A Note About Power Function

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Abstract

In this paper described some new view and properties of the power function, the main aim of the work is to enter some new ideas. Also described expansion of power function, based on done research. Expansion has like Binominal theorem view, but algorithm not same.

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1 Introduction

A method based on derivative of n-rank difference between numbers of such sequence, in this paper described the conclusion, that difference, which equals to power of the numbers, is constant. Going from it we should to explain the power function as series of derivative of some function. In this paper rank of difference is written as k, also difference rank is maximal rank of differential equation, which also described, Δx, which often used, just shows, that start numbers have linear nature and should be constantly for using such method. We have the power function of the form:

$$f\left(x_k\right) = x_k^n$$

$$0 < x \in R, \;\; n \in N$$

Have a distribution law for x

$$x_k = k\Delta x$$

Where

$$\Delta x = x_{i+1} - x_i$$

Annotation 0: Let be next numbers sequence: $x_k = \{1, 2, 3, 4, 5, 6, 7, ..., N\}$ As we can see the difference $x_i - x_{i-1} = 1 \rightarrow \Delta x = 1$ and numbers have next law $x_k = k$. Let introduce next example: $x_i = \{0, 3, 6, 9, 12...N\} \rightarrow x_i = 3i$. We have a set of values, done by next form functions:

$$\Delta_i^1 = x_{i+1}^n - x_i^n,$$

$$\Delta_i^k = \Delta_{i+1}^{k-1} - \Delta_i^{k-1},$$

Where k is rank of delta function, in case of:

$$k = n$$

the set has a distribution of the form(for each i indexes is constant):

$$\Delta_i^n = n!(\Delta x)^n$$

Let enter the follow sequence of numbers:

$$\begin{array}{llllll}
x_k & x_k^n & \Delta_i^1 & \Delta_i^2 & \Delta_i^3 & ......\Delta_i^n \\
x_0 & x_1^n & x_1^n - \; x_0^n & \Delta_1^1 - \Delta_0^1 & \Delta_1^2 - \Delta_0^2 & ....\Delta_1^{n-1} - \Delta_0^{n-1} \\
x_1 & x_2^n & x_2^n - \; x_1^n & \Delta_2^1 - \Delta_1^1 & \Delta_2^2 - \Delta_1^2 & ....\Delta_2^{n-1} - \Delta_1^{n-1} \\
x_2 & x_3^n & x_3^n - \; x_2^n & \Delta_3^1 - \Delta_2^1 & \Delta_3^2 - \Delta_2^2 & \\
x_3 & x_4^n & x_4^n - \; x_3^n & \Delta_4^1 - \Delta_3^1 & & \\
x_4 & x_5^n & x_4^n - \; x_3^n & & & \\
x_5 & x_6^n & & & &
\end{array}$$

if

$$\Delta x \rightarrow 0$$

we have next n rank delta view:

$$d^n y = n! dx^n$$

Assume that:

$$f^{(n)}(x) = n!(\Delta x)^n$$

Then:

$$d(f^{(n-1)}(x)) = n!(\Delta x)^n dx \rightarrow f^{(n-1)}(x) = \int n!(\Delta x)^n dx = n!(\Delta x)^n \int dx = n!(\Delta x)^n x + C_1$$

After we may deduce $f^{'}(x)$:

$$f^{'}\,(x) = n!(\Delta x)^n \frac{x^{n-1}}{(n-1)!} + \Delta_0^{n-1} \frac{x^{n-2}}{(n-2)!} + C_2(i) \frac{x^{n-3}}{(n-3)!}) + C_3(i) \frac{x^{n-4}}{(n-4)!} + \ldots + C_{n-1}\,(i)\,,$$

where $C_k(i)$ - the constant generating functions given by the equation:

$$C_i\,(x) = \Delta_0^{n-i} - \left(\Delta_0^{n-i} - \left(\Delta_1^{n-i} - f^{(n-i)}\,(1)\right)\right) i$$

The general idea of the method is to get $f^{'}(x)$, which when used with sum operator returns correct value of number to power with number of steps $f(n)$, if we have $f^{(n)}(x)$ so that intermediate values $f^{(k)}(x)$ correspond to the values of delta $f^{(k)}(x) = \Delta_x^k,\ k \in [1, ...n-1]$. The derivative of some rank k is basic equation for $\Delta_x^k,\ f^{(k)}(x) = \Delta_x^k$.

