An Epistemic Relativity Theory

Ramzi Suleiman

University of Haifa

Al Quds University

Please address all correspondence to Dr. Ramzi Suleiman, University of Haifa, Haifa 31509, Israel. Email: suleiman@psy.haifa.ac.il, Mobile: 972-(0)50-5474- 215.

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Abstract

Einstein's theory of special relativity (SR) theory dictates, as a *force majeure*, an ontic view, according to which relativity is a true state of nature. For example, the theory's solution to the famous twin paradox prescribes the "traveling" twin returns truly and verifiably younger than the "staying" twin, thereby implying the “traveling” twin returns to the future.

Here I propose an *epistemic* view of relativity, according to which relativity results from a *difference in knowledge about Nature* between observers who are in motion relative to each other. Utilizing this postulation, together with SR's first axiom, I construct an epistemic relativity theory (ER) for the dynamics of moving bodies in inertial systems. I show that ER solves the twin paradox such that the rejoining twins age equally. Tests of the theory's time transformation show that although the theory contradicts the Lorentz invariance principle (LI), it accords nicely with the results of the famous Michelson-Morley experiment and the well-known Frisch and Smith muon decay experiment. More important, the theory accounts for the linear Sagnac effect, which disobeys starkly both LI and SR. It also precisely predicts the results of several recent neutrino velocity experiments conducted by OPERA and other collaborations. I explain why the experimental setups of the linear Sagnac and the neutrino velocity experiments qualify them as stringent falsification tests for the LI and SR.

In another paper cited here I show that application of ER to cosmology and astrophysics, proves quite potent in providing plausible answers to key cosmological questions, including dark matter and dark energy, and the evolutionary timeline of the nucleosynthesis of chemical elements.

The theory's prediction concerning the kinetic energy density as a function of velocity reveals that for approaching bodies, the dependence of energy on velocity is similar, although not identical, to the one prescribed by SR. However, for departing bodies, the theory predicts kinetic energy density will monotonically increase with velocity up to a maximal value, after which it will monotonically decrease more steeply, reaching zero at a velocity equal to the velocity of light. Most strikingly, the breakdown of the acknowledged relationship between energy and velocity is predicted to occur at velocity $v = \Phi c$, where c is the velocity of light and Φ is the famous Golden Ratio (≈ 0.618). This result echoes a recent finding demonstrating that the quantum criticality of cobalt niobate atoms exhibits Golden Ratio symmetry. Another peculiar Golden Ratio symmetry of the predicted energy density indicates that at v = - Φ c (approaching bodies), the energy density is equal to 1+ Φ (= $\frac{1}{\Phi}$) ≈ 1.618. No less surprising, we find the maximal energy density at v = Φ c, relative to the rest-frame energy density is $\Phi^5 \approx 0.09016994$, which precisely equals Hardy's probability of entanglement. The emergence of these results from a deterministic relativity theory based on SR's first axiom plus an axiom specifying light as information carrier is puzzling. One possible explanation is to attribute their emergence to mere coincidence. However, given the many confirmed predictions of the theory, including in cosmology, and the unlikelihood of such a coincidence actually happening, this explanation is highly improbable. Another possibility worth pursuing is that ER reveals more than one thread for a possible connection between an epistemic view of relativity and quantum mechanics, with the Golden Ratio symmetry playing a key role.

Keywords: Relativity, Ontic, Epistemic, Michelson-Morley, Muon decay, Sagnac Effect, neutrino velocity, OPERA, Golden Ratio

1. Introduction and Propositions

A fundamental and still debated question about the nature of quantum mechanics is whether the wave function is a state of nature (*ψ*-ontic) or a state of knowledge about Nature (*ψ*-epistemic). The ontic view of quantum states has a long history in the interpretation of quantum mechanics. Schrödinger initially interpreted the quantum state as a tangible physical wave, and this view continues to be the one most physicists and philosophers of science adopt [e.g., 1-3]. The epistemic view, although less common than the ontic view, also has a long tradition and has recently gained more supporters [e.g., 4-7]. Fuchs [8] notes that Einstein was the first to unambiguously state why the quantum state should be viewed as information about reality and not as ontic, one-to-one correspondence with reality. Einstein's view about the incompleteness of quantum theory, most known from the famous EPR paper [9], has been expressed more vigilantly in his correspondence with Schrödinger and with other scientists. In a letter to P. S. Epstein, 10 November 1945, Einstein wrote: "I incline to the opinion that the wave function does not (completely) describe what is real, but only a (to us) empirically accessible maximal knowledge regarding that which really exists" (A. Einstein, [10], quoted in [11]).

