the Shubnikov group (5). Note that $SU_L(2)$ is a local symmetry, because isospins can be rotated separately over each point of the Minkowski base space. A more detailed description of this phase transition is given in (2.3.14) and (2.3.11).

Since II happens after I, i.e. at lower temperature, one naturally expects Λ_R somewhat larger than Λ_r (i.e. $R < r$) in agreement with fig. 2 and the Bethe-Slater curve fig. 3. As argued in (2.5.3) both scales are of order $\lesssim \Lambda_P$ and much larger than the scale Λ_F where the isospins align. Note that while Λ_F approximately corresponds to the critical point of transition III, the values of the exchange integrals J and therefore the iso-magnetic behavior are determined at distances r and R.

To describe II in the framework of the Landau approach to phase transitions one should consider density fluctuations $D \exp(i\vec{p}\vec{x})$ within the gaseous/fluid assembly of tetrons (or of tetrahedral 'molecules') and use D as the order parameter of the coordinate alignment.

For an ordinary crystal these fluctuations can be identified with phonons; in the general relativistic framework of a spacetime crystal considered here they correspond to gravitational excitations. This will become clearer below where elastic deformations of the crystal will be identified with metrical changes.

Since the density perturbation adds to the uniform density of the tetrahedron gas/liquid, there is no symmetry under changing sign of the density wave, and so the Landau free energy expansion allows for a cubic term

$$\Delta F = \alpha(T - T_c)D^2 + \beta D^3 + \gamma D^4 \qquad (16)$$

where $T_c \sim \Lambda_r$ is the critical temperature for phase transition II.

The appearance of the cubic term is characteristic for a first order phase transition where a second minimum, which develops in the potential when the temperature is lowered, for some time remains higher than the minimum at $D = 0$ of the gas/liquid phase, and furthermore the two minima are separated by a potential wall. When the temperature drops below the critical value, there is a discontinuity which is not present in second order transitions.

The latent heat associated with this discontinuity is released very suddenly and can be used to explain the extreme acceleration needed for cosmic inflation[13, 14]. It provides all the necessary ingredients for the inflationary process to start and to eventually stop, once the coordinate alignment of tetrahedrons is completed and the energy is exhausted. Most of the initial molecular energy has then been transferred to the elastic energy of the crystal. However, some of it survives in the form of excitations of the crystal and will be converted into ordinary matter, as the temperature goes further down.

In place of quarks and leptons, whose existence is tied to the isospin ordering fig. 2, shortly after crystallization other excitations are more important, like the internal coordinate vibrations discussed in (2.3.20) and [3], or excitations of the tetron-antitetron bonds discussed in (2.1.9) and (2.1.10). Most prominent among the latter are the gauge bosons and the scalars of the 2HDM sector (2.1.14).

Because of the dominance of the electroweak bosons this cosmological era is often called 'radiation dominated' or 'electroweak'. At temperatures far above the Fermi scale all these excitations are effectively massless states transforming under the local $SU(2) \times U(1)$ symmetry, and they dominate the universe all the way down to the electroweak SSB (=isospin alignment). More details about the tetron model view on this era can be found in (2.1.12), (2.1.13), (2.3.7) and (2.3.8).

As well known, in general relativity a non-vanishing energy momentum tensor leads to a curvature of the spacetime continuum. Sakharov was one of the first to interpret this on the basis of metric elasticity[64, 66, 67, 68], i.e. he showed that general relativity is equivalent to the description of a 4-dimensional elastic continuum.

Furthermore, he thought gravity forces might be explainable from a microscopic structure which in some sense is analogous to the molecular structures responsible for material elasticity in low-energy physics. It is in fact well known that the Einstein equations are generalizations of the Cosserat elastic theory[46] to 3+1 dimensions[68]. In this spirit I want to interpret the hyper-crystal as a *discrete micro-elastic spacetime continuum* – while maintaining the rigid tetrahedral structure within the internal spaces.

In order for that to work I will assume that in addition to the isospin interactions discussed before there are elastic inter-tetrahedral coordinate interactions responsible for the forces of gravity. These elastic forces between tetrahedrons may arise as relic interactions from the saturated bonds within a tetrahedron, and are naturally much smaller than all particle physics(=isospin vector) interactions.

From the point of view, that there is connection between the tetron model and the forces of gravity, it appears likely that Λ_R and Λ_r of figures 2 and 3 are related to the Planck scale. As will be argued in (2.5.3) they are actually both of the order $\lesssim \Lambda_P$.

The short version of this argument goes as follows: the Planck length L_P arises as a lower limit on Δx in the 'generalized' Heisenberg uncertainty relation[48], which includes the effects of gravity. On the other hand in the tetron model *all* known particles are interpreted as crystal excitations, with an extension of at least one lattice spacing r. Since every physical experiment necessarily uses these particles, its resolution cannot be better than r. This strongly suggests $r \gtrsim L_P$, or equivalently $\Lambda_r \lesssim \Lambda_P$. Note that for such extremely small lattice spacings, spacetime effectively looks like a continuum and the violation of Lorentz symmetry cannot be experimentally observed.

In such an environment, where the inter-tetrahedral coordinate interactions are of micro-elastic type, the metric defined by the elastic medium will react to any kind of mass/energy transfer by attaining a curvature and/or acceleration. For example, the energy released during crystallization will immediately blow up the distance between the tetrahedrons thus inflating the volume of the hyper-crystal spacetime.

What was called the 'hyper-crystal' before, therefore is a 'Planck lattice' in the sense of [15, 16]. However, due to the elasticity property, the lattice spacing / Planck length $r = L_P(t)$ varies with (cosmic) time, with 1.6×10^{-35} m being its present value.

As a consequence of this statment at least one of the fundamental constants c, h and G must vary with time, too. This is because the Planck length can be written as

$$L_P = \sqrt{\frac{\hbar G}{2\pi c^3}} \tag{17}$$

Due to the 'crystallographic' origin of our universe – with the lattice spacing and the elasticity properties depending on temperature, pressure etc – it turns out that in fact all of them are time dependent. Even more: these quantities are only defined within the hyper-crystal but not in the whole R^{6+1}. The details of this viewpoint will be worked out in a separate publication.

2 Questions and Answers

In this section a list of questions and answers is presented which arise in connection with the tetron model. The more important ones will be specially marked and reviewed in the summary section 3.

2.1 Questions about the gauge sector

According to section 1 the universe is interpreted as a fiber bundle over Minkowski space R^{3+1} with fibers given by the iso-magnetic tetrahedrons fig. 1. The electroweak gauge fields are to be interpreted as connections in that fiber space, i.e. they help to define what parallel alignment in between different fibers means.

2.1.1 How can such a model lead to a local gauge theory?

The internal 3-dimensional space which hosts the tetrahedrons is naturally endowed with a $SU(2)_L \times U(1)_F$ symmetry:
–the $SU(2)$ factor arises from the rotational symmetry of the internal spin vectors before their alignment and was explained in section 1.
–the U(1) factor corresponds to tetron number conservation, which on the level of quarks and leptons translates into the $B-L$ quantum number, cf. (2.1.8).
As explained in section 1 these groups act as *local* symmetries, because their elements can be chosen different for different points of the Minkowski base space. Connections can be defined for the $SU(2)_L \times U(1)_F$ bundle, which are to be interpreted as gauge bosons. As shown in question (2.1.11), there is a mixing of the $U(1)_F$ field with W_z, with mixing angle equal to the Weinberg angle. After the mixing the corresponding local gauge fields are given by the observed $W^{\pm}$, Z and γ.

2.1.2 Are the electroweak bosons and the Higgs field composite?

The answer is yes, but one should specify how this works out in detail.
–The most straightforward possibility is that they are composites of mignon antimignon pairs. However, as will be seen in (2.3.22), such a pairing is more appropriate to describe chiral symmetry breaking in QCD. Furthermore, such a construction would make the top-quark content dominate the boson sector of the SM, similar to top condensate models. Since $m_t \gg m_W$, this is usually not considered a convincing scenario.
–Secondly, one could be tempted to insist they are fundamental objects, because they are connections of the basic $SU(2) \times U(1)$ fiber bundle in the sense of differential geometry. As such they could have been induced by curvature dynamics of the full R^{6+1} geometry. I do not think this is a very attractive option, because the flat R^{6+1} knows nothing about the dynamics within the curved hyper-crystal bundle. Furthermore, the Higgs field as a necessary add-on to account for the SSB has not a simple interpretation in the pure differential geometric framework.
–I adhere to the idea that they must be excitations of tetron-antitetron bound states, i.e. directly arise from the nature of the tetron interactions in the crystal. Even more, the dynamics is such that these excitations can only form and travel inside the hyper-crystal and cannot exist outside of it. In other words, it is assumed that the tetron matter making up the crystal is so strongly bound that tetrons cannot be split off, not even in pairs. This requirement is dictated by the no-dissipation concept, cf. (2.5.10). Namely, one has to take care that such pairs do not leave the crystal and dissipate into R^{6+1}, because otherwise energy would not be conserved inside the crystal.
'Travelling' of such a pair is meant in the sense of a quasi-particle, i.e. the binding hops from one pair to another, while the tetrons themselves stick to their place in the crystal. Equivalently, one may describe it as a density wave bilinear in $\bar{\psi}$ and ψ that travels through the crystal.

2.1.3 Can a stable and massless particle like be photon be composite?

Yes. Masslessness of the photon is protected by the U(1) gauge symmetry. As long as this symmetry holds, the photon remains massless.