2 Example for x^3

Let be next set of numbers:

x_i	x_i^3	Δ_i^1	Δ_i^2	Δ_i^3
0	0	1	6	6
1	1	7	12	6
2	8	19	18	6
3	27	37	24	6
4	64	61	30	
5	125	91		
6	216			

As we can see the last difference for each i is became constant and equals to 3!, described an algorithm, when

$$\Delta x = 1$$

Let white the third rank delta next:

$$f^{'''}\,(x) = 3!$$

Second delta should be accordingly next:

$$f^{''}\,(x) = 3! \int dx = 3!x + C_1$$

$$C_1 = \Delta_0^2$$

Then first delta is:

$$f^{'}\,(x) = \int (3!x + C_1)\, dx + C_2 = 3! \frac{x^2}{2} + \Delta_0^2 x + C_2(i)$$

After we should to make conclusion (description is in Annotation 1) of $\mathrm{C}_2\,(x)$:

$$C_2\left(x\right) = 1 - (1 - (7 - f_1^{'}(1)))x = 1 - (1 - (7 - 9))x = 1 - 3i$$

So, the the first derivative is:

$$f^{'}\left(x\right) = 3x^2 + 6x + (1 - 3i)$$

Let enter next equalization: $i = x \to f_k^{'}\left(x\right) = 3x^2 + 3x + 1$

So expansion for x_i^3 has follow view:

$$x_i^3 = \sum_{x=0}^{x_i - 1} f_k^{'}\left(x\right)$$

2.1 Example for x^4

Consider the next numbers:

x	x^4	Δ_i^1	Δ_i^2	Δ_i^3	Δ_i^4
0	0	1	14	36	24
1	1	15	50	60	24
2	16	65	110	84	24
3	81	175	194	108	
4	256	369	302		
5	625	671			
6	1296				

Let write fourth delta as $f^{(4)}\left(x\right) = 4!$

Third delta is:

$$f^{'''}\left(x\right) = 4! \int dx = 4!x + C_1$$

Constant 1 is next:

$$C_1 = \Delta_0^3$$

By integrating of third derivative we obtain second delta function:

$$f^{''}\left(x\right) = \int (4!x + C_1)dx = 4!\frac{x^2}{2} + \Delta_0^3 x + C_2(i)$$

Next we should to obtain Constant 2 function:

$$C_2(i) = 14 - (14 - (50 - f^{''}(x)))i = 14 - 12i$$

$$f^{'}\left(x\right) = \int \left(4!\frac{x^2}{2} + C_1 x + C_2\left(i\right)\right) dx = 4!\frac{x^3}{3} + \Delta_0^3 \frac{x^2}{2} + C_2\left(i\right) x + C_3\left(i\right)$$

$$C_3\left(i\right) = 1 - (1 - (15 - f^{'}(1)))i = 1 - 10i$$

After integration we obtain the first derivative:

$$f^{'}(x) = 4x^3 + 18x^2 + 14x - 12ix + 1 - 10i$$

If we equal $i = x$ derivative is changed follow way: $f_c^{'}(x) = 4x^3 + 18x^2 + 4x - 12x^2 + 1$

As described upper, we may to obtain next expression:

$$x_i^4 = \sum_{x=0}^{x_i-1} f_c^{'}(x)$$

2.2 Example for x^5

Let be next distribution:

x	x^5	Δ_i^1	Δ_i^2	Δ_i^3	Δ_i^4	Δ_i^5
0	0	1	30	150	240	120
1	1	31	180	390	360	120
2	32	211	570	750	480	
3	243	781	1320	1230		
4	1024	2101	2550			
5	3125	4651				
6	7776					