Remarkably, a parallel question regarding the nature of Einstein's theories of relativity has never been seriously raised, at least not in theory. The neglect is most probably due to the fact that special and general relativity theories, dictate, as a *force majeure*, an ontic view of relativity. For example, the solutions to the twin paradox in both theories prescribe the "traveling" twin returns truly and verifiably younger than the "staying" twin, thereby implying the "traveling" twin returns to the future.

Because Einstein's model of the universe has been hardly challenged, no one seems to have found any utility in challenging the ontic view of relativity. Nonetheless, one should be allowed to ask what would be the aftermath of adopting an epistemic view of relativity (R-epistemic) rather than the standard view of relativity as a state of nature (R-ontic). The epistemic approach the present paper takes could be stated as follows:

Relativity is the result of a difference in knowledge about Nature between observers who are in movement relative to each other.

Given the above definition, two questions arise: Why should any difference arise in knowledge between the two observers? And how would each observer know what knowledge the other observer acquires about some measurement? To answer these questions, we must define the way in which information about some physical measurement transfers from one observer to another. For rendering the proposed approach useful, I postulate that *information translated from one observer to another is carried by light or another wave of equal velocity*, c.

Under this specification, it becomes obvious that any information about time intervals, spatial distances, and so forth between observers who are at rest relative to each other will be kept unchanged. However, information translated between two observers who are in motion relative motion will suffer certain modulation. To summarize, the proposed epistemic theory of relativity (ER) rests on the two following axioms:

1. The laws of physics are the same in all inertial frames of reference (SR's first axiom).

2. All translations of information from one frame of reference to another are carried by light or electromagnetic waves of equal velocity (information-carrier axiom).

2. Epistemic Relativity Transformations

Let us first inspect the modulation of the time interval of a given event taking place in a frame of reference F', while departing with constant velocity *v* with respect to an observer in another frame of reference *F* (see Figure 1).

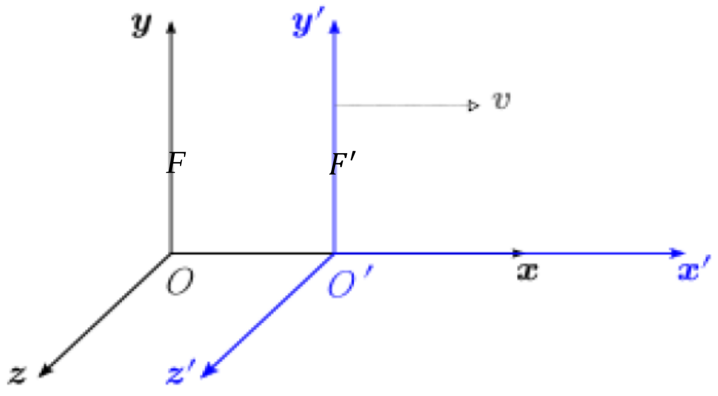


Figure 1: Observers in two reference frames moving with velocity *v* with respect to each other.

Assume that at the "moving" frame F', a certain event started exactly at the time of departure (t=t'= 0). Assume that promptly at the termination of the event, the observer in the "moving" frame measures the time (denote it by t'), and with no delay, sends a signal to the observer in the "staying" frame in order to indicate the termination of the event. Also assume that with the arrival of the signal, the "staying" observer promptly registers his/her termination time, denoted by *t*. The termination time *t*, registered by the "staying" observer, is equal to *t'*, the termination time registered by the "moving" observer, plus the time the wave signal took to cross the distance *x* in *F* that the "moving" observer has crossed relative to the "staying" observer, from the moment the event started (*t*= 0) until it ended (t=*t*). The time in *F* that the wave signal took to cross the distance *x* is equal to $\frac{x}{c}$, where *c* is the velocity of the wave signal relative to the "staying" observer.