Nb: masslessness implies stability of the photon.

2.1.4 Is the photon really composite?

No in the QED7 model advocated in [1], where the ordinary photon is part of a '7-dimensional' photon, responsible for the iso-magnetic interactions.But yes within the 'no-dissipation' hypothesis advocated in this article, cf. (2.5.10). The latter has the advantage that energy is conserved for all processes inside the crystal, so no compactification of internal spaces is needed. The only particles which are not excitations are the tetrons, the building blocks of the crystal. These, however, are bound with energies $> 10^{10}$ GeV.

The photon being an internal excitation cannot be scattered away from the hyper-crystal. Since according to (2.2.3) one has $Q(U) = 0$, the photon is a $\bar{D} - D$ excitation of D-tetrons which are themselves bound within the hyper-crystal

$$A_\mu \sim e\, Q(D)\bar{D}\gamma_\mu D \tag{18}$$

As discussed before, the no-dissipation hypothesis (2.5.10) may have rather challenging consequences. If the photon is not a fundamental particle, it is difficult to believe that the Lorentz symmetry valid within the hyper-crystal is a fundamental property of the original full R^{6+1} spacetime. Lorentz structure probably comes into being only when the crystal is formed and holds only inside of it.

2.1.5 What happens on the microscopic level when a mignon and an antimignon annihilate into an electroweak gauge boson?

Assume the 2 mignons are in 2 neighboring tetrahedrons. When the gauge bosons are formed, the mignon vibrations disappear and are replaced by an internal density vibration of a bound $\bar{\psi}$-ψ pair involving 2 tetrons from the neighboring tetrahedrons. This is in contrast to the suggestion made in [1] where these pairs were made up of free tetrons floating around. The latter idea has been abandoned because the binding energy of a tetron in the crystal is too large, of order Λ_P, and it would furthermore allow energy in the form of $\bar{\psi}$-ψ pairs to dissipate away from the hyper-crystal.

The excitations of bound pairs of tetrons behave trivially under $A_4 + S(S_4 - A_4)$

Shubnikov transformations, i.e. the information about the discrete tetrahedral structure is washed out, because mignon and antimignon compensate each other in that respect. What remains is the transformation property under $SU(2)_L \times U(1)_F$. Since ψ is an isospin doublet, the product of ψ and $\bar{\psi}$ leads to $2 \otimes 2 = 3 + 1$, i.e. a triplet (the weak bosons) and a singlet (the B-L photon).
These serve as connections in the fiber space. As such they are useful to define what alignment of adjacent tetrahedrons means.

2.1.6 Why can mignon couplings be understood as gauge couplings?

The mignons are dynamical sections in the $SU(2)_L \times U(1)_F$ fiber bundle described above. In order to keep up gauge invariance they are naturally endowed with gauge couplings to the connections. As for the couplings of the fundamental tetron fields $\psi = (U, D)$ I refer to (2.2.3).

2.1.7 What is the meaning of the initial $U(1)_F$ symmetry?

On the tetron level it is tetron number, on the mignon level it is $B - L$. For more details see (2.1.8).

2.1.8 How do the electric charges of mignons arise?

As explained in (2.1.16) there are no W_R bosons. Nevertheless, $SU(2)_L \times U(1)_F$ is the complete gauge symmetry for mignons. This is no contradiction because it is meant in the sense that all $V + A$ coupling terms to mignons necessarily vanish. However, this does not forbid to formally introduce a right handed isospin quantum number via $I_3 = I_{3R} + I_{3L}$.
Furthermore, $F = B - L = B + \bar{L}$ is the appropriate fermion number to choose for mignons (8), with $F(l) = -1$ for leptons and $F(q) = 1/3$ for quarks. The mixing among the neutral gauge bosons can then be described by introducing the unbroken generator Q as

$$Q = I_3 + \frac{F}{2} \qquad (19)$$

so that

$$Q(u) = \frac{1}{2} + \frac{F(q)}{2} \qquad Q(d) = -\frac{1}{2} + \frac{F(q)}{2} \tag{20}$$

$$Q(\nu) = \frac{1}{2} + \frac{F(l)}{2} \qquad Q(e) = -\frac{1}{2} + \frac{F(l)}{2} \tag{21}$$

2.1.9 What is the tetron content of the Higgs field and of the SM vev?

We start from the basic idea that all observed scalars and vector bosons are excitations of tetron-antitetron bound states induced by correlations between tetrons of *neighboring* tetrahedrons, cf. questions (2.1.2) and (2.1.5).
Some of the underlying correlations must be responsible for the electroweak SSB, because one of the tetron-antitetron excitations is the Higgs particle. Since it is to support the radial alignment of tetrahedrons fig. 2, it should be identified as

$$H \sim \bar{U}_\star U_\star \tag{22}$$

where $U_\star$ is the 'radial' U iso-spinor introduced in (10) corresponding to an isospin vector $\vec{Q} = U_\star^\dagger \vec{\tau} U_\star$ pointing outward as in fig. 1. The point is that the content of the Higgs particle is in one-to-one correspondence with the vev needed to stabilize the alignment of isospins in fig. 2, and isospin vectors pointing outward correspond to radial spinors $U_\star$ while those pointing inwards corresponds to $D_\star$.
According to these considerations the SM Higgs doublet Φ must be of the form

$$\Phi \sim \begin{pmatrix} -\bar{D}_\star(1+\gamma_5)U_\star \\ \bar{U}_\star(1+\gamma_5)U_\star \end{pmatrix} \tag{23}$$

i.e. not as in ordinary $SU(2)_L \times SU(2)_R$ symmetric NJL theories[32, 33] but formally similar to top-color models[26] – provided the use of radial isospinors is understood. The implication of this formula on the vev and on the NJL structure inherent in the SM will be discussed in (2.1.14) and (2.3.11).

2.1.10 What is the tetron content of the weak gauge bosons?

The answer to this question depends on whether one is talking about the ordered or about the symmetric phase. In the symmetric phase, e.g. shortly after the hypercrystal was formed, there is only the coordinate tetrahedron but no tetrahedral 'star'-configuration of isospin vectors as in fig. 1. Therefore radial spinors (10) should play no role. The photon is given by (18) and the $U(1)_F$ tetron number gauge boson by

$$B_\mu \sim g' \; F(\psi) \; [\; \bar{U}\gamma_\mu U + \bar{D}\gamma_\mu D \;] \tag{24}$$

where g' is the $U(1)_F$ gauge coupling. Similar formulas hold for the $SU(2)$ gauge bosons.

It is interesting to note that in the ordered phase, i.e. at temperatures below Λ_F, eqs. (18) and (24) formally keep their validity, U and D naturally being replaced by $U_\star$ and $D_\star$.

2.1.11 Can γ-Z mixing and the measured value of the Weinberg angle be understood in the tetron model?

Yes. In (2.2.3) it is shown that $F(\psi) = -1$ and $Q(D) = -1$. Using this input one can directly infer from (18) and (24) that the weak mixing angle at the unification/crystallization point Λ_r must be 45 degrees, i.e. $\sin^2(\theta_w) = 1/2$. The form of the Z-boson is

$$Z_\mu \sim -\frac{e}{\sin(\theta_w)\cos(\theta_w)} \; [I_3(U)\bar{U}\gamma_\mu U + I_3(D)\bar{D}\gamma_\mu D + Q(D)\sin^2(\theta_w)\bar{D}\gamma_\mu D \;] \tag{25}$$

which at Λ_r reduces to $Z \sim \bar{U}\gamma_\mu U$, i.e. at the unification point the Z consists only of U-tetrons. Note I do not distinguish left- and right-handed tetrons in these equations, because in the philosophy followed in this paper the SU(2) gauge bosons a priori contain lefthanded as well as righthanded tetrons. It is only the internal chirality of the configuration fig. 1 that prevents the $V + A$ component to become active, cf. (2.1.15), (2.1.18), (2.1.8) and [1].

In the next subsection (2.1.12) the prediction $\sin^2(\theta_w) = 1/2$ at Λ_r will be shown to agree with the present experimental value provided one uses 3 ingredients: i) the evolution of the SM beta function as given in [50], ii) eq. (26) and iii) a value of the unification scale relatively close to Planck scale.

2.1.12 What is meant by 'unification scale' in the framework of the tetron model?

In the tetron model the natural electroweak unification scale is given by the energy Λ_r at which the hyper-crystal is formed from tetrahedral 'molecules' via the phase transition II. As argued in sections 1 and (2.5.3) this scale corresponds to the lattice spacing r in fig. 2 and is naturally of order $\lesssim \Lambda_P$. As shown in (2.1.11), at Λ_r the value of the Weinberg angle must be 45 degrees. This corresponds to a relation between the $U(1)$ and $SU(2)_L$ gauge couplings

$$g'(\Lambda_r) = g(\Lambda_r) \tag{26}$$

Note that (26) goes beyond the SM because the gauge group $SU(2)_L \times U(1)_F$, even in the form of a U(2) group, is not simply connected and therefore no relation between the values of g and g' can be predicted. In contrast, in the tetron model a prediction is possible and given by (26). This is based on the observation that the original U(1) gauge symmetry is tetron number and that the photon (18) should be of D-content only.

Using the SM beta funtions[50] one can extrapolate g and g' from their measured values at m_Z to ultrahigh energies in order to see for which values of Λ_r eq. (26) can be satisfied. Since there is no diminishing factor 3/5 in (26) like in GUT inspired models[69], Λ_r comes out to be nearly equal to the Planck scale instead of $\Lambda_{GUT} \approx 10^{15}$GeV.

