Maxwell's equations, Einstein's Special Theory of Relativity and the Lorentz transformation: a historical review

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Abstract: The article shows that a lot of misunderstanding and confusion has been originated after Einstein published his article about the STR. Fundamental errors, manipulative mathematics and an unfounded believe in the transformation formulas of the STR caused this chaos.

Maxwell's equations
The remarkable property of the Maxwell equations is that they don't show any relation with the source of an EM-field, notwithstanding the fact that one can calculate, by means of the wave equations for the electrical and magnetic field, the propagation velocity of such a field. The outcome for vacuum is the well-known propagation velocity $c=1/\sqrt{\varepsilon_0 \mu_0}$. So in first instance it looks like $c$ doesn’t have or need a reference. The background for this apparent non-physical property is the following.
Maxwell lived from 1831 until 1879, which is the last part of the, centuries long, period during which the standard opinion was that the intangible medium ether “filled” the whole universe. It was assumed to be in absolute rest and be necessary for the propagation of EM-fields. Moreover, the opinion was that the propagation velocity of an EM-field would only be $c$ w.r.t. this ether, so independent of the velocity of the source w.r.t. this ether.

N.B. Sound has exactly the same property regarding its propagation velocity in, for example, water and air. It is independent of the velocity of its source w.r.t. that medium.

In 1818 Fresnel deduced the mathematical expression for the velocity of light in a moving tangible medium. This expression shows the so-called drag coefficient of Fresnel. He deduced this expression, based on the same opinion as described above, assuming that the ether is not any more at absolute rest w.r.t. a moving transparent tangible medium. As he described it: the ether is dragged by a small fraction of the velocity between ether and transparent medium. Fizeau experimentally proved the correctness of Fresnel's equation in 1851. So why would any physicist doubt about this ether!

Michelson and Morley lived in the same century as Maxwell, Fresnel and Fizeau. They developed their well-known experiment based on the just described opinion in the period 1880-1890. The consequence of this opinion namely is that the earth also has to have a measurable velocity in the universe w.r.t. this ether.

No wonder that the whole physical world was astonished by the negative outcome of their experiment!
**Einstein’s Special Theory of Relativity**

Einstein abandoned the ether together with the introduction of his STR. But at the same moment he made a fundamental error: he replaced the ether by an equivalent system “in rest”. The quotes have been added by him. The explanations in his article of 1905 clearly show that his system “in rest” has exactly the same properties as the ether he rejected.

By manipulating his mathematics at one point, he succeeded in presenting consistent transformation formulas. ‘Consistent’ regarding the property of these formulas that, after transforming the coordinates x and t from system S to System S’, the original coordinates in S are found applying again the same formulas with the appropriate variables. The mentioned manipulation concerns the variable x, being a constant in S at the start of his mathematics, defined as \( x = \beta c t \) at the point of manipulation and again this constant after that. See: Ref [2].

**The chaos**

At a certain moment the community of physicists seemingly discovered Einstein’s fundamental error, regarding his system “in rest”. Instead of publishing this error loud and clear, it slinky changed Einstein’s hypothesis. The system “in rest” changed to a much more idiotic hypothesis: the velocity of light in vacuum from that moment on became c w.r.t. whatever reference / inertial system!

I found one article that tries to present the mathematics that lead to that same ‘consistent’ transformation formulas as Einstein generated, notwithstanding the fact that he based them on a fundamental different hypothesis! It is written by Professor S. Bentvelsen, in Dutch.  

It shows even more manipulative mathematics than the one of Einstein!

**Lorentz transformation**

One of the frequently used arguments that the STR is a correct theory is the claim that the Maxwell equations are invariant under the Lorentz transformation. The background is the following.

What is meant here with the Lorentz transformation is specifically and only the **velocity**

transformation: 

\[ v' = (v_x - v_R) / (1 - v_R v_x / c^2) \]

with \( v_R \), the relative velocity of S and S’, \( v_x \) the velocity of x in S and \( v_x' \) the velocity of \( x' \) in S.

It can be deduced simply from the coordinate and time transformation as produced by Einstein. See the attachment.

That means that this velocity transformation is not deduced by Lorentz and thus has a misleading name. Lorentz developed his own time and coordinate transformation, most times called contraction formulas. They look like Einstein’s formulas, but are significant different. See Ref [1].

Einstein: \( t' = \beta (t - v_R x / c^2) \) and \( x' = \beta (x - v_R t) \)

Lorentz: \( t' = \beta (t/\beta - v_R x / c^2) \) and \( x' = \beta x \).

The velocity transformation shows that if \( v_x \) in S equals c, \( v_x' \) in S’ also equals c.

However this property is in contradiction with the second hypothesis of Einstein:

*Each light beam moves in the coordinate system “in rest” with the specific velocity c, independent of the fact whether this light beam has been emitted by a body at rest or a moving body.*

Further on Einstein calculates the time periods that light needs to travel along rod \( r_{AB} \) as
\( r_{AB} / (c-v_R) \) on the way forth, resp. \( r_{AB} / (c+v_R) \) on the way back \( r_{AB} \). The rod moves with velocity \( v_R \) w.r.t. the system "in rest". The light is reflected at point B of this rod. The rod is effectively an inertial system w.r.t. which the velocity of light should be \( c \), according to the velocity transformation, but is clearly not the fact in Einstein’s theory.

The property of the velocity transformation must have led to the situation that Einstein’s original hypothesis has been changed to the one that says that the velocity of light is \( c \) w.r.t. whatever reference and is used to claim that the Maxwell equations are invariant under this transformation. Given the fundamental contradiction of the 'Lorentz' transformation with Einstein’s hypothesis, together with the presented manipulative mathematics, it must be concluded that the claim is worthless.

**Conclusion:**
The Maxwell equations don’t have any relation with the STR, nor with the 'Lorentz' transformation and can perfectly be applied without these by taking the proven theory that the propagation velocity of light in vacuum is only \( c \) w.r.t. its source.

References:
[1]: Electromagnetic Phenomena in a system moving with a velocity smaller than any velocity than that of light. Prof. H.A. Lorentz, Royal Dutch Academy of Sciences 6 (1904), pages 809-831

**Attachment: Derivation of the velocity transformation**

Einstein’s transformation formulas are:

\[
x' = \beta(x-v_R t)
\]

\[
t' = \beta(t-v_R x/c^2)
\]

\[
\beta = 1/\sqrt{1-(v_R/c)^2}
\]

with \( x \) and \( t \) defined in S, \( x' \) and \( t' \) defined in S' and with \( v_R \) the relative velocity of S and S'. Replacing \( x \) by \( v_x t \), representing a velocity \( v_x \) of \( x \) in S, results in:

\[
x' = \beta(v_x - v_R) t
\]

\[
t' = \beta(1 - v_R v_x / c^2) t
\]

Defining \( v_x' \) as the velocity of \( x' \) in S’ results in:

\[
v_x' = x'/t' = (v_x - v_R) / (1 - v_R v_x / c^2)
\]

If \( v_x = c \) then \( v_x' \) is also \( c \), being fundamentally in contradiction with the hypothesis on which Einstein based his STR: the velocity of light is only \( c \) in the system "in rest"!

**N.B.**
Einstein’s mathematical manipulation is that he used in only one equation: \( x = c t \).
In the rest of his theory \( x \) is a constant in S.
Without this manipulation he would never have found his transformation formulas!