

## Fundamental of Rate Theory for CMNS

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**Abstract:** The condensed matter nuclear reactions (CMNR) are thought to happen for trapped H(D) particles within some chemical (electro-magnetic) potential well with finite life time. As the life time is much longer than the collision time of two-body interaction of free particles, CMNR reaction rates are significantly (on the order of 19-20 in magnitude) enhanced if we compare with estimated reactions rates by the two-body collision formula. The basis of CMNR rate theory is reviewed in this paper by extracting essence of the TSC theory tools developed until now. Derivation of Fermi's golden rule with nuclear optical potential, rate formulas by Born-Oppenheimer wave function separation, estimation of bracket integral of inter-nuclear strong interaction rate, estimation of time dependent barrier penetration probability by the HMEQPET method for dynamic D(H)-cluster condensation/collapse process, and DD fusion power levels as functions of inter-nuclear d-d distance and effective existing (life) time are given. A DD fusion power level of 10 kW/mol-dd-pairs is possible for a 1 pm inter-nuclear d-d distance with 10 ato-seconds life time. The level of 2.8 nano-mol 4D/TSC formations/sec may release 10 kW neutron-free heat-power with  $^4\text{He}$  ash.

**Keywords:** condensed matter nuclear reaction, trapped D(H) particles, finite life time, enhanced fusion rate, Fermi's golden rule, time-dependent barrier factor, 1pm d-d distance, 10 kW heat power

### 1. Introduction

This paper reviews the fundamental aspect of rate theory for the development of cold fusion (CMNR) theories [1-15]. The basic nuclear reaction rate formulas for trapped state D(H) particles in condensed matter and some examples on estimated reaction rates and heat-power levels are shown.

Despite the importance of reaction rate estimation in modeled CMNS (condensed matter nuclear science) theories, only a few authors have treated nuclear reaction rates properly (see many theory papers in list [1]). Some theories have borrowed rate formulas of two body collision process which is for the case of nuclear reactions by the random free particle motion as in plasma and gas phase or beam-target interactions. Intrinsically the CMNS nuclear reactions (CMNRs) should happen between trapped particles of protium and deuterium (H or D) in negative potential wells organized by the ordering of condensed matter such as periodic lattice, mesoscopic nano-particle and surface deformed/fractal conditions. Such trapped H(D)-particles should have finite lifetime or existing time in negative potential wells and are keeping mutual inter-nuclear distances for finite time-intervals before nuclear fusion reactions. Therefore the application of collision cross section formulas which treat instantaneous

interactions (within about  $1.0E-23$  s collision time, for instance) is not proper for our CMNS nuclear reactions which may happen within much longer time interval of dynamic (or time-dependent pseudo-trapped state) and hence rate of CMNR is drastically enhanced if we compare with the estimated reaction rate by the two-body collision formulas. We need therefore to use formulas based on the Fermi's first golden rule for rate estimation, due to the finite life time of trapped particles [4]. This paper recalls the procedure and formulas for the fundamental of rate theory for CMNS, as has been used in the TSC theory development [2-15]. The outline is as follows:

- 1) Why two-body collision theory cannot be used for nuclear reactions of CMNS is explained. It is shown that the collision rate theory underestimates drastically (on the 19-20 orders of magnitude) fusion rates of trapped deuterons in condensed matter, in comparison with more rigorous rates by the Fermi's first golden rule for trapped deuterons.
- 2) The quantum mechanical (QM) formulation of rate calculation by the Fermi's first golden rule is given with nuclear optical potential and chemical Coulomb potential. The scaling of imaginary part component of nuclear optical potential is revisited [8].
- 3) Formulas with EM (electro-magnetic) trapping potential well and nuclear interaction potential are derived by the adiabatic approximation, namely the Born-Oppenheimer approach [4-8].
- 4) Collapsing condensation of D(H)-cluster is needed to have observable nuclear reaction rates by overcoming Coulombic repulsion, namely drastically enhancing barrier penetration probability of trapped particles in condensed matter. Time-dependent analysis is needed here. The HMEQPET method calculation of barrier factors as a function of d-d inter-nuclear distance is re-shown after the references [4-7].
- 5) It is shown that inter-nuclear d-d distance of D-cluster/molecule should be smaller than 1.0 pm to have d-d fusion rate with experimentally claimed heat-power levels. This issue is explicitly given first time in this paper, though it has been given implicitly in the references [5, 6] already.
- 6) Some example model/calculations are given for D(H)-cluster condensation/collapse processes and final nuclear products (but details in the references [4-7]).

This paper only treats the initial state interaction of three steps (initial, intermediate and final states) of nuclear reaction process, since the fusion reaction rates for explaining the observed anomalous heat levels of cold fusion (CMNR) experiments are to be determined by the initial state interactions. The studies on the intermediate and final states interactions (reaction products) are given, for the TSC theory, in the references [8-10, 13]. For modeling the sites of D(H)-cluster, such as 4D/TS, in condensed matters are given in the references [4, 12, 14, 15]. The overall background of theoretical modeling for condensed cluster fusion is given in the reference [4] and very briefly in the references [3, 7].

As the motivation of this paper was for a tutorial review of fundamental tools used in the TSC theory development [2-15], description of mathematical derivations of used formulas is not repeated in the text description and instead the slide-type figures are used to explain the process of quantitative analyses in easily visible way. To understand mathematical derivations of used physical formulas, please visit original papers [5, 6, 8, 11, 13].

## 2. Brief Review of Two-Body Collision Rate Formulas

The some detail explanation of QM derivation of cross section and reaction rate for two-body free particle collision process is summarized in the reference [8].

A very basic image of two body collision in random motion of free particles is given in Fig.1.