One may rightfully ask whether the SM beta functions are really applicable up to such high energies or whether they get appreciable corrections from other crystal excitation like the 2HDM Higgs partners discussed in (2.1.14), or from phinons and isospin density waves which appear at higher energies, cf. (2.3.20).

2.1.13 Is there a connection between the tetron model unification scale and the scales relevant for the standard cosmological model?

Yes. In the usual cosmological terminology there is the scale at which inflation ends, and this scale is usually identified as the temperature below which the radiation dominated epoque starts. This era can be described as an equilibrium of effectively massless electroweak gauge bosons.

In the tetron model, inflation is associated with the release of latent heat at crystal formation time. The end of inflation is the time when crystallization(=the inflation period) has finished, and the unification of the electromagnetic and the weak interactions is naturally interpreted as happening at this point. It is the time at which our 3+1 dimensional universe started to exist. According to the analysis in (2.1.12) and (2.5.3) this roughly corresponds to Planck scale energies, and therefore in the present model the electroweak era starts shortly after the Planck scale.

2.1.14 Is there a connection between the isospin interactions (12)+(13) of the tetron model and the SM Lagrangian?

Yes, there is. To explain this in detail, one first has to notice that the mignon vibrators appearing in (12) are defined in terms of $\psi^\dagger$, while the components of the Higgs doublet appearing in the SM Lagrangian are given in terms of $\bar{\psi}$. Even more, while the vibrators are supposed to 'live' within one tetrahedron, the Higgs excitations according to the philosophy discussed in (2.1.2) extend over two of them.
In accordance with this observation, two types of internal vectors should be distinguished:
–isospin vectors $\vec{Q}$ of type (2) and (4) which are the carriers of the isospin waves (mignons). They correspond to the internal angular momentum of tetrons *within one tetrahedron* and are the 'charges' of the conserved internal Noether currents. Their excitation spectrum leads to the quark and lepton flavors because they arrange according to the Shubnikov group within one tetrahedron.
–fields like the $\vec{\pi}$ component of the Higgs doublet, which involve $\bar{\psi}$ instead of $\psi^\dagger$. Together with the vev $\langle\bar{\psi}\psi\rangle \neq 0$ and the Higgs particle $H = \bar{\psi}\psi - \langle\bar{\psi}\psi\rangle$ they are important for the pairing process between tetrons and antitetrons of *neighboring tetrahedrons* which in the tetron model is responsible for the SSB.
To understand this in more detail consider the SM Higgs potential with one doublet

$$V_{SM}(\Phi) = -\mu^2\Phi^+\Phi + \lambda(\Phi^+\Phi)^2 = -\frac{1}{2}\mu^2(\sigma^2 + \vec{\pi}^2) + \frac{1}{4}\lambda(\sigma^2 + \vec{\pi}^2)^2 \qquad (27)$$

where $\sigma = \Lambda_F + H$. This potential naturally describes the alignment of neighboring tetrahedrons and anti-tetrahedrons in fig. 2, although in fig. 2 not the $\vec{\pi}_i$ are drawn but the $\vec{Q}_i$. The point to notice is that two of the $\vec{\pi}_i$ are in parallel iff all the corresponding $\vec{Q}_i$ are. Therefore the pairing force $\sim \mu^2 \vec{\pi}_i \vec{\pi}_j$ implied by (27) exactly corresponds to a 'ferromagnetic exchange coupling' of strength μ^2 in the SSB interaction (13).

There is one drawback in this argument, and this concerns the number of d.o.f. While the SM Higgs doublet only has 4 real d.o.f., the isospin vibrators in the form of $\vec{Q}_L$ and $\vec{Q}_R$ contain 8. According to (4) these can be given as

$$\psi^\dagger \psi \qquad \psi^\dagger i\gamma_5 \vec{\tau} \psi \qquad \psi^\dagger i\gamma_5 \psi \qquad \psi^\dagger \vec{\tau} \psi \tag{28}$$

I have included $\psi^\dagger \psi$ and $\psi^\dagger i\gamma_5 \psi$ in this list albeit their vibrations do not correspond to mignons, but to isospin density fluctuations, cf. (2.3.20).

Equivalently, one may write these as

$$U^\dagger U \quad D^\dagger D \quad U^\dagger D \quad D^\dagger U \quad U^\dagger \gamma_5 U \quad D^\dagger \gamma_5 D \quad U^\dagger \gamma_5 D \quad D^\dagger \gamma_5 U \tag{29}$$

or

$$U_R^\dagger \psi_L \pm c.c. \qquad D_R^\dagger \psi_L \pm c.c. \tag{30}$$

In contrast, the 4 d.o.f. of the Higgs doublet are given in (23). To account for the remaining 4 one should add a second scalar doublet to the dynamics, e.g. in the form

$$\Phi' \sim \begin{pmatrix} -\bar{D}_\star (1+\gamma_5) D_\star \\ \bar{U}_\star (1+\gamma_5) D_\star \end{pmatrix} \tag{31}$$

to supplement (23). This corresponds to adding a pseudo-scalar iso-scalar η and a scalar iso-vector triplet $\vec{v}$ to the theory.

Together, Φ and Φ' form the basis for an extended SM with 2 Higgs doublets. Such models are usually abbreviated as 2HDM, and have been extensively discussed in the literature[52, 53, 54].

The argument about the ferromagnetic alignment induced by the negative mass term $-\mu^2 \vec{\pi}\vec{\pi}$ in the potential can be extended to the 2HDM model where the potential contains a term $\sim \vec{v}\vec{v}$ in addition. This term, however, must not give an appreciable contribution to the SSB interaction (13), because otherwise the b-quark mass would come out to be of order Λ_F. In other words, Φ' must not take part in the SSB; the 'mass term' $\sim \vec{v}\vec{v}$ has to have a positive coefficient and correspondingly

$$\langle \Phi' \rangle = 0 \tag{32}$$

for the second Higgs doublet and therefore no q/l mass contributions from Φ'. This feature smartly agrees with the property of the inert version[53, 54] of the 2HDM model. It is interesting to note that in that model the η or the v_z (depending on which mass is smaller) is a serious dark matter candidate, because there are no couplings between q/l mignons and Φ'. For further details see (2.5.12).

2.1.15 How can the chiral nature of the weak bosons be ensured?

The iso-magnetic tetrahedral structure in fig. 1 violates internal parity, the state with opposite internal parity being given by a system where the 4 internal spin vectors show inwards instead of outwards. As shown in [1] and (2.1.18) this internal parity violation triggers the violation of external parity needed for the weak interactions, provided the interaction among tetrons stems from a common interaction in the full R^{6+1} space, cf. (34).

2.1.16 Are there SU(2)$_R$ gauge fields $\vec{W}_R$ in addition to SU(2)$_L$?

No. Z and W are originally connections of an SU(2) bundle. According to the discussion in (2.1.18) it is only the formation of the chiral structure fig. 1 together with the octonion (i.e. R^{6+1}) origin of the interaction which forces them to couple to left-handed mignons only. Without the internal chirality fig. 1 the weak interactions would be vectorlike.

2.1.17 If there is no $\vec{W}_R$, why is there η?

The 2HDM model (2.1.14) with η and $\vec{v}$ naturally accompanies the vibrations of $\vec{Q}_L$ and $\vec{Q}_R$. 2HDM models do not need a $\vec{W}_R$-field.

2.1.18 Is there a fundamental interaction which gives rise to such iso-magnetic structures? Can the parity violation of the weak interactions be explained from first principles?

Before I start to discuss this question, note it is not about the coordinate forces which are responsible for the formation of (tetrahedral molecules and) the hyper-crystal at scale Λ_r, but only about the isomagnetic forces relevant for the isospin vector alignment at the Fermi scale.

The scenario advocated in this paper consists of fundamental 6+1 dimensional spinor fields ψ ('tetrons'), transforming as (1), which form tetrahedral structures and a hyper-crystal a la fig. 2, in which the quarks and leptons propagate as internal spin waves. The Higgs and the observed vector bosons are excitations of tetron-antitetron bound states, and the system as a whole naturally gives rise to the $SU(2)_L \times U(1)_F$ gauge symmetric SM. For the appearance of the strong interactions see (2.3.22), and the role of gravity is discussed in section 2.5.

The isomagnetic tetron interactions are claimed to derive from the octonion structure inherent in 6+1 dimensions. The octonions form the unique non-associative, non-commutative and normed division algebra in 8 dimensions, and their imaginary units provide for 7 of the 21 generators of SO(6,1). They are closely related to the Dirac matrices Γ_μ in 6+1 dimensions[69]. The 6+1 dimensional vector current $\bar{\psi}\Gamma_\mu\psi$ arises more or less directly from the octonion product of two octonions corresponding to the spinors $\bar{\psi}$ and ψ.

When the hyper-crystal is formed and R^{6+1} spacetime decomposes into Minkowski and internal space, the Γ_μ split into SO(3,1) Dirac matrices γ_μ and a remainder according to[69, 40]

$$\Gamma_{1-7} = (\gamma_{1-4}, \gamma_5 \tau_{x,y,z}) \tag{33}$$

where x, y and z denote the internal coordinates. This splitting has its physical origin in the coordinate interactions of the tetrons, which lead to the formation of the hyper-crystal, and mathematically it parallels the splitting of an octonion into 2 quaternions.