Thus, the termination time *t* registered by the "staying" observer is equal to

$$t = t' + \frac{x}{c} \ . \qquad .... (1)$$

On the other hand, the distance x is equal to

$$x = v\, t \ . \qquad .... (2)$$

Substituting *x* from (2) in (1), we get

$$t = t' + \frac{vt}{c} \ , \qquad \text{.... (3)}$$

or

$$\left(\frac{t}{t'}\right)_{ER} = \frac{1}{1-\frac{v}{c}} = \frac{1}{1-\beta} \ , \qquad \text{.... (4)}$$

where $\beta = \frac{v}{c}$.

Notably, eq. (4) is fundamentally different from the famous prediction of SR [12]:

$$\left(\frac{t}{t'}\right)_{SR} = \gamma = \frac{1}{\sqrt[2]{1-\left(\frac{v}{c}\right)^2}} \ . \qquad \text{.... (5)}$$

Figure 2 depicts the comparison between the two predictions.

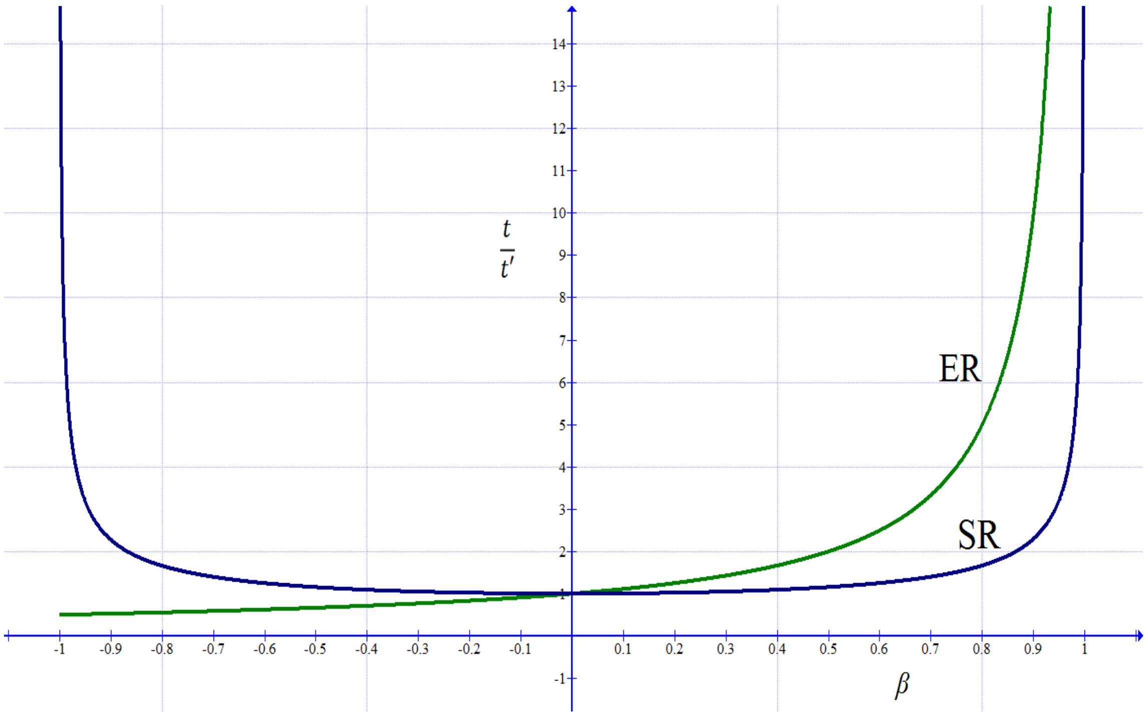


Figure 2: Time transformation in ER and SR

As the figure shows, for positive β values (F'departing from F), the predicted pattern of dependence of $\frac{t}{t'}$ on β is similar to the one predicted by SR, although the time dilation predicted by information modulation $(\frac{t}{t'})_{ER}$ is larger than the time dilation predicted by SR. Conversely, for negative β values (F' approaching F), the relative time $\frac{t}{t'}$ as a function of β depicts time contraction and not time dilation, as predicted by SR.