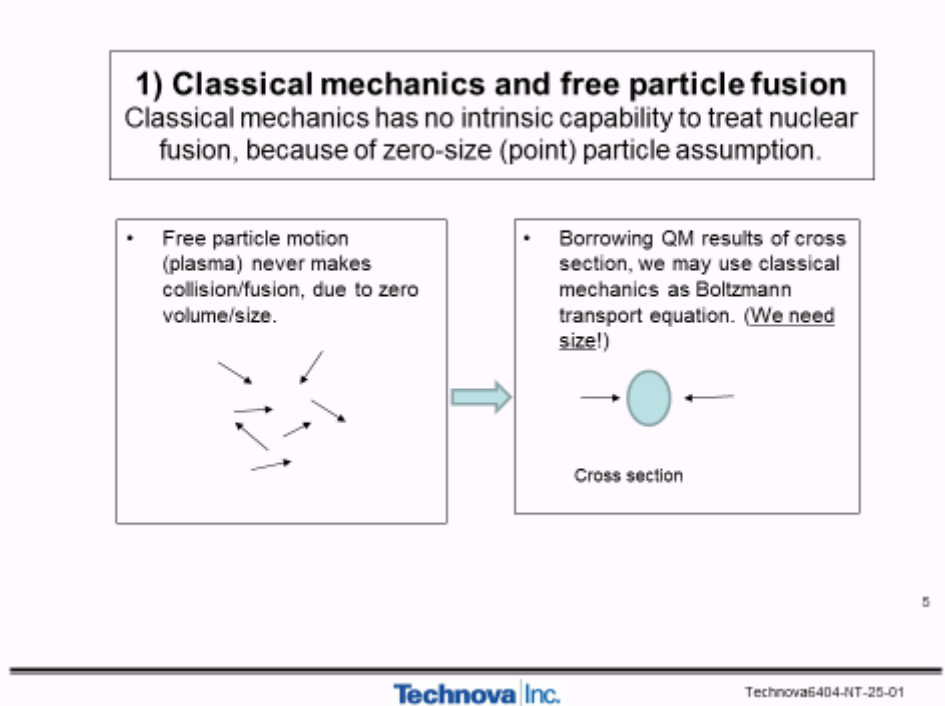


Fig.1: A simple image of two-body collision and cross section for random motion

As shown in the left-inserted figure of Fig.1, no collision ever happens in random motion of Newtonian classical mechanics particles, since the particles sizes are regarded as zero, namely points. To cause any collision in classical mechanics, we need to assume finite sizes of particles. Borrowing the simple image of nucleus as closely packed nucleons (neutrons and protons for virtual instantaneous QCD oscillating states inside nucleus by exchanging quarks with gluons) to be  $R_0 A^{1/3}$  in fm size, we can estimate the geometrical cross section of collision in the view of classical mechanics particle motion. Here  $R_0$  is 1.2 fm and  $A$  is the mass number (sum of nucleon numbers,  $A = N + Z$ ,  $N$  for neutrons and  $Z$  for protons). We know that the geometrical cross sections do not agree with measured data (mostly by accelerator beam with target) and we have had to accumulate evaluated data with comparison of measured data and QM cross sections formulated by the squared collision amplitude of scattered particle QM wave function (see a simple derivation of QM cross section formulas in reference [8]). Usually, the QM collision cross section becomes much larger than the classical geometrical cross section mainly due to the spread wave nature of QM particle wave functions. The second important aspect is that the classical geometrical cross section does not have capability to treat nuclear fusion process which should make the sub-atomic (inter-nuclear) level transition of structure in quark-gluon interaction. The sub-atomic

level transition is made by the strong force interaction or the weak force interaction, resultantly to cause mass deficit (defect) that contributes to the nuclear excited state of intermediate compound state and

**Fusion Rate Formula for Collision Process<sup>4)</sup>  
of two free particles**

- $T = \langle \Psi_f | H_{int} | \Psi_i \rangle$   
= <Initial State Interaction>  
x<Intermediate Compound State>  
x<Final State Interaction>
- **Cross Section**  $\sim T^2 \rho(E')$
- $\rho(E')$ : final state density
- **Reaction-Rate( $\sigma v$ ):**  $(4\pi^2/h)vT^2 \rho(E')$
- <Initial> = <El. EM Int><Strong Int>
- <Final>=BRs to Irreversible Decays

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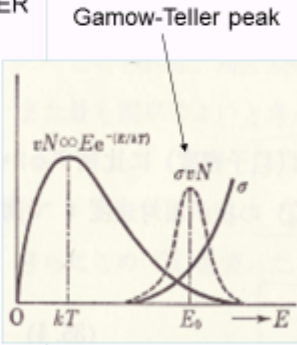

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Fig.2: A summary slide of two-body collision cross sections and transition matrix

### The Case of Hot Plasma Fusion<sup>7)</sup>

- **Confinement of high kinetic energy deuterons (plasma) in a very large scale (torus) room**, like Tokamak magnetic field confinement.
- Average kinetic energy of d-d (or d-t) reaction for ITER is aimed to be about **10keV** ( $E_k$ ).
- **<Macroscopic Fusion Rate> =  $\langle Nd(E_k)^2 v \sigma_{dd}(E_k) \rangle$**   
 $N_d$ : deuteron density,  $v$ : relative d-d velocity,  
 $\sigma_{dd} = (S(E_k)/E_k) \exp(-\Gamma_{dd})$ : fusion cross section,  
 $E_k$ : relative d-d kinetic energy  
 $\Gamma_{dd}$ : Gamow factor
- Free particle motion and collision process:  
 $N_d(E_k) = N \cdot (E_k/T^2) \exp(-E_k/T)$ : Maxwell-Boltzmann distr.

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Fig.3: Fusion reaction rate formulas for hot plasma fusion

the released particle kinetic energies after the final state strong/strong, strong/electro-magnetic or strong/weak force-driven transitions [10]. As the fact, we need to use fully QM formalisms to know

reaction rates of ‘hither-to-unknown’ nuclear reactions in condensed matter.

In Figures 2 and 3, we show brief summary of QM two-body cross section formulas and fusion rate estimation for deuteron electron plasma. Cross section is given to be proportional to the square of transition matrix  $T$  for the free particle collision process (Fig.2). The averaged fusion rate of free-gas like motion of D-gas and D-plasma is given by effective Gamow peak for the Maxwellian average of two-body d-d cross section which is a product of astrophysical  $S$ -factor (intrinsic inter-nuclear strong interaction factor) and barrier factor (QM tunneling probability for Coulombic interaction between deuterons) [3]. The screening effect (decreasing repulsive force between deuterons) by existing negative charge of electrons in gas and plasma state is usually estimated by using the Thomas-Fermi free gas model. Since the screened potential by using the Thomas-Fermi model does not have negative well of particle trapping, its application to the D(H) particles trapped in some kind of chemical potential of condensed matter becomes a very rough approximation, as we show in the following of this paper.

Fig.4 shows the image of charge screening effect comparing the bare Coulomb potential, the Thomas-Fermi free gas potential and the pseudo-chemical potential of dynamic condensed matter.

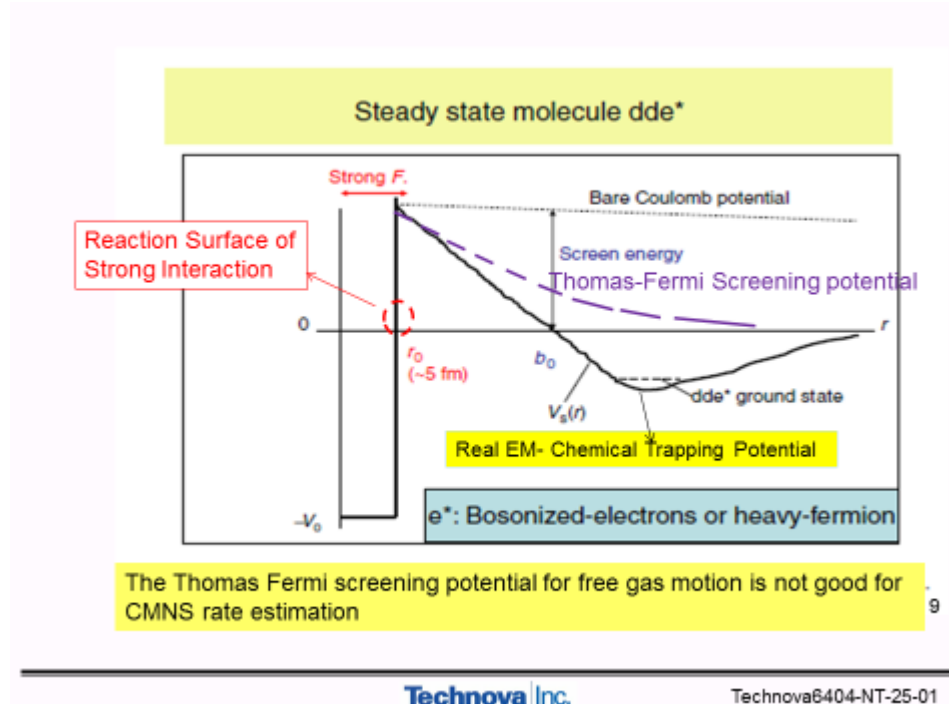


Fig.4: Comparison of charge-screening effects to cause nuclear fusion enhancement between the bare Coulomb potential, the Thomas-Fermi potential and the pseudo-potential of d-d pair chemical trapping in condensed matter. To estimate fusion rate, we estimate Gamow integral of barrier factor from  $b_0$  point to fusion interaction surface distance (about 5 fm of d-d distance).

In intuition, we may have understanding that the pseudo-potential of D(H) particle trapping in (dynamic conditions of, as explaining later) condensed matter may give much more enhanced screening effect than the Thomas-Fermi model of screened potential effect.