Starting from (33) one can try to derive the parity violation of the weak interaction. The important point to notice is the appearance of the product $\gamma_5\vec{\tau}$ in the internal part of (33). In principle, the presence of such a coupling corresponds already to a parity violating behavior, both in internal and Minkowski space, because γ_5 signals axial behavior in Minkowski space and $\vec{\tau}$ does the same job for the non-relativistic internal space.

Any 6+1 dimensional vector coupling $\bar{\psi}\Gamma_\mu\psi$ reduced to internal space will induce such a term. However, for this to actually become perceivable, an additional appropriate 'chiral situation' has to be provided, again both in internal and Minkowski space. In Minkowski space this can be achieved, for example, by using polarized beams or if there is a second vertex with a γ_5-coupling in the Feynman diagram of the process.

An analogous requirement must be met in the internal space. In other words, a configuration with a handedness must be present, in order to pick up a non-vanishing contribution from the axial coupling, and this in the case at hand is given by the local chiral ground state structure fig. 1.

As a matter of fact, the non-relativistic circumstances of the internal R^3 space make it a similar situation as one has in optical activity of molecules, where in addition to a circularly polarized photon there must be a handed molecule in order to produce a non-vanishing effect. According to (33) a 4-tetron interaction of two vector currents will induce a term

$$\vec{\pi}\vec{\pi} = [\bar{\psi}\vec{\tau}\gamma_5\psi][\bar{\psi}\vec{\tau}\gamma_5\psi] \tag{34}$$

which agrees with the term of the Higgs potential (27) responsible for the alignment of isospin vectors and on the level of isospin vectors reproduces the Heisenberg ansatz (13). As discussed in connection with fig. 3, the sign of the coupling must be anti-ferromagnetic for inner- and ferromagnetic for inter-tetrahedral distances. The latter is in accord with the negative mass term of the Higgs potential.

An important question is whether there is a renormalizable interaction in 6+1 dimensions containing the 6+1-dimensional vector current $\bar{\psi}\Gamma_\mu\psi$ as a necessary ingredient. In [1] a 6+1 dimensional QED model was proposed as a candidate. According to that scenario, 6+1 dimensional photons and tetrons would be the fundamental field content of the original R^{6+1}. This model, however, is not compelling. It may suffer from the problem of energy dissipation into the internal dimensions, cf. (2.5.10). In the present paper I advocate the gauge bosons to be tetron-antitetron bound states which do not exist in the original R^{6+1} but arise only when the (3+1)-dimensional hyper-crystal is formed, cf. (2.1.2) and (2.1.4).

2.2 Questions about the fundamental fermion ψ

According to (1) the tetron matter fields ψ which form the sites of the lattice fig. 2 transform as 8 under SO(6,1) and decompose into an isospin doublet when the hyper-crystal is formed. In this section some further properties of the tetrons are elucidated.

2.2.1 What is the use of introducing an additional level of matter?

There are several good reasons to do so:
i) the existence of 3 families of quarks and leptons with altogether 24 states (plus the corresponding mass and mixing values) strongly suggests that they are not truly elementary objects.
ii) a material origin for the observed internal symmetry groups of the SM is highly desirable. Traditionally, they are pasted into the theory as purely abstract groups, corresponding to a rather static behavior of the representation spaces. This line of thinking started with Heisenberg's invention of isospin SU(2), went over to SU(3) of color and ended with the (SUSY) GUT groups. The present model works quite different, isospin symmetry being obtained by extending spacetime by 3 internal spatial dimensions in which an independent dynamics takes place.
iii) spontaneous symmetry breaking is introduced in the SM in a more or less ad hoc way by adding a scalar field to a system made up of fermions and gauge bosons. This is similar to and extends the Ginzberg-Landau model for super-conductivity to a local relastivistic non-abelian symmetry. However, as well known from many branches of physics, a material background is required for a phase transition and SSB to occur. In the tetron model the breaking of $SU(2)_L$ is associated to the alignment of the (material) internal tetrahedrons over Minkowski space as shown in fig. 2[1]. For the interpretation of the Higgs field as a Cooper-pair like tetron-antitetron pair see (2.1.9).

2.2.2 What is so interesting about the 8 of SO(6,1)?

This question is discussed in (2.1.18) and (2.5.5).

2.2.3 What are the couplings / charges of the tetrons?

After the hyper-crystal is formed a tetron ψ decomposes into its isospin components U and D. This fact fixes the weak charges

$$I_3(U) = +\frac{1}{2} \qquad I_3(D) = -\frac{1}{2} \tag{35}$$

Using (19) one finds

$$Q(U) - Q(D) = 1 \qquad F(\psi) = Q(U) + Q(D) \tag{36}$$

where F is the U(1) charge and given by tetron(=fermion) number. Therefore it must be the same for both types of tetrons, i.e.

$$F(\psi) = F(U) = F(D) \tag{37}$$

Tetrons do not have a color charge because they are not involved in interactions of triplets of the Shubnikov group. Therefore, it is appropriate to normalize F in analogy with leptons instead of quarks, i.e. to put

$$F(\psi) = -1 \tag{38}$$

Eq. (36) then leads to

$$Q(U) = 0 \qquad Q(D) = -1 \tag{39}$$

In other words, the normalization (38) is equivalent to defining the U direction in iso-spinor space to be the one which is electrically neutral.

2.2.4 How large is the mass of a single tetron?

Difficult to say. Naively, if gauge bosons and Higgs scalars are considered $\bar{\psi}$-ψ bound states, the natural guess for m_ψ would be in the range of 40 to 60 GeV. However, in truth they are not bound states but excitations of the $\bar{\psi}$-ψ bonds within the crystal. Furthermore, tetrons are so tightly bound within the hyper-crystal with an extremely large binding energy which has been released during inflation/crystallization that a mass value cannot seriously be given.

2.2.5 How large is the binding energy of a tetron ψ within the hypercrystal. Can it be ionized?

No. The binding energies are extremely large, of the order of $\Lambda_r \lesssim \Lambda_P$, cf. (2.5.3) and the discussion after (16).

2.2.6 Why not use a molecular model instead of a crystal?

Even though Lorentz invariance can be established for lattice spacings of order L_P, it may seem difficult to imagine that the world is a kind of crystal, with every spacetime point occupied by 4 tetrons. - So why not use a model where excitations of tetrahedral molecules in an otherwise empty space are used to explain the q/l spectrum? The 'molecules' would extend into internal dimensions and have a frustrated anti-ferromagnetic structure as in fig. 1. Even an explanation of the SSB as a re-arrangement within the molecules below a certain temperature is feasible.
However, with such a picture one would run into all the known problems of classic composite models[31]. The killing argument certainly is, how one and the same molecule can sometimes have a mass larger than 100 GeV and sometimes be as light as neutrinos.

2.2.7 Why are the scales Λ_R and Λ_r so much larger than the quark and lepton masses?

Roughly speaking because the energies Λ_F involved in isospin alignment are much smaller than the energies Λ_r needed for the coordinate formation of the crystal. R and r are the length scales for which the iso-magnetic exchange integrals have to be calculated, i.e. the *abscissa* in the Bethe-Slater curve fig. 3. On the other hand the values of the exchange integrals J are drawn on the *ordinate* of fig. 3 and always $\leq \Lambda_F$.

2.3 Questions about the local tetrahedral structure and the nature of the SSB?

To obtain the correct mass spectrum for quarks and leptons not only the internal geometry but also other features of the model have to fixed.

2.3.1 Are the tetrahedrons formed by 4 or 8 tetrons, i.e. how many vibrators are needed on each lattice site?

4 tetrons are enough. Naively, one would think that 4 tetrons can give rise to only $4 \times 3 = 12$ excitations of their isospin vectors. However, there are 2 vibrators $\vec{Q}_L$ and $\vec{Q}_R$ for each tetron, and for 4 tetrons this gives $2 \times 4 \times 3 = 24$ states. Depending on the helicity of the q/l to be constructed only initial states $\langle \vec{Q}_L \rangle \neq 0$ or $\langle \vec{Q}_R \rangle \neq 0$ have to be considered. However, to obtain their masses one has to consider both $\langle \vec{Q}_L \rangle \neq 0$ and $\langle \vec{Q}_R \rangle \neq 0$.
In any case there are 24 vibrators $\delta\vec{Q}_{Li} \neq 0$ and $\delta\vec{Q}_{Ri} \neq 0$ to be considered, and this leads to the desired 24 states (8). It is interesting to note that in effect, even a purely left-handed q/l has a tiny amount of vibrating right-handedness in it.

2.3.2 What exactly is the local ground state of the system? Do all these isospins point outwards?

Yes, they all point outwards, as shown in fig. 2. There is no questioning about the direction of antitetrons because the relativistically covariant isospin vectors (3) are used which comprise both particles and antiparticles. As pointed out in (2.1.9) the definition of 'pointing outwards' is tied to the definition of $U_\star$ in (10).

2.3.3 Is the binding which makes up the Higgs field due to isospin or is it due to tetron coordinate interactions?

It is due to both. The Higgs particle (22) relies on the alignment of $U_\star$ isospinors and is therefore natural part of the iso-magnetic interactions. On the other hand, the Higgs (as well as all other scalar and vector fields) is an excitation of the tetron-antitetron bonds in the crystal, and therefore controlled by the coordinate interactions.

2.3.4 What are the unbroken symmetries of the model?

The Shubnikov group (5) and $U(1)_Q$. The unbroken Shubnikov group has only singlet and triplet representations and leads exactly to the flavor spectrum of quarks and leptons (8) needed in the SM.