As we can see : $f^{(5)}(x) = 5!, d(f^{(4)}(x)) = 5!dx \to f^{(4)}(x) = 5! \int dx + C_1$ Next let put $C_1 = \Delta_0^4$:

$$f^{(3)}(x) = \int (5!x + \Delta_0^4)dx = 5!\frac{x^2}{2} + C_1 x + C_2(i)$$

$$C_2(i) = 150 - 60i$$

$$f^{''}(x) = \int (5!\frac{x^2}{2} + C_1 x + C_2(i))dx + C_3(i)$$

$$C_3(i) = 30 - 80x,\ f^{'}(x) = \int dx(\int (5!\frac{x^2}{2} + C_1 x + C_2(i))dx + C_3(i)) =$$

$$= 5x^4 + 40x^3 + 75x^2 - 20x^3 + 30x - 80x^2 + C_4(x)\ |_{i=x}$$

where $C_4(x) = 1 - \sum_{x=1}^{n-1} 30x^2 + 20$

3 Operating with float numbers

The first derivative, which we use to expanse power function, has such type: $f^{'}(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + a_3 x^{n-3} + \ldots + a_n$ where $a_1 ... a_n \in N$ and used with $f(x) = x^n$, $x, n \in N$. So, if we want to make expansion for float number with i number of symbols after point we should to revision our function next way: $f_r^{'}(x) = b_0 x^n + b_1 x^{n-1} + b_2 x^{n-2} + b_3 x^{n-3} + \ldots + b_n$ where $b_k = a_k / 10^{in}$ we have only to change coefficients, of the function according the power n and number of

symbols after point i . Number of iteration is placed in range $(1,\ (10^{-in}n-1))$
And expression seen follow:

$$x^n = \sum_{k,i=0}^{(10^{-in})n-1} f_r^{'}(k,i),\ x_i \in R\ and\ float$$

4 Examples of sequences

In this annex will be displayed the sequences accordingly start distribution of the type: $x_i = k\Delta x$. For example let be next set for $f(x_i) = x_i^4,\ x_i = 2i,\ \Delta x = 2.$

$f(x_i) = x^4,\ x_i = 2i$:

x_i	x_i^4	Δ_i^1	Δ_i^2	Δ_i^3	Δ_i^4
0	0	16	224	576	384
2	16	240	800	960	384
4	256	1040	1760	1344	384
6	1296	2800	3104	1728	384
8	4096	5904	4832	2112	384
10	10000	10736	6944	2496	
12	20736	17680	9440		
14	38416	27120			
16	65536				

$\rightarrow \Delta_i^4 = 4! \cdot 2^4 =$ $24 \cdot 16$. Next let show the example for float numbers:

$$f(x_i) = x^3,\ x_i = 1/10i, \Delta x = 0,1 :$$

x_i	x_i^3	Δ_i^1	Δ_i^2	Δ_i^3
0,1	0,001	0,007	0,012	0,006
0,3	0,008	0,019	0,018	0,006
0,4	0,027	0,037	0,024	0,006
0,5	0,064	0,061	0,030	0,006
0,6	0,125	0,127	0,036	0,006
0,7	0,216	0,169	0,042	0,006
0,8	0,343	0,217	0,048	
0,9	0,512	0,271		
1	1			

$\rightarrow \Delta_i^3 = 3! \cdot \Delta x = 3! \cdot 0,1^3$

5 Expressions in use

$$x_i^3 = \sum_{x=0}^{x_i-1} 3x^2 + 3x + 1\}\ and : 3^3 = \sum_{x=0}^{2} 3x^2 + 3x + 1 = 1 + (3+3+1) + (12+6+1) = 27$$

$$x_i^4 = \sum_{x=0}^{x_i-1} 4x^3 + 18x^2 + 4x - 12x^2 + 1\ and : 2^4 = \sum_{x=0}^{1} 4x^3 + 18x^2 + 4x - 12x^2 + 1 = 1 + 15 = 16$$

6 Conclusion

In this paper extended a topic about expanse of basic type power function with linear distribution of start numbers. Method is need to be revised, explained only the basic idea, future researches in this side will improve it. Theoretically, if we have some random numbers sequence we could to present it as $x_k = k\Delta x, \Delta x = f(k)$, in that case $f(k) = ln(k)$, or $f(k) = exp(k)$ or some other function. It could give us a new interpolation methods.

$$Dedicated - to - Valerie - Oprya$$

with love

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