### 3. Rate Formulas for Trapped D(H) Particles in Condensed Matter

A simple image of QM state of d-d pair trapped in a chemical potential well is illustrated in Fig.5.

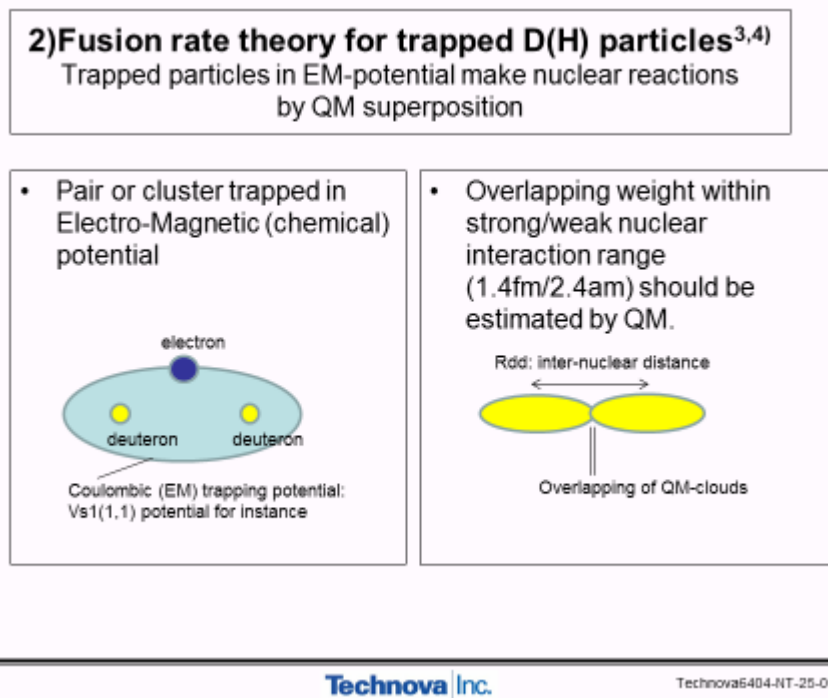


Fig.5: A simple image of trapped d-d pair in QM chemical potential well (left figure) and a very small domain of strong interaction for overlapping QM particle density functions for d-d pair (right figure)

A simple case of d-d pair trapped in a chemical potential is a  $D_2^+$  ion, namely a d-e-d three body system, as illustrated by the left figure of Fig.5. The chemical trapping potential is  $V_{s1}(1,1)$  as given in references [5, 6, 8, 12]. For treating the very fast dynamic process of TSC condensation/collapse motion, time variation of TSC trapping potentials [5, 6, 12] were approximated (substituted) with effective-mass variation of  $V_{s1}(m, Z)$  pseudo potentials [6]. Here the  $V_{s1}(m, Z)$  pseudo potential is defined as the d-d (or p-p) pair trapping potential of EQPET (electronic quasi-particle expansion theory[8])  $dde^*$  molecule with effective mass  $m$  and effective charge  $Z$ . The selection of  $(m, Z)$  is not necessarily a set of integers, but continuous numerical values can be used to follow the continuous variation of elapsed time of condensation. Examples of  $V_{s1}(m, Z)$  pseudo potentials used for time-dependent condensation motion will be shown in the next section. The  $V_{s1}(1,1)$  potential is the real one of  $D_2^+$  molecule which is stable in nature; that means its infinite life time at ground state ( $R_{dd}(gs)$  is 138 pm with electron mean kinetic energy 13.6 eV [5]). However, the pseudo d-d trapping potential for  $dde^*$  pseudo molecule is a kind of pure mathematical tool to describe an adiabatically discretized time-dependent trapping potential with very small time mesh [5, 6]; this means the very fast time variation of TSC-like cluster condensation motion can be numerically simulated by using  $V_{s1}(m, Z)$  potential with the empirically related substitution from time-step to discretized- $m$  with as practically small as possible time mesh [5, 6]. In

every discretized time-mesh at every time-step, we need to calculate the weight of superposed wave functions of two deuterons (as shown in the right figure of Fig.5) that is the time-dependent barrier penetration probability for getting to the interaction surface ( $R_{dd} = \text{ca. } 5\text{fm}$ ) of strong nuclear interaction domain.

**Nuclear Optical Potential for reaction rate formula and QM probability density balance equation**

- Forward Equation:
 

$$i\hbar \frac{\partial \Psi}{\partial t} = \left[ -\frac{\hbar^2}{2M} \nabla^2 + V + iW \right] \Psi$$
- Adjoint Equation:
 

$$-i\hbar \frac{\partial \Psi^*}{\partial t} = \left[ -\frac{\hbar^2}{2M} \nabla^2 + V - iW \right] \Psi^*$$
- $\Psi^*_{x(1)} - \Psi_{x(2)}$ :
 

$$i\hbar \left( \Psi^* \frac{\partial \Psi}{\partial t} + \Psi \frac{\partial \Psi^*}{\partial t} \right) = i\hbar \frac{\partial \Psi \Psi^*}{\partial t} = i\hbar \frac{\partial \rho}{\partial t}$$

$$i\hbar \frac{\partial \rho}{\partial t} = -\frac{\hbar^2}{2M} [\Psi^* \nabla^2 \Psi - \Psi \nabla^2 \Psi^*] + i[2W\rho] = -i\hbar \text{div} \vec{j} + i[2W\rho]$$

Particles absorption by nuclear fusion

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Fig.6: Derivation of the balance equation of particle-pair QM density function by using the forward and the backward Schroedinger equations with nuclear optical potentials

**Fusion Rate Formula by Fermi's Golden Rule**

$$\langle \text{FusionRate} \rangle = \frac{2}{\hbar} \langle \Psi_f | W(r) | \Psi_i \rangle$$

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi + [V_{nr}(r) + iW(r)] \Psi + V_c(r) \Psi = E \Psi$$

Nuclear Optical Potential

Coulomb Potential

$$\Psi(r) = \Psi_n(r) \cdot \Psi_c(r)$$

Inter-nuclear wave function

EM Field wave function

**Born-Oppenheimer Approximation**

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Fig.7: The fusion rate formula by Fermi's first golden rule

Now we have to be cautious about the fact that the superposition of two deuteron wave functions does not mean fusion reaction yet. To cause a fusion reaction event, there should occur the transition of sub-atomic (inter-nuclear) structure of superposed two deuteron's admixture; namely the fusion transition from the d+d pair to the intermediate transit compound particle  ${}^4\text{He}^*(\text{Ex})$  should occur. The interaction Hamiltonian for this transition is written by the component proportional to the imaginary part  $W(r)$  of nuclear optical potential [8]  $V(r) + iW(r)$ . To derive a QM averaged fusion transition rate, we shall treat the balance equation of particle QM density function  $\rho(r) = \Psi \Psi^*$  with the forward wave function  $\Psi$  and the adjoint (backward) wave function  $\Psi^*$  of particle-pair, as shown in Fig.6.

The right-hand most term of the bottom equation of particle QM density balance corresponds to the rate of nuclear fusion [4, 8]. The fusion rate formula is therefore become the top most equation of Fig.7.

Our real fusion rate calculation has to be done time-dependently by using every substituted EQPET pseudo potential  $Vs1(m, Z)$  at every small time step. In Fig.8, we illustrate an image of time-dependent pseudo potential for EQPET molecule  $dde^*(m, Z)$ . In numerical calculations for ground state eigenvalue search of  $dde^*(m, Z)$  with  $Vs1(m, Z)$  potential, we applied the variational principle [6, 11] with Gaussian d-d pair wave function (this is approximation for wave function form). As the elapse of condensing time of TSC-like symmetric D(H)-cluster becomes to approach the collapse time, pseudo potential at the discretized small time step becomes deeper and virtual deuteron kinetic energy becomes larger, by reflecting in the Heisenberg Uncertainty Principle (HUP) [11].