2.3.5 Can one calculate Fermiskala, W/Z-Masse, Higgs mass, Weinberg angle from first principles?

The origin of the Weinberg angle was discussed in (2.1.11). The Fermi scale and the Higgs mass arise from iso-magnetic exchange and pairing interactions, as discussed in (2.1.9). Knowing the fundamental theory (2.1.18) they are calculable from 6-dimensional exchange integrals.

2.3.6 Did gauge bosons exist at temperatures above the Fermi scale?

Yes, they did. In the tetron model gauge bosons are particle-antiparticle excitation states of crystally bound tetrons. Shortly after the crystal was formed at Λ_r they came into being and made up for the radiation dominated epoque, as it is called by cosmologists, cf. (2.1.13).

2.3.7 Did quarks and leptons exist at temperatures above the Fermi scale?

No, because from their very nature they require the existence of the iso-magnetically ordered state fig. 2. When the universe cooled down and the hyper-crystal began to form, there was only the coordinate alignment of tetrahedrons but no alignment of isospin vectors. In that stage, at temperatures above the Fermi scale, no quarks and leptons existed, because iso-magnetic waves could not travel through the crystal. Only phinons (2.3.20) and scalar and vector bosons (as excitations of the tetron-antitetron bonds) existed. This era is usually called radiation dominated, cf. (2.1.13).

2.3.8 Then why can quarks and leptons be produced at energies above the Fermi scale?

In collider experiments they can exist at energies much larger than 1 TeV because the alignment of isospins is stable much beyond Λ_F in the fully ordered hyper-crystal, i.e. in our universe. This is due to a collective hysteresis effect in which the crystal stabilizes itself by the concerted action of all aligned tetrahedrons to maintain the isomagnetic ordering. As a result, quarks and leptons can be produced and propagate normally, even in cases where energies locally exceed the critical energy Λ_F.

2.3.9 Why is $\bar{\psi}$ involved in the order parameter and not $\psi^\dagger$, wheras the total 'iso-magnetization' $\vec{\Sigma}$ eq. (14) is like in ordinary magnetic models?

$\vec{\Sigma}$ (with its factors ψ and $\psi^\dagger$) refers to one single tetrahedron. It is an inner-tetrahedral quantity which only indirectly influences the SSB. In contrast, the SSB is described by the alignment of two neighboring tetrahedrons over physical space, more precisely a tetrahedron and an 'anti-tetrahedron'. Thus, the order paramenter should be built from ψ and $\bar{\psi}$.
Furthermore, one has $\langle\vec{\Sigma}\rangle = 0$ for the ground state fig. 1, so it is no candidate for the order parameter anyhow.

2.3.10 Why not use $\langle\vec{\pi}\rangle$ as an order parameter?

To answer this question consider 2 neighboring tetrahedrons A and B with tetrons ψ_{Ai} and $\bar{\psi}_{Bi}$, and $i = 1, 2, 3, 4$ counting the tetrahedral sites. The aligned tetrahedral 'star' configuration of these 2 tetrahedrons not only implies $\langle\vec{\Sigma}_{A,B}\rangle = 0$ for each tetrahedron separately. but also $\langle\vec{\Pi}\rangle = 0$, where $\vec{\Pi}$ is defined as $\vec{\Pi} = \sum_i \vec{\pi}_i$ and $\vec{\pi}_i = \bar{\psi}_{Bi}\vec{\tau}\psi_{Ai}$. This is because one can show that $\vec{\pi}$ vectors of 2 adjacent tetrahedrons are parallel, iff the corresponding isospin vectors $\vec{Q}$ are.

2.3.11 Is the vev $\langle \bar{U}U + \bar{D}D \rangle$ or $\langle \bar{U}U \rangle$ or what?

The vev is given by $\langle \bar{U}_\star U_\star \rangle$ in accordance with (22) where $U_\star$ is the 'radial' iso-spinor introduced in (10) corresponding to an isospin vector $\vec{Q} = U_\star^\dagger \vec{\tau} U_\star$ pointing outward as in fig. 1. This is precisely what is needed to stabilize the alignment of isospins pointing outward as in fig. 2 because isospin vectors pointing outward correspond to radial spinors $U_\star$ while those pointing inwards corresponds to $D_\star$.

2.3.12 So what is the microscopic interpretation of the Higgs particle?

As discussed in (2.1.2) the Higgs is neither fundamental nor a bound state of mignons, but an excitation of tetron-antitetron pairs which are themselves bound within the hyper-crystal.

2.3.13 Should one consider separate tetrahedrons for anti-tetrons, with isospin vectors pointing inward?

This question is justified, because anti-fermions usually react to magnetic forces with an opposite sign. However, using isospin vectors (2) one is treating the problem in a relativistically covariant way. As can be seen in (3), the isospin vectors contain particle as well as well as antiparticle contributions.

2.3.14 Is there a difference between the SM SSB and a ferromagnet, apart from the fact that the SM SSB takes place in internal space? Is the symmetry breaking in the tetron model really spontaneous?

Both cases (ferromagnet and tetron structure) are similar in that at high energies / temperatures the directions of (iso)spins are oriented randomly with an associated SU(2) Heisenberg symmetry, and this defines the symmetric state.
In an uni-axial ferromagnet an accidental magnetization axis usually appears spontaneously, based on a thermodynamic potential

$$V_{FM}(\vec{M}) = -a\vec{M}^2 + b\vec{M}^4 \tag{40}$$

where $\vec{M}$ is the total magnetization and the minimum of the potential is at $\langle \vec{M}^2 \rangle = a/2b$.

In the case at hand the crystallization process at scale Λ_r is accompanied by a coordinate alignment of all tetrahedrons, i.e. there is a spontaneous selection of one global internal coordinate system for all tetrahedrons. This coordinate alignment, however, happens much prior to the alignment of isospins at the Fermi scale and has not much to do with it.

When the temperature decreases towards Λ_F, the anti-ferromagnetic tetrahedral 'star' configurations fig. 1 are taken where the isospin vectors avoid each other as far as possible. Note there is an infinite SU(2) symmetric set of such 'star' configurations just as in a ferromagnet there is an infinite set of possible magnetized states corresponding to all possible magnetization directions in R^3. The difference as compared to a ferromagnet is that not only the stars over one tetrahedron have to be included but also those over all the other tetrahedrons over Minkowski space, with their independent SU(2) degeneracies, and this makes the problem a local gauge symmetric one.

The SSB consists in the simultanous selection of one among all the possible star configurations over all Minkowski base points – namely the one with $\langle \Phi \rangle \sim (0, 1)$. This corresponds to a vev for the $U_\star$ and $\bar{U}_\star$ isospinor component, i.e. the one with an isospin vector pointing outward in radial direction and the corresponding isospin vector in the neighboring 'anti'-tetrahedron pointing in the same direction. The choice of $(0, 1)$ - and of U - is notational convention and, in the framework of the gauge theory, corresponds to choosing a certain gauge (the so called unitary gauge). There is again a similarity to the situation in a ferromagnet where some axis is selected by the spontaneous magnetization, and the coordinate system is then 'gauged' in such a way that this axis is called the z-axis $\sim (0, 0, 1)$.

One may ask what role the coordinate alignment of tetrahedrons at the crystallization point $\Lambda_r \lesssim \Lambda_P$ plays in this game, because it seems plausible that the state, where the tetrahedrons of coordinate and isospin both point in the same radial directions, is energetically preferred. (This geometry is in fact depicted in figures 1 and 2.)

A similar situation is sometimes encountered in ordinary uni-axial ferromagnets in cases when the coordinate backbone of the crystal prefers one specific magnetization direction, so that the ferromagnetic phase transition is not really spontaneous. This effect can be modeled by adding a tiny explicit symmetry breaking contribution to the potential (40) by hand. At high temperatures due to thermal fluctuations this structural / coordinate effect is not relevant. However, it becomes relevant near the Curie temperature where it fixes the magnetization direction.

In the present case, however, this possibility needs no consideration. The reason is that $\Lambda_R \lesssim \Lambda_P$ is so large as compared to Λ_F, that the granular internal coordinate structure is not noticed by the isospin vectors (nor by any human experiment). From the perspective of the isospin vectors it looks as if they are sitting on an internal coordinate structure which is rotationally invariant. They only feel the anti-ferromagnetic aversion towards their 3 fellows within one tetrahedron.

As a consequence of these considerations all 'star' configurations are energetically equivalent, and the symmetry breaking is spontaneous.

2.3.15 What are the relevant scales in the model?

There are essentially only 2 scales: the Fermi scale and the Planck scale.

The crystallization process with coordinate alignment of tetrahedrons but erratic directions of isospin vectors corresponds to scales Λ_r and Λ_R slightly below Λ_P, while Λ_F is the scale where the isospin vectors align.

In view of $\Lambda_F \lll \Lambda_P$ and the fact that Λ_F is a measure for the length of the isospin vectors, one cannot say that figures 1 and 2 reflect the true size proportions very well.

2.3.16 What is the geometrical meaning of these scales?

r and R can be interpreted as lattice spacings; originally they correspond to the length of the bond between any two tetrons. The Fermi scale measures the 'length' of isospin vectors of the ground state fig. 1.

2.3.17 Do GUT theories play any role?

At best as effective theories describing the approximate behavior of excitations present at ultra-high energies, just as the SM describes the mignon and gauge boson behavior. However, it is difficult to justify why there should be other SSBs in the model besides those described in (2.3.15) and in section 1. I see no reason for the proliferated Higgs sector characteristic for most GUT theories.