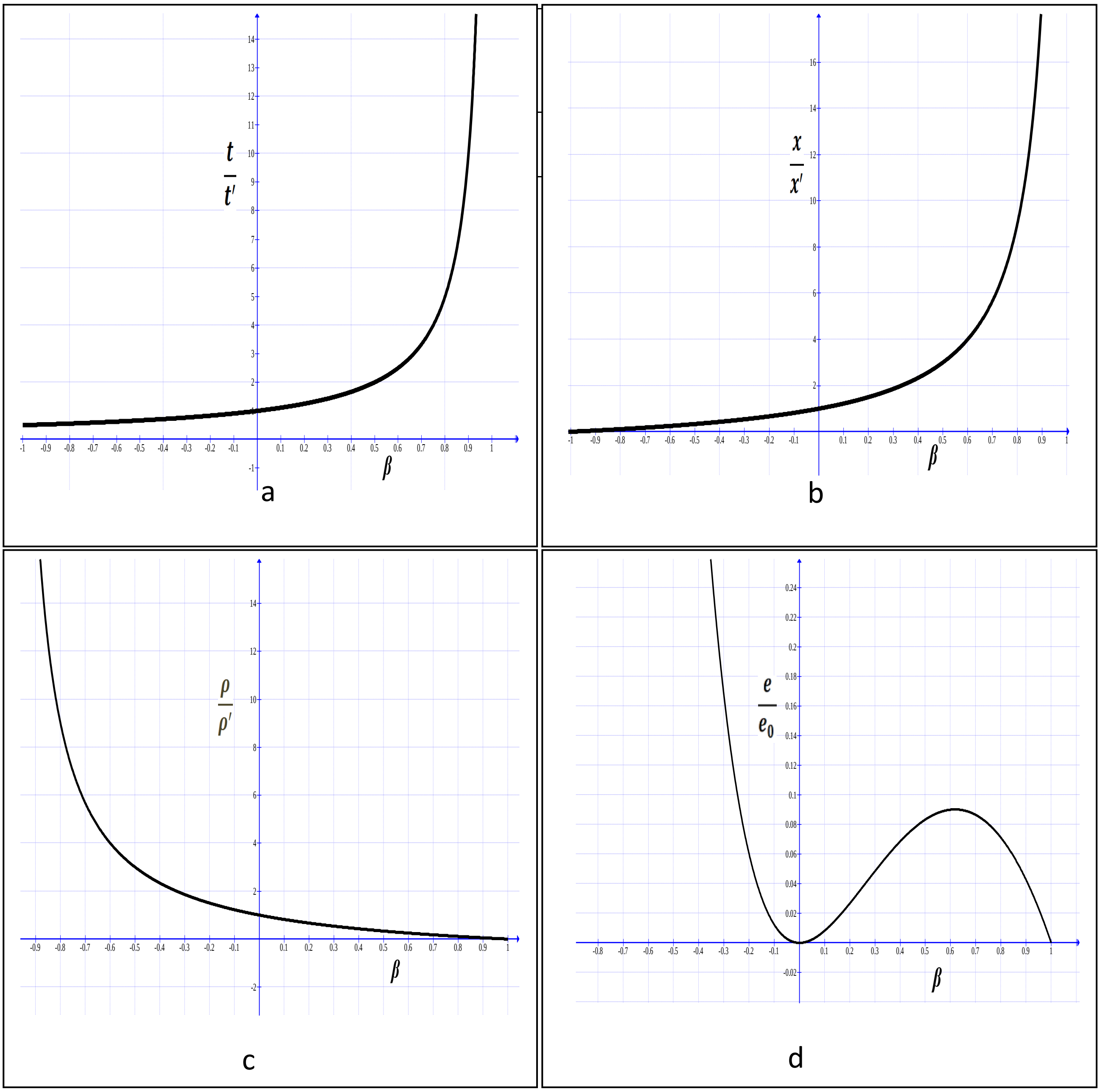
Note that equation (4) closely resembles the Doppler formula [13, 14]. The Doppler Formula predicts a red- or blueshift depending on whether the wave source is departing or approaching the observer. Similarly, eq. (4) predicts that the time duration of an event on a moving frame is dilated or contracted depending on whether the frame is departing or approaching the observer.

Appendix A details the derivations of ER's transformations for distance and for mass and kinetic energy densities. Table 1 and Figures 3a-3d depict all the transformations. I shall discuss here in more details the transformations of time and kinetic energy and contrast their predictions with relevant experimental findings**.**

Table 1

Epistemic Relativity Transformations

Physical Term	**Relativistic Expression**
Time	$\frac{t}{t'} = \frac{1}{1-\beta}$
Distance	$\frac{x}{x'} = \frac