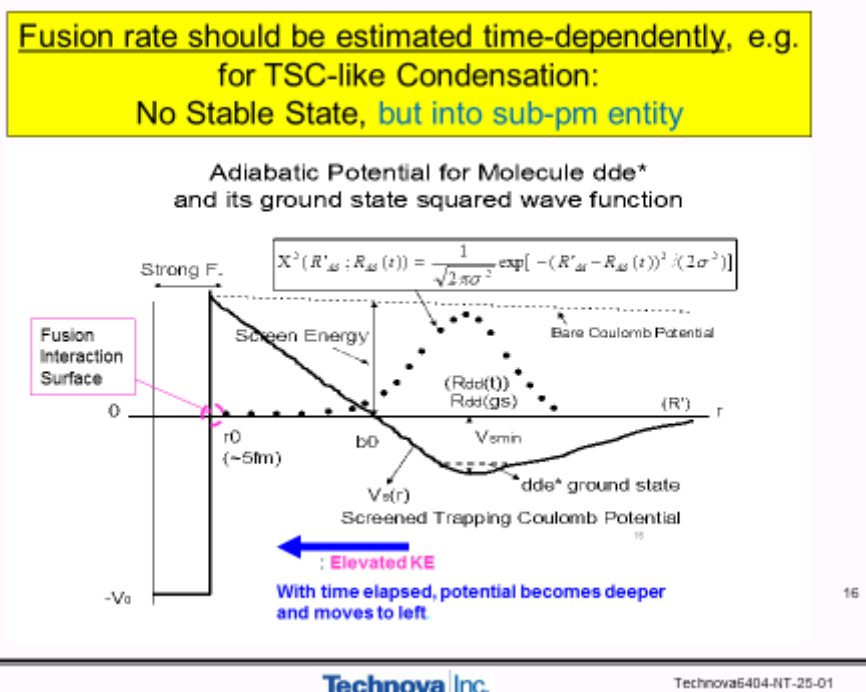


Fig.8: Illustration of time-dependent adiabatically approximated pseudo potential of d-d pair trapping

In the HMEQPET method, we adiabatically treat a quasi-steady state of EQPET pseudo-particle  $dde^*$



within a very small discretized time interval; step-dependent time-steps of 0.05 fs to 0.0000001 fs were used in actual calculations by the QM Langevin calculation code [6, 12]. We needed to choose as very small time mesh as 0.0000001 fs when condensation motion approaches very closely to the collapse time  $t_c$ . By applying the adiabatic HMEQPET method for simulation, however, we make it possible to treat only steady state Schroedinger equations for every  $dde^*(m, 2)$  pseudo-molecule.

A steady state Schroedinger equation, with two components of nuclear optical potential and chemical (or EM) d-d pair trapping pseudo potential, can be again adiabatically solved by the Born-Oppenheimer approximation, as shown in Fig.9.

**Adiabatic QM Equations for Initial State Interaction**

**Inter-Nuclear QM Schroedinger Equation:**

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi_n(r) + [V_{nr}(r) + iW(r)] \Psi_n(r) = E_n \Psi_n(r)$$

**Outer-Nuclear QM Schroedinger Equation for Electro-Magnetic Field:**

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi_c(r) + V_c(r) \Psi_c(r) = E_c \Psi_c(r)$$

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Fig.9: Born-Oppenheimer approximation is applied for separately calculating the inter-nuclear wave function and the outer-nuclear (chemical or EM-field) wave function of d-d pair

Then, the fusion rate formula is converted to be more practical one as shown in Fig.10. We calculate the inter-nuclear fusion rate, namely the bracket integral of the imaginary part  $W(r)$  of nuclear optical potential, being independent of outer chemical EM (Coulombic) field. The second term of bracket integral corresponds to the so called barrier penetration factor (or QM tunneling probability), for which we can approximately use the WKB Gamow integral treatment [8]. We should be cautious to the domain of bracket integrals for both cases. The domain of bracket integral must be within the domain of strong (or weak) nuclear interaction range as approximately given with the  $4\pi R_0^2 \lambda_\pi$ , where  $\lambda_\pi$  is Compton wave length of pion (for strong interaction, 1.4fm) or weak W- boson (weak interaction).

Fusion Rate Formula by Fermi's First Golden Rule with Born-Oppenheimer Approximation

Inter-nuclear fusion rate      Barrier Factor

$$\langle FusionRate \rangle = \frac{2}{\hbar} \langle \Psi_{nf} | W(r) | \Psi_{ni} \rangle_{Vn} \cdot \langle \Psi_{cf} | \Psi_{ci} \rangle_{Vn}$$

$Vn \approx 4\pi R_n^2 \hat{\lambda}_\pi$  : Effective Volume of Nuclear Strong (Weak) Interaction Domain

$\hat{\lambda}_\pi$  : Compton wave length of pion (1.4 fm) (weak boson: 2.5 am)

$R_n$  : Radius of Interaction surface of strong (weak) force exchange

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Fig.10: Practical fusion rate formula by Born-Oppenheimer approximation for the Fermi's golden rule

Estimation of inter-nuclear fusion rates for 2D, DT, 3D and 4D fusion was empirically done [6, 8].

**<W> value Estimation**

• Using  $Tn - (PEF)^{\frac{1}{2}}$  in S-value analysis:

Cluster	<W> (MeV)
DD	0.008
DT	0.115
3D	1.93
4D	62.0

$\langle W \rangle = \langle \psi_{nf} | W(R) | \psi_{ni} \rangle$

Inter-nuclear wave functions are governed by the real part of optical potential  $V(R)$ , having Woods-Saxon type well shape that makes inter-nuclear wave functions very localized near around  $V(R)$ . Therefore,  $\langle W \rangle$  becomes approximated by the surface sticking at  $R=R_0$ .

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Fig. 11: Estimated bracket integral of  $W(r)$  for two-body and multi-body fusions

Estimation of the barrier factors is given in the next section.

#### 4. Effect of Life Time of Trapped D(H)-Particles in Dynamic Pseudo-Potentials

According to our past studies on dynamic motions of D(H)-molecules and clusters [4, 5, 12], we have recognized that feasibly enhanced DD fusion rates as large as experimentally claimed anomalous heat-powers do not exist for steady molecules of  $D_2^+$ ,  $D_2$ ,  $D_3^+$  or their transient time-dependent motion of formation. However, we have found that three-dimensionally symmetric (or orthogonally coupled electron wave-function and deuterons wave-function in the three dimensional space) D(H) cluster such as 4D(H)/TSC, 6D(H)/RDC, 8D(H)/RDC may have no steady ground states but may make ultimate-condensation/collapse-type one-through motion time-dependently [5, 6, 12]. The case of 4D/TSC condensation/collapse motion in 4 sub-divided steps is copied in Fig.12. Here, TSC is the tetrahedral symmetric condensate and the RDC is the Rhombic dodecahedron condensate, respectively.