2.3.18 Is there a unification of electroweak and strong couplings?

No. In the tetron model the strong forces are on a less fundamental footing than the electroweak interactions. For further details see (2.5.11) and (2.3.22). The unification scale of electromagnetism and weak has been discussed in (2.1.12).

2.3.19 What about domain walls?

There are no domain walls in the hyper-crystal. To understand this statement, one should first realize that one has to distinguish i) domains arising at the crystallization point (coordinate alignment of tetrahedrons) from ii) those arising at the Fermi scale (tetrahedral alignment of isospins).
i) In an ordinary crystal, one expects the appearance of domains with different ordering directions arising from concurrent nucleation of crystal germs in different points of space. In principle, this is also true for the R^{6+1} space under consideration. The reason why they are not present in the case at hand has to do with the fact that the hyper-crystal grows into and occupies only a quasi 3-dimensional subspace of R^6. Therefore it intersects with other hyper-crystals from concurrent nucleation points, which grow into other 3-dimensional subspaces of R^6, in at most 1 point (because the intersection of 2 almost flat 3-dimensional submanifolds in R^6 in general is just 1 point). The result of the other nucleations will be different hyper-crystals, i.e. different worlds whose intersection with our universe will consist of at most one point, showing up as a defect within our hyper-crystal structure.

2.3.20 Are there excitations of the hyper-crystal besides the known quarks, leptons and scalar and vector bosons?

Yes. An incomplete listing includes:
–phinons. They are the analogs of phonons in a solid and have been described under more general circumstances in [3]. In the case at hand there are 12 phinon states,

that can be arranged according to representations of the permutation group S_4. Just as mignons they travel as quasi-particles through the hyper-crystal. Phinon masses are expected to be much larger than mignon (quark/lepton) masses. While the mignon spectrum is lying at and below Λ_F, the phinon spectrum is expected to be at and below the crystallization energy $\lesssim \Lambda_P$. Note, this is not a very accurate characterization, in view of the fact that neutrino masses are so tiny with respect to the Fermi scale. Note further, phinons are internal coordinate vibrations, and thus have to be distinguished from gravitational waves. An interesting question is whether mignon-phinon scattering is possible.
–isospin density waves: they are to be distinguished from phinons and from mignons. The vibrators in this case are similar to the isospin vectors (4), however without the factor of $\vec{\tau}$, i.e. given by $\psi^\dagger(1 \pm \gamma_5)\psi$. As far as I can judge these excitations correspond to a fourth family of fermions, i.e. a lepton-like and a quark-like isospin doublet, probably higher in mass, because they are not related to the other families by the Z_3 family quantum number inherent in the Shubnikov group $A_4+S(S_4-A_4)$. In particular, the fourth 'neutrino' is expected to be much heavier than the known neutrinos, because its mass is not suppressed by internal angular momentum conservation.
–one should mention scalar fields other than the Higgs bosons. They are the components of the second Higgs doublet (31) and the most promising candidates for dark matter, cf. (2.5.12).

2.3.21 Could quark and leptons be phinons?

or in other words: what is the advantage of using mignons with $A_4 + S(S_4 - A_4)$ symmetry over phinon excitations with symmetry group $A_4 \times Z_2$ as advocated in [3]?
The answer is that many of the attractive features of mignons are absent, like the explanation for the Higgs mechanism, for tiny neutrino masses etc.

2.3.22 Is it possible to understand the dynamics of the strong interactions from within the tetron approach?

I don't have a final answer to this question. From its very construction the tetron model is concerned mainly with the symmetries and interactions of electroweak physics. The colors of quarks arise merely as a byproduct, because they are interpreted as the 3 d.o.f. of the Shubnikov triplet representation. It is therefore clear that QCD with a color gauge group and SU(3) color triplets does not directly arise in the tetron model.

In section 1 it was argued that the phase transitions of a 6+1 dimensional spacetime filled with a condensing tetron gas supplies all relevant physics for the early universe and it may even account to understand the forces of gravity. As for the latter the suggestion is that it may arise from elastic forces between tetrahedrons which are the remnants of more fundamental tetron-tetron coordinate interactions and induce curvature and/or torsion effects on Minkowski space.

One would like to interpret the strong interactions in a similar spirit, namely assuming there are no other interactions in the universe besides the ones among tetrons. The strong interaction structure does not correspond to any part of the $SU_2 \times U_1$ bundle connection, because this allows only for the 4 electroweak gauge bosons. One may note, however, that a Shubnikov invariant mignon-mignon interaction is to be expected among the triplets, transforming as

$$T \times T = A + A' + A'' + 2T \tag{41}$$

As shown in [5] this structure can be embedded into a SU$_c$(3) algebra where the color indices correspond to the 3 d.o.f. of the triplet representations in (8). Unfortunately, it is not clear whether gluons and QCD gauge interactions can really arise from this line of reasoning.

2.3.23 Is there a connection between the QCD vacuum and the electroweak condensate?

A prominent feature of the strong interactions is chiral symmetry breaking, i.e. the appearance of a non-vanishing quark condensate $\langle \bar{u}u + \bar{d}d \rangle \approx -(0.1GeV)^3$ which breaks $SU(2)_L \times SU(2)_R$ to isospin $SU(2)_V$. The main difference as compared to

the electroweak case is that all these groups are global, not local symmetries.
While the QCD condensate is formed by triplets of mignons, i.e. of certain isospin excitations, the Higgs condensate corresponds to an alignment of isospin vectors. While the former extends over the strong interaction scale ~ 1 GeV, the latter is related to the Fermi scale.

2.4 Questions about the quark and lepton mass spectrum and the CKM and PMNS mixing matrices

Quark and lepton masses can be calculated as excitation frequencies of mignons – a rather straightforward procedure where the results are obtained quite naturally by considering isomagnetic interactions among the tetrons of one or two tetrahedrons only. Analytic mass formulas can be given in terms of internal Heisenberg and Dzyaloshinskii-Moriya exchange couplings[2].

2.4.1 Using exchange couplings instead of Yukawas – isn't one replacing one set of unknown parameters by another set and one effective theory (the Standard Model) by another one (the internal Heisenberg model)?

No, because the internal couplings can in principle be calculated from first principles as exchange integrals over internal space, just as in ordinary magnetism the exchange couplings of the Heisenberg model are in principle calculable from exchange integrals of electronic wave functions over physical space. What one needs to know is the underlying 6+1 dimensional dynamics of tetron interactions.

2.4.2 Why are mignons fermions?

After all, they are constructed from excitations of 'bosonic' isospin operators (2), and this could lead one to believe that they are bosons, just like magnons in ordinary ferromagnets are bosonic quasi-particles.
However, it is important to distinguish the behavior in internal space from that in Minkowski space. While mignons transform as Shubnikov singlets A and triplets T

(i.e. *not* as projective representations) w.r.t. internal space, it is not hard to see that they are Dirac fermions w.r.t. Minkowski space.

The point to note is that mignons are not bound states of tetrons but eigenmodes of their excitations. As such they are not tensor products but linear combinations of (fluctuations of) tetron fields ψ_a^α (where $a = 1, 2$ is the internal and $\alpha = 0, 1, 2, 3$ the Dirac index of the tetron). Since each mignon is a vibration of *one* isospin eigenmode, one concludes that it must be a Dirac particle w.r.t. the Minkowski base space. Using the 'bosonic' isospin vectors is merely a tool to separate the proper isospin triplet vibrations of $\vec{Q} = \psi^\dagger \vec{\tau} \psi$ from singlet density vibrations $\psi^\dagger \psi$, cf. (2.3.20). For example, an isospin vector vibration

$$\vec{Q} \sim (\cos(\omega t), \sin(\omega t), 0) \tag{42}$$

originally corresponds to a vibration

$$\psi \sim (\cos(\omega t/2), i \sin(\omega t/2)) \tag{43}$$

of the tetron ψ field.

Note that only the internal d.o.f. are given in (43). The Dirac spinor d.o.f. remain unchanged and make up for the fermion properties of the mignon. Looked at from 'below', i.e. from the Minkowski base space, the tetron excitations are Dirac fermions. If such an excitation travels through the crystal as a quasi-particle, it can be either L or R, particle or anti-particle.

2.4.3 Why are there exactly the quark and lepton states of the 3 generations?

The unbrocken (Shubnikov) symmetry group has only singlet and triplet representations and leads exactly to the flavor spectrum of 24 quarks and leptons as given in (8).

2.4.4 Are there other ground states than the tetrahedral one which yield the appropriate quark and lepton spectrum (8)?

No. I scanned other geometries with 8 iso-magnetic vibrators and found that for most systems mignons appear in 2-dimensional representations[9, 10], not useful for the q/l spectrum of particle physics.

2.4.5 Should one really use covariant isospin vectors (3) containing both particle and anti-particle contributions as vibrators?

Yes, one should. It is important for the vanishing electric charge of the hyper-crystal, for the formation of gauge bosons out of tetron-antitetron pairs, and in general to maintain relativistic covariance. Technically it is important for the mass calculations corresponding to mignon mass terms $\langle 0|T(\bar{q}q)|0\rangle$.

2.4.6 Why not use only $\vec{Q} = \vec{Q}_L + \vec{Q}_R$ as vibrators instead of $\vec{Q}_L$ and $\vec{Q}_R$ separately?

This would not reflect the true number of d.o.f. of a tetron.
Furthermore, the independent vibrations of $SU(2)_L$ vibrators $\vec{Q}_L$ are extremely important to inherit the nature of the SSB and the top mass in the model, cf. (13). Note that for the calculation of mignon masses from propagators $\langle 0|T[\bar{q}_R(y)q_L(x)]|\rangle +$ *c.c.* one should imagine $\vec{Q}_R$ to be based on the neighboring 'anti-tetrahedron' at y.

2.4.7 Is it okay to start the mass calculations using chiral $SU(2)_L \times SU(2)_R$ symmetry quantities $\vec{S} = \vec{Q}_L$ and $\vec{T} = \vec{Q}_R$?

Yes. This is just the way the SM works. Non-zero fermion masses are developed via SSB starting from a massless, i.e. $SU(2)_L \times SU(2)_R$ symmetric chiral theory.

2.4.8 Can the simple Heisenberg interaction (12) really explain the full q/l mass spectrum with its extreme hierarchies?

No. The masses of some of the fermions get contributions from other physical sources, namely
–the top mass is dominated by a contribution of order Λ_F which stems from the symmetry breaking *inter*-tetrahedral interactions (13). Physically it arises because the top quark corresponds to the 3 eigenmodes which 'disturb' the global ground

state in the strongest possible way. This disturbance is also responsible for the hierarchy observed in the CKM matrix elements.
–only strange-, charm- and muon-mass are dominated by anti-ferromagnetic exchange couplings within one tetrahedron, and thus can be obtained from the *inner*-tetrahedral Heisenberg exchange couplings alone.
–down-quark, up-quark and electron are left massless by the Heisenberg and DM interactions (12) and (13). They get their relatively small masses from energetically favored torsion contributions[2], i.e. vibrations of the internal spin vectors of the generic form $d\vec{Q}/dt \sim \vec{Q}$.
–neutrino masses are protected by internal angular momentum conservation, i.e. by the internal rotational symmetry, cf. (2.4.9).

2.4.9 How can the smallness of neutrino masses be understood?

Neutrinos are interpreted as internal Goldstone modes of the breaking of internal SO(3) by the formation of the discrete structure fig. 1. The associated conserved Noether charge is given by the total internal angular momentum $\vec{\Sigma}$ defined in (14) which implies the existence of 3 zero-frequency modes. All this is analogous to how magnons are interpreted as Goldstone modes in ordinary magnetism, except that one is considering physical processes in the internal spaces.
While in ordinary ferromagnets after magnetization a U(1) symmetry about the z-axis survives, in the given frustrated configuration fig. 1 all three SO(3) generators give rise to Goldstone bosons, to be identified as the internal magnons corresponding to the 3 neutrino species.

2.4.10 Neutrinos are fermions. How can they be Goldstone modes?

One has to distinguish the dynamics in internal from that in physical space. In physical space the neutrinos are fermions, but neutrinos are Goldstone bosons w.r.t. the dynamics in internal space, because in internal space they are described by (bosonic) excitations of the total internal angular momentum $\vec{\Sigma}$ defined in (14) which is the conserved quantity associated with the internal rotational symmetry.
As discussed before, the representations A and T in (8) are ordinary representations

of the Shubnikov group, not projective representations, so all quarks and leptons are 'bosonic' w.r.t. the internal dynamics, cf. (2.4.2).

2.4.11 How do neutrinos obtain their tiny non-zero masses?

As Goldstone modes neutrinos are strongly protected to getting masses. However, as proven in [2] the observed non-zero neutrino masses can be generated on the phenomenological level by tiny torsional interactions which violate (15). These can also be used to accommodate appropriate PMNS mixing values. Physically the existence of such interactions is a signal for the activity in isospin space of small anisotropic forces.

2.4.12 What is the difference between neutrinos as Goldstone modes and the longitudinal pseudo-Goldstone modes $\vec{\pi}$ of the weak gauge bosons?

The pseudo-Goldstone bosons $\vec{\pi}$ arise from the breaking of the *local* $SU(2)_L$, when all tetrahedral isospin structures over Minkowski space get aligned according to fig. 2. In contrast, the neutrinos are Goldstone modes of the breaking of internal SO(3) symmetry by the tetrahedral structure within one single fiber.

2.4.13 Why are neutrinos a part of isospin doublets while $\vec{\pi}$ (and $\vec{W}$) is an isospin triplet?

First of all one has to distinguish representations of the Shubnikov group from those of isospin symmetry. Both ν and $\vec{W}$ behave as singlets under the Shubnikov group. This, however, is all they have in common. While neutrinos and charged leptons represent vibrations of $\vec{Q}_L + \vec{Q}_R$ and $\vec{Q}_L - \vec{Q}_R$, respectively, within a *single* tetrahedron, the pseudo Goldstone bosons $\vec{\pi}$ corresponding to the breaking of local $SU(2)_L$ signal the alignment of *neighboring* tetrahedrons in fig. 2.

2.4.14 How is isospin realized on the mignon level?

Calculating the masses of quarks and leptons from vibrations $\delta\vec{S}$ and $\delta\vec{T}$ of the ground state fig. 1, the isospin of mignons essentially corresponds to an exchange of the roles of $\vec{S}$ and $\vec{T}$, i.e. of the $SU(2)_L$ and the $SU(2)_R$ sector. This is because due to the mass term of the form $\bar{\psi}_R\psi_L + c.c.$ a charge conjugation transition (instead of a parity transformation) should be used to accomplish this exchange, and such a transformation is necessarily accompanied by an exchange of isospins U and D via[65]

$$(U, D) \rightarrow (-D^*, U^*) \tag{44}$$

Note, this formula takes into account only internal and no Dirac d.o.f. It agrees with the behavior under internal time reversal (6), so that the Shubnikov symmetry in the present case can in fact be formulated by using charge conjugation and without introducing the concept of an internal time.
To be more explicit, the transition between 2 isospin partners can be obtained by $\vec{S}_i \rightarrow -\vec{T}_i$ and $\vec{T}_i \rightarrow \vec{S}_i$ on the tetrahedral sites i. In the actual calculation of mignon eigenstates, it turns out that the top-quark is predominantly an $\vec{S} = \vec{Q}_L$ excitation while the b-quark is $\vec{T} = \vec{Q}_R$. On the other hand the neutrinos are given by vibrations of the conserved quantity $\sum_i(\vec{S}_i + \vec{T}_i)$, while charged leptons are essentially given by vibrations of the $\sum_i(\vec{S}_i - \vec{T}_i)$ combination.
The connection between the attributes 'left-handed' and 'up-type' (and similarly 'right-handed' and 'down-type') is of fundamental importance in the tetron model. It relies on the chiral structure of the internal ground state fig. 1 and the octonion induced form of the tetron interactions (2.1.18). Furthermore, it is at the heart of the tetron model explanation of weak parity violation (2.1.18) and of the large value of the top quark mass from (13) as compared to the other q/l masses.
It is interesting to note that according to (6) the internal reflection operators which exchange the elements of A_4 and $S_4 - A_4$ comprise isospin transformations, charge conjugation as well as transitions between left and right.
Note further that for the actual construction of quarks and leptons as mignons one has to use radial spinors (10) in (44).

2.4.15 Do pairs of ordinary and true Shubnikov representations form isospin doublets?

No. A true Shubnikov representation is a representation of $A_4 + S(S_4 - A_4)$ which is not a representation of S_4. True Shubnikov representations are labeled by an index s in the following. Analyzing the representations appearing in (8) one finds that A and A_s, T and T_s arise in the combination

$$\begin{aligned} A(\nu_e) + A(\nu_\mu) + A_s(\nu_\tau) \quad &+ \quad T(u) + T(c) + T_s(t) + \\ A(e) + A(\mu) + A_s(\tau) \quad &+ \quad T(d) + T(s) + T_s(b) \end{aligned} \tag{45}$$

According to this result an isospin doublet is not given by a pair (A, A_s) or (T, T_s) of an ordinary representation and a Shubnikov representation. Instead, the rows A, A, A_s and T, T, T_s correspond to particles of the 1., 2. and 3. family.

2.4.16 Why is top so heavy, why not bottom?

The top quark corresponds to the vibrations of $\vec{\Sigma}_L = \sum_i \vec{Q}_{Li}$. This vector plays a special role due to the nature of the SSB. Basically this is so because of the relation between internal and external parity[1] and because nature has chosen to break internal parity, i.e. prefers the state fig. 1 with all internal spins pointing outwards over the one with isospins pointing inwards. The top-mignon is the rotational vibration where all isospin variations act against the SSB alignment in the strongest possible way.

In contrast, the b-quark is mostly an excitation of the $\vec{Q}_{Ri}$, and therefore gets only a small contribution from the interaction (13).

2.5 Questions about the global crystal structure and cosmology

As seen in fig. 2, there is an internal discrete tetrahedral structure at smallest distances which is able to build up the global structure of the universe by exact repetition. The following questions shed light on the nature of this repetition.

2.5.1 Is physical space discrete, i.e. is there a granular structure of physical space in addition to the discrete tetrahedral structure in internal space?

Most probably yes. Although the discrete structure of physical space is not compelling and the distance r between two tetrahedrons could be identically zero without contradicting any particle physics experiment, the discussion in section 1 suggests that r has a tiny non-vanishing value of the order of the Planck length.