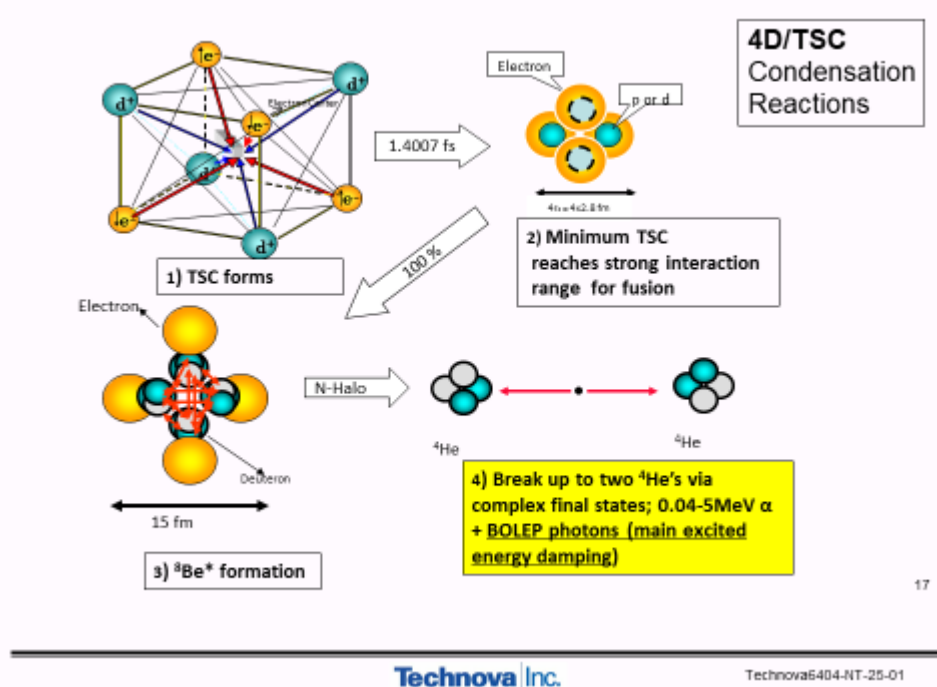
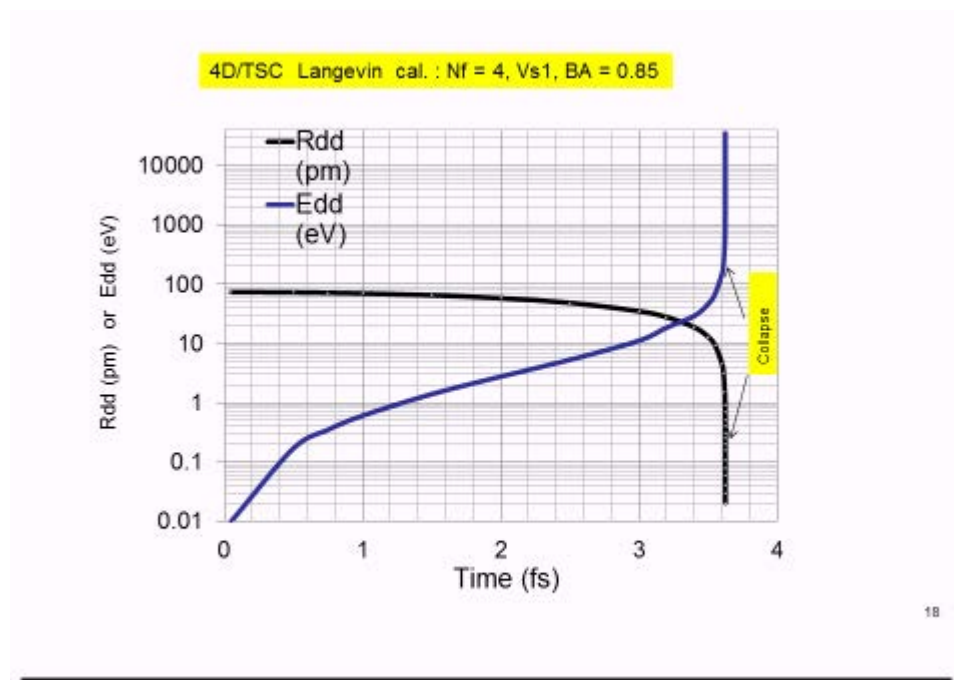


Fig.12: The 4 steps of 4D/TSC condensation; 1) TS formation, 2) collapse, 3)  ${}^4Be^*$  intermediate compound, and 4) the final state decay processes to produce low energy helium and BOLEP photons

The corresponding QM Langevin code calculation with  $Vs1(1,1)$  potential for every d-d pair in 4D/TSC is shown in Fig. 13. To see the detail behavior near to the collapse time, the log-log plot of inter-nuclear d-d distance  $R_{dd}$  and mutual kinetic energy of d-d pair is given in Fig.14, by reversing the flow of time. We have estimated already that 100% 4D fusion will take place until the condensed  $R_{dd}$  reaches at 20 fm [6]. However, the following discussions will be made for arguing the possibly enhanced d-d fusion rate by this kind of dynamic condensation/collapse motion, for understanding general feature of fusion rate enhancement as a function of quasi-life time (effective existing time interval) of d-d pair and also as a function of second parameter of d-d distance  $R_{dd}$ .

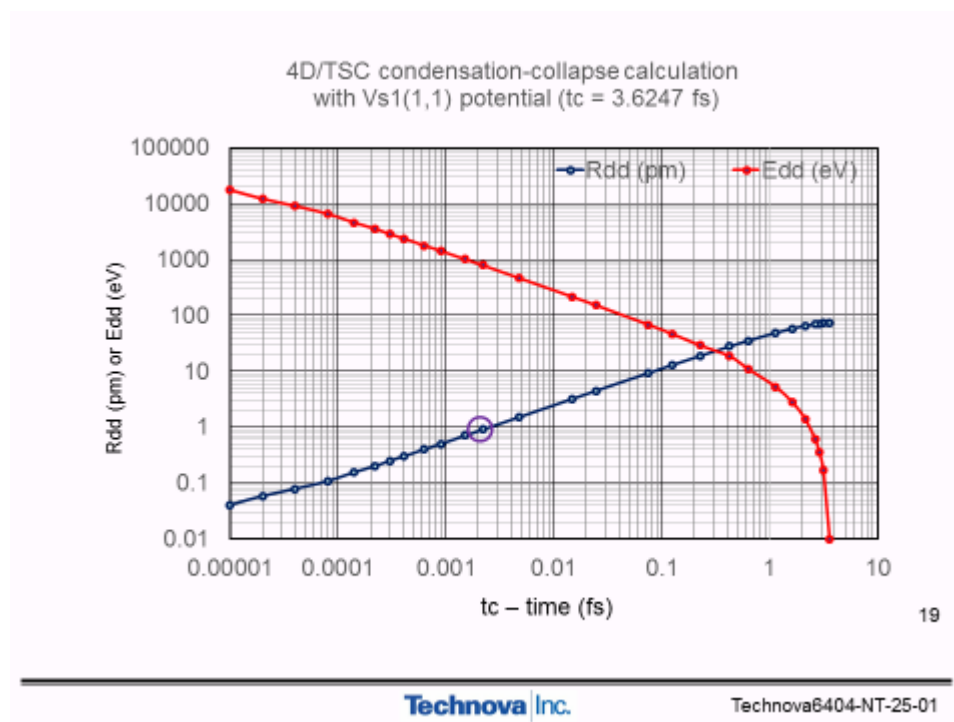


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Fig.13: 4D/TSC condensation/collapse motion calculated by the QM Langevin code [12]



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Fig.14: The log-log plot of 4D/TSC condensation as a function of reversed time direction from the collapsed time  $t_c$  (3.6247 fs for this case) to the starting point of 4D/TS formation time ( $t = 0$ ). When  $R_{dd}$  reaches at 35 fm, relative deuteron kinetic energy of d-d pair is increased to the level of ca. 20 keV.

Just for example, we put a marker for  $R_{dd} = 1$  pm region (see red circle in Fig.14). The condensation motion is changing continuously. However, according to our HMEQPET adiabatic analysis with very small discretized time meshes, we see the effective existing time of d-d pair with diminished size of 1 pm  $R_{dd}$  distance has about 2 as (ato second =  $1.0E-18$  second) “pseudo life time” of  $dde^*(m, 2)$  pseudo EQPET molecule. What we want to know now is the rough measure of DD fusion power per one molar d-d pairs ( $6.023E+23$  pairs). To know this, we need to estimate the value of barrier factor for the  $R_{dd} = 1$  pm adiabatic state.