2.5.2 How can such a granular structure be compatible with Lorentz invariance?

The answer is similar as in models of the 'Planck lattice'[15, 16], of spacetime foam[7] or loop quantum gravity[8]. As long as the discrete units are tiny enough, e.g. of the order of the Planck length, neither violations of rotational symmetry or of the principle of equivalence can be detected[51].

2.5.3 Why is the Planck length the natural lattice spacing for the hyper-crystal?

The short answer: the Planck length arises as a lower limit on Δx in the generalized Heisenberg uncertainty relation (46). On the other hand in the tetron model all known particles including the photon are interpreted as crystal excitations with an extension of at least one lattice spacing r. Since every physical experiment necessarily is using one of these particles, its resolution cannot be better than r. This strongly suggests $r = L_P$.

An extended answer: according to the Kopenhagn interpretation of quantum mechanics Planck's constant is due to an uncertainty necessarily arising from the process of measuring. In the present model all known particles (quarks, leptons and vector bosons) are interpreted as excitations on the Planck lattice hyper-crystal and thus will always extend over at least one lattice spacing r. Therefore measurements involving physical particles cannot be more accurate than r. This uncertainty is assumed to imply the quantum mechanical uncertainty and to fix the value of Planck's constant in the following way:

In ordinary quantum mechanics there is a fixed dimensionful quantity h which relates the canonical Fourier variables of frequency to energy and inverse distance to momentum, i.e. it transforms the spacetime quantities x and t into physically 'active' quantities $p = \hbar k$ and $E = \hbar\omega$. This is the reason why h appears in the uncertainty relation $\Delta x \Delta p > \hbar$ which otherwise would just be the Cauchy-Schwarz inequality valid for the Fourier analysis of waves, i.e. $\Delta x \Delta k > 1$ with no dimension on the r.h.s.

The 'generalized' uncertainty relation includes gravitational effects of the photon on a test particle[48] so that the Heisenberg relation is modified to

$$\Delta x > \frac{\hbar}{\Delta p} + L_P^2 \frac{\Delta p}{\hbar} \tag{46}$$

This result can be derived e.g. by extending the 'Heisenberg microscope' thought experiment (which imagines a photon to measure x and p of a probe particle) to include gravitational effects of the photon[48].

Eq. (46) implies that there is a minimum value of

$$\Delta x \sim L_P \tag{47}$$

corresponding to a photon with wavelength L_P. Usually, this is interpreted in such a way that at smaller distances/wavelengths everything dissolves into quantum fluctuations and the laws of physics do not have any meaning any more.

Although it is conceivable that similar features hold true over the whole of R^{6+1}, in the present model this statement strictly speaking refers only to the physical laws within the crystal, for the following reason: since the photon is assumed to be composite, the minimal wave length of photons that can be produced roughly corresponds to the distance r between 2 tetrahedrons, cf. fig. 2.

This is then automatically the minimum length scale that can be resolved by the photon. On the other hand according to (47) this scale is given by L_P. Therefore we conclude $r = L_P$, a result which agrees with the idea discussed above, that Minkowski space has a discrete structure, the lattice constant being given by the Planck length to be identified as the distance between 2 tetrahedrons.

2.5.4 How is physical space defined within the 6+1 dimensional world? How is it distinguished from the internal dimensions?

The aligned tetrahedrons define a 3-dimensional subspace of R^6. Everything orthogonal gives physical space. Note that the crystal does not grow into the internal dimensions. This has to be added to the list of desired properties of the tetron interactions which do not allow tetrahedrons to be stacked on top of each other in fig. 2. This is the reason why internal dimensions need not be compactified, cf. (2.5.10).

2.5.5 Where do the 6 spatial dimensions come from?

The tetron spinor ψ can be interpreted as an octonion field living in 6+1 dimensions. Octonions form a rather unique mathematical structure[41, 42, 43]. They are the next thing to use when complex numbers (used for amplitudes in quantum mechanics) and quaternions (used for rotations and spinors in physical space) are not to enough to describe physical phenomena. This idea has been applied in (2.1.18) to write down the fundamental 4-tetron interaction. The splitting of R^6 into an internal space and physical space corresponds to a splitting of an octonion into two quaternions.

2.5.6 How does all of this fit into inflationary cosmology?

In the tetron model inflation is associated to the crystallization process with an accelerated expansion due to the release of crystallization energy, cf. (2.1.12) and the discussion after (16).

2.5.7 How can this energy lead to metric velocities larger than the speed of light?

The answer is simply that at that stage there was no light, because in the tetron model photons are described as excitations of the hyper-crystal and have appeared only after the crystallization came to an end. What counts during the inflationary period is the maximum velocity allowed in the full R^{6+1} space.

2.5.8 What are the tetron model answers to the flatness and the horizon problem?

They are similar to those in ordinary inflationary models:
–flatness problem (the question why the universe is almost flat everywhere): due to the exponential expansion – triggered by the release of crystallization energy – the universe is much larger than anticipated.
–horizon problem (the question why the universe looks almost the same everywhere): all parts of the universe were causally connected at the time when the hyper-crstal was born. Due to the subsequent exponential expansion they have lost their causal contact.

2.5.9 Are the internal spaces compact or infinite?

They are infinite, no assumption about compactification of internal spaces needs to be made. The reason why we cannot step into the internal dimensions is because the hyper-crystal (=our universe) is restricted to a 3+1 dimensional 'surface' in R^{6+1}. We encounter only excitations on this surface and cannot exist away from it. Neither can energy dissipate into the internal dimensions, cf. (2.5.10).
In other words, going away from the 'surface', internal space is empty. Exception: other hyper-crystals may have condensed at big bang times. In general these will lie skewed with respect to the one we live in and form separate 'universes'. For geometrical reasons two hyper-crystals can meet in at most one point, cf. (2.3.19).

2.5.10 If internal space is infinite, how can one avoid dissipation of energy into the internal dimensions?

If internal space is infinite then matter and energy may in principle disappear into it. This can happen in the form of particles which move in the direction orthogonal to the hyper-crystal. However, in (2.1.2) and (2.1.4) I have taken the viewpoint that all observed particles including the photon are excitations of the crystal and as such cannot exist away from it. Furthermore, the tetrons, from which the crystal is made, are assumed to be so strongly bound, that they can be split off the hyper-crystal only by supply of Planck scale energies.

2.5.11 Was there a GUT era in the early universe where electroweak and strong couplings were united?

No, because there is no GUT – cf. (2.3.17). In the tetron model the strong force has a different origin than the electroweak one, and thus any kind of 'unification' seems unlikely.
The proper history of the universe starts with the end of the crystallization process (=the inflation era), at which point electromagnetism and weak forces are united, cf. (2.1.12). In the standard terminology this is the starting point of the radiation dominated era (with photons, effectively massless W/Z and dark matter as the dominant excitations). At the end of the electroweak era (at temperatures of order Λ_F) there is the alignment of isospin vectors corresponding to the electroweak phase transitiion which gives masses to q/l and W/Z.

2.5.12 Are there dark matter candidates in the model?

Yes, there are several possibilities:
-further original internal excitations of the crystal, like phinons, cf. (2.3.20) and [3]. However, for them to be reasonable dark matter candidates, one has to answer the question, why their interactions with mignons are small.
-the pseudoscalar η arising in the 2HDM ansatz (2.1.14). Such a possibility is widely discussed in the literature [53, 54] provided the η is inert, i.e. it does not interact with quarks and leptons. This condition can be fulfilled in the present model because of (32). Note that right after inflation there is the radiation dominated era where the inert scalar is copiously produced together with a soup of many other tetron-antitetron bound states (photon, W/Z, Φ and Φ').

3 Conclusions

The present review is devoted to a model which tries to give a microscopic meaning to physical phenomena usually described by the Standard Model of elementary particles. By introducing an additional level of matter one is able to understand and to calculate known particle properties (like the quark and lepton masses and mixings)

from first principles and furthermore to make predictions for future experiments. Most prominent among the latter are:
–the existence of a fourth family of quarks and leptons with a very massive neutrino, as discussed in (2.3.20).
–the existence of a second Higgs doublet similar as in inert 2HDM models[53, 54], as discussed in (2.1.14).

The different viewpoints which have been presented in the preceding sections have supplied a set of important requirements as to the nature of tetron interaction. We know already
–that tetrons have a tendency to pair formation with anti-tetrons of neighboring tetrahedrons in the crystal.
–that tetron bonds are extremely short, of the order $\gtrsim L_P$.
–that they get saturated in tetrahedral configurations.
–that these configurations form 'flat' crystal structures, i.e. there is no stacking of tetrahedrons on top of one another, no growth of the hyper-crystal into internal directions.
–that isospins within the quartets of tetrons are maximally frustrated (fig. 1).
–that once tetrons are in such a saturated hyper-crystal configuration, there is a left-over elastic force among the internal tetrahedrons, which gives rise to the gravitational interactions.

After discussing particle physics properties, the implication of the tetron model on phase transitions in the early universe have been elucidated. This has led to the idea that besides the
–isomagnetic interactions among aligned isospin vectors which are relevant for particle physics
one should consider 2 other forces:
-strong rigid coordinate forces among tetrons fixing the form and extension of the (internal) tetrahedral structures and their arrangement into the hyper-crystal.
-weak elastic forces between the tetrahedrons, which are the basis of the gravitational interactions.
All 3 types of forces are assumed to derive from one universal interaction among the tetrons, and it was suggested that one should use octonion multiplication as a guideline which eventually will lead to the correct renormalizable theory in 6+1

dimensions.

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