The calculation process of time dependent barrier factor is shown in Fig.15.

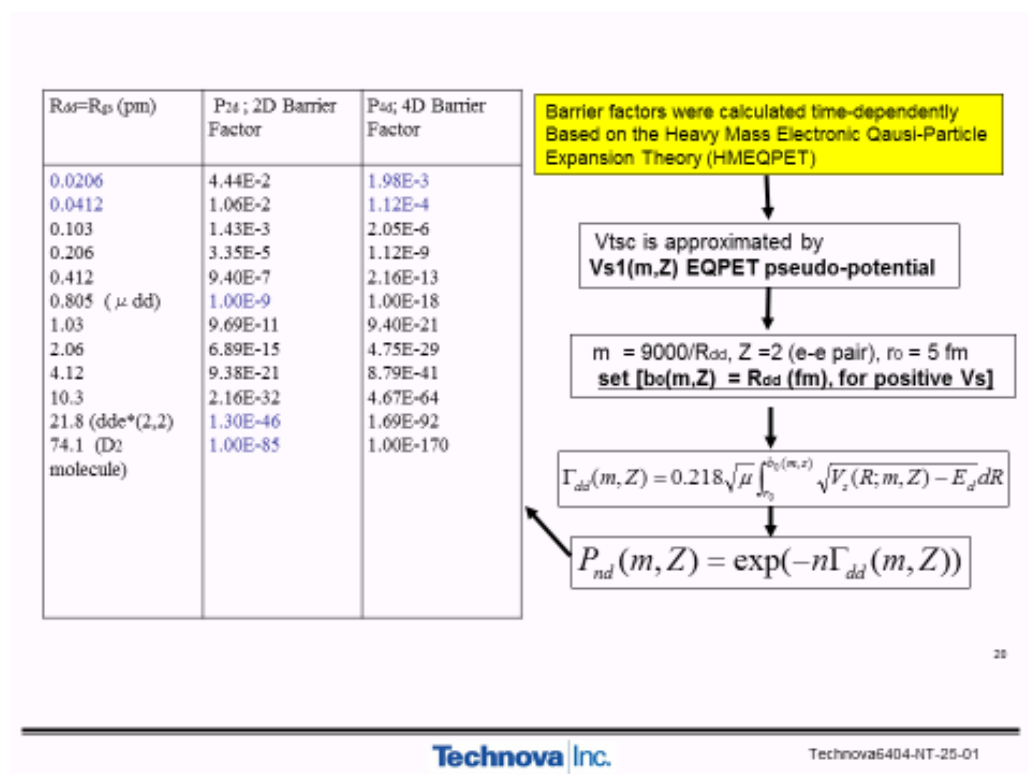


Fig.15: The HMEQPET method of calculating time-dependent barrier factors as a function of  $R_{dd}$  or condensation time

Implement of Gamow integral calculation with a pseudo potential  $Vs1(m, Z)$  needs a fix-up technique to erase negative values of potential well region inside the square root of Gamow integral. Before showing that, some examples of used pseudo potentials are shown in Fig.16. A  $dde^*$  adiabatic pseudo molecule with the ground state d-d distance 100 pm with -36 eV negative well is given by the  $Vs1(0.4, 2)$  EQPET potential. A  $dde^*$  adiabatic pseudo molecule with the ground state d-d distance 10 pm with -394 eV negative well is given by the  $Vs1(4,2)$  EQPET potential. A  $dde^*$  adiabatic pseudo molecule with the ground state d-d distance 1 pm with -4000 eV negative well is given by the  $Vs1(44, 2)$  EQPET potential. Here  $Z=2$  is assumed and fixed. As discussed in detail in the reference [8], anti-parallel spin coupling with reversed momenta for two electrons locating on a square domain of a  $D_2$  molecule-like

face of 4D(H)/TSC under condensation/collapse motion were thought to make a kind of “transient Cooper pair” on each square face of TSC cube. The pseudo electronic quasi-particle  $e^*(2, 2)$  is of approximate “bosonized state” of electrons in the Platonic symmetry of condensate [8, 6].

To erase negative values in the square root of Gamow integral, we have introduced the fix-up technique as illustrated in Fig.17.

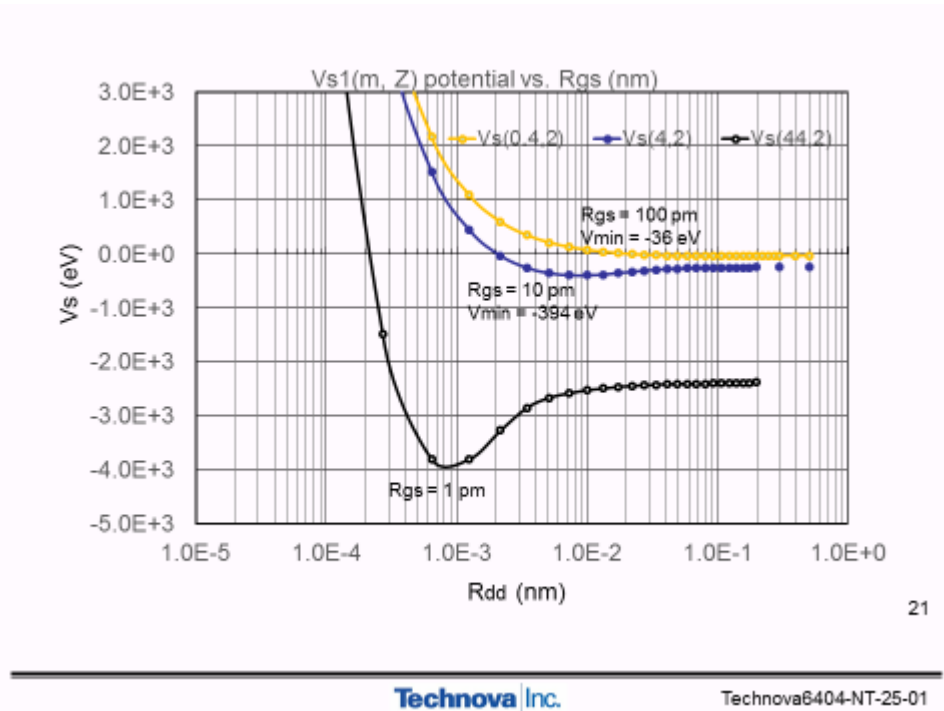


Fig.16: Some examples of used pseudo potentials to approximate real dynamic cluster trapping potentials as  $V_{tsc}$

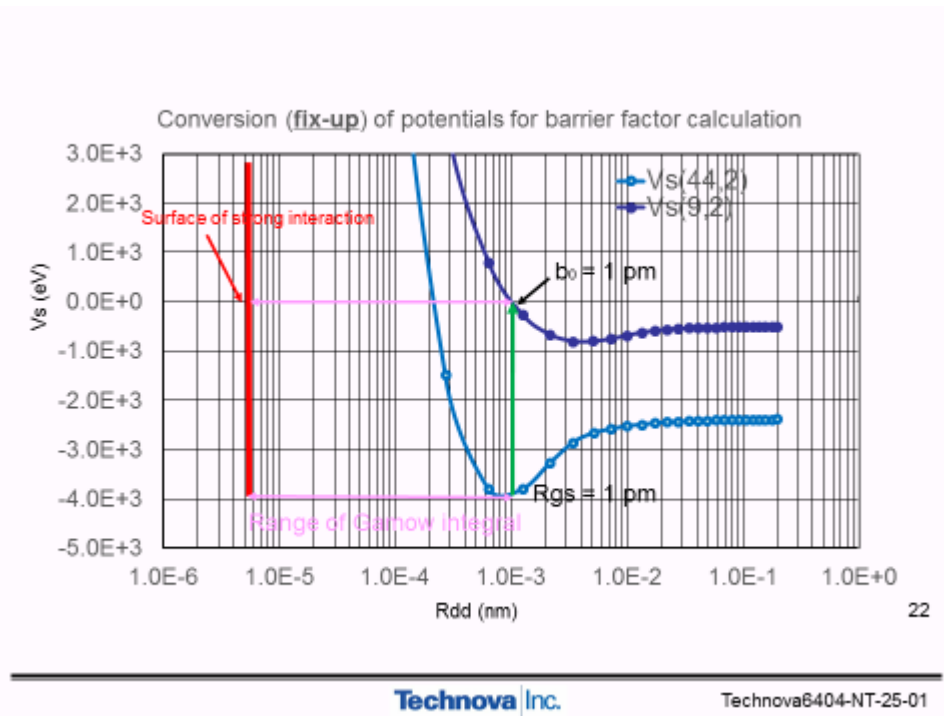
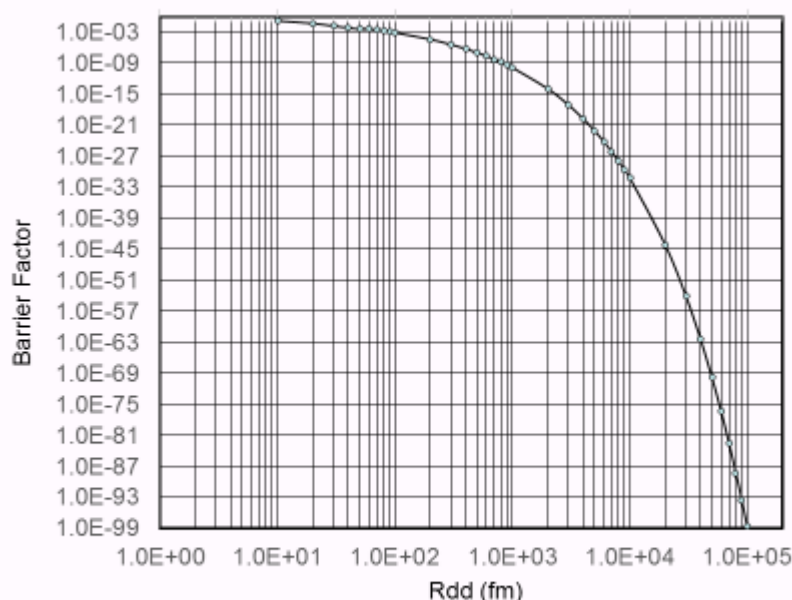


Fig.17: Fix-up technique to erase negative values in the root of Gamow integral

Instead of carrying out Gamow integration with the deep negative well of  $Vs1(44, 2)$  pseudo potential for  $R_{dd} = 1$  pm dynamic pseudo molecule state, we use  $Vs1(9, 2)$  pseudo potential to implement Gamow integral calculation from the  $b_0$  point (potential zero crossing point) of 1 pm to the strong interaction surface range about 5 fm. We have one-to-one empirical relation for the  $R_{dd}(gs)$  to  $b_0$ -corresponding potential for time-varying  $m$  value [6].

Calculated barrier factors by the HMEQPET method are plotted in Fig.18 as a function of d-d distance.



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Fig.18: Calculated barrier factors as a function of inter-nuclear d-d distance of pseudo dde\* molecule

To see observable heat level by d-d fusions,  
inter-nuclear d-d distance should become  
Less than  $R_{dd}$  of 1 pm with feasible life-time:

at  $R_{dd} = 1$  pm and life-time =  $3.0E-18$  s (TSC-cal):

<Fusion yield> = (barrier-factor) x <Wh/2> x <life-time>

=  $(1.0E-10) \times (1.5E19) \times (3.0E-18)$

=  $4.5E-9$  (fusion/d-d)

Order of Macroscopic Fusion Rate:

$6.023E23$  (d-d/s) x  $4.5E-9$

= ca.  $3E15$  f/s/mol-dd = ca. **3 kW/mol-dd**

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Fig.19: Slide to show how to calculate heat power of DD fusion with parameters of life-time and d-d distance

The estimation of DD fusion power level as a binary-variable function of  $R_{dd}$  and life-time was done by a calculation procedure as shown by the slide of Fig.19. Illustration of DD fusion power levels vs. parameters of life time and d-d distance by graph is given in Fig.20.

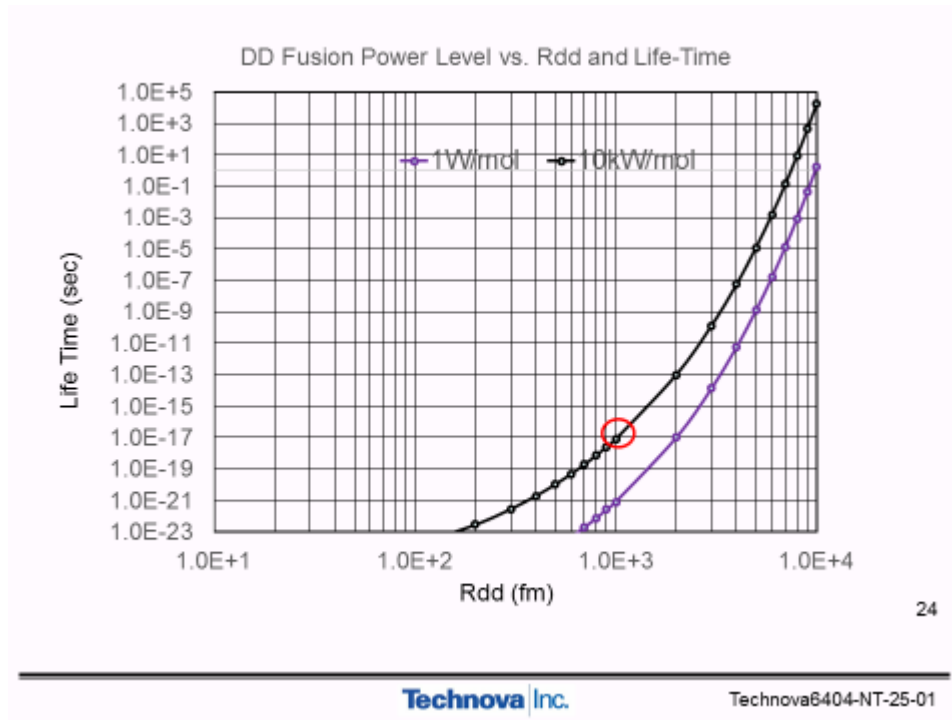


Fig.20: Scaling curve of heat-power level per one molar d-d pairs as functions of life-time and d-d distance

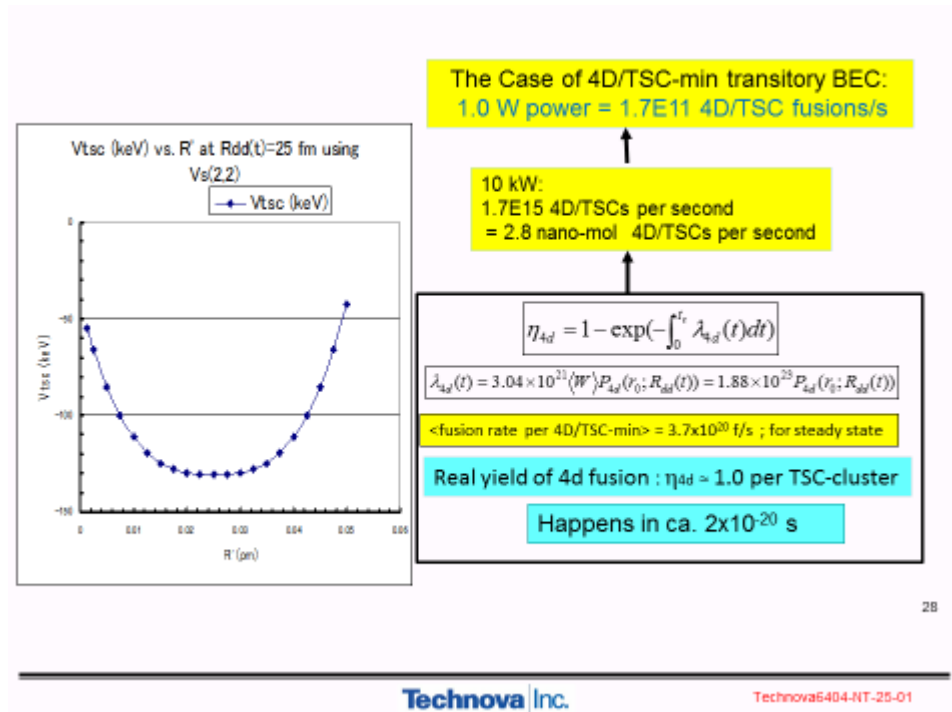


Fig.21: Heat power level of 4D/TSC fusion reaches 10 kW level by 2.8 nano-mol TS formation per second



## Collision Rate Formula UNDERESTIMATES fusion rate of steady molecule/cluster

Cluster	R <sub>dd</sub> = R <sub>gs</sub> (pm)	Barrier Factor	Steady Cluster d-d Fusion Rate (f/s)	Steady Cluster 4d Fusion rate (f/s)	Fusion Rate for d- d collision formula (f/s)
D <sub>2</sub>	74.1	1.0E-85	2.4E-66		3.6E-86 (4.0E-72)*
dde*(2,2)	21.8	1.3E-46	3.2E-27		1.0E-46 (1.0E-31)*
ddμ	0.805	1.0E-9	2.4E+10		1.5E-9 (1.0E+8)*
4D/TSC- minimum	0.021	1.98E-3		3.7E+20	

\* Frequency of d-d pair oscillation by QM-Langevin calculation was considered. 25

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Table-1: Calculated fusion rates by trapped state DD pair, compared with those by two-body collision theory

We see that with  $R_{dd} = 1\text{pm}$  and  $10$  as  $(1.0\text{E}-17\text{ s})$  life time the DD fusion rate reaches  $10\text{ kW/mol-dd}$  level and we meet the similar conditions of experimentally claimed anomalous excess heat-power.

However, according to the author's CMNR/Cold-Fusion studies until now, DD fusion cannot explain the claimed  ${}^4\text{He}$  production per  $24\text{ MeV}$  energy release without hard gamma rays and neutrons [3-11].

The author has been asserting that 4D/TSC fusion (see Fig.12) may be the real source of  ${}^4\text{He}/23.8\text{MeV}$  nuclear events in CMNR experiments using deuterium loading systems. In this case, as shown in Fig.21,  $10\text{ kW}$  heat-power level may be attained by the TS (tetrahedral symmetry) state formation rate of  $2.8$  nano-mol per second in condensed matter. The rate theory for TS state formation is of pure solid-state, surface physics and chemical problems, and is yet to be studied [4], although we have proposed some models of TS formation on catalytic surface SNH (sub-nano hole) [7, 14, 15], lattice defects (vacancies) and non-linear QM oscillation of deuterons trapped in a global mesoscopic potential [15] of nano-metal-particle.

Finally, we show Table-1 of calculated fusion rates for steady dde\* molecules, including muonic dd molecule and 4D/TSC-final-collapsed state. Evidently, the collision theory drastically underestimates fusion rates of trapped DD pairs in negative chemical potential well which is reasonable for condensed matter nuclear reactions. The underestimation is very large order as 19-20 orders of magnitude. When we make corrections with ground state oscillation frequency (ca.  $1.0\text{E}14\text{ Hz}$  for  $\text{D}_2$  molecule), the underestimation is still large as ca. 6 orders of magnitude.

We have already pointed out that a dynamically diminished size ca.  $1\text{ pm}$  of d-d pair with life time of ca.  $10\text{ ato s}$  will meet the feasible condition that can explain the excess-heat-power levels of

experimental claims by many authors (see papers by Fleischmann-Pons, M. Miles, M. McKubre, V. Violante, Y. Arata, and so on in the list of [1]). Looking at Fig.20 again, we see that the  $R_{dd} = 10$  pm state of diminished d-d size requires a life time of ca.  $1.0E+4$  seconds, namely 2.78 hours for the quasi-steady (isomeric) dde\* molecule. Such an isomeric d-d quasi-molecule state has never been observed in condensed matter science (CMS), since the transition from the ca.  $R_{dd} = 100$  pm state of usually considered d-d pair state in PdD-like lattice to the  $R_{dd} = 10$  pm state with long life time will emit ca. 360 eV photons to be observable, according to referring pseudo ground state levels of trapped d-d pair in  $Vs1(0.4, 2)$  and  $Vs1(4, 2)$  pseudo potentials as shown in Fig.16. Of course, deuterium gas, namely  $D_2$  molecule with  $R_{dd} = 74.1$  pm has no possibility to observe any DD fusion event in our life time (at longest 110 years), because its fusion rate is so small as  $(2.4E-66) \times (6.023E+23) = 1.44E-42$  f/s/mol. The muonic DD fusion has a great possibility in principle, as 100% d-d fusion yield takes place in ca. 100 ps after a muonic dd molecule is formed [6, 12], but intense generation of muonic flux by high energy accelerator is not economical. Until now, the author has found a plausible/feasible mechanisms of high power level CMNR events only in the dynamic condensation/collapse motion of such D(H)-clusters as TSC and RDC [4].

## 5. Conclusions

For the condensed matter nuclear reaction (CMNR) as known as cold fusion which has been studied in emerging Condensed Matter Nuclear Science (CNMS), we need to take life-time of D(H)s in chemical trapping potential well into account properly to formulate reaction rate formulas.

We cannot confidently use formulas for two-body collision theory for CMNR, as they drastically underestimate fusion rate as we have shown in this work. The underestimation is as huge as 19-20 orders of magnitudes. We understand that, because of much longer life-time (or existing time) of diminished size d-d pair trapped dynamically/transiently in condensed matter, a d-d pair can have very enhanced chance to cause stochastic DD fusion.

Fermi's first golden rule should be used for rate formulas for CMNR. We have shown a summary review of reaction rate formulas for CMNR based on the Fermi's golden rule. For applying the Fermi's golden rule to dynamic/time-dependent D(H)-cluster condensation/collapse motion, we have developed some mathematical tools such as the QM-Langevin code for analysis of time-dependent condensation motion and the HMEQPET code for time-dependent QM tunneling probability estimation with eigenvalue search of pseudo potential trapping (the variational principle is applied).

Feasibly enhanced cluster fusions may happen only for collapse states of dynamic condensation: 4D/TSC, 6D/RDC, 8D/RDC for examples, as studied in our recent works.

Therefore, we need time-dependent fusion rate calculation to estimate real fusion yield per D(H)-condensing-cluster formation. We have done some elaborations for the deuterium (D) related CMNRs based on the strong nuclear reaction mechanisms, and extension (not described in this paper) for protium

(H) related CMNRs.

If inter-nuclear d-d distance for condensate becomes less than 1.0 pm, we can expect reaction rates with ‘observed heat-power levels’. The inter-nuclear d-d distance of 10 pm state of d-d pair in condensed matter lattice dynamics is not short enough to observe experimentally claimed excess-heat power level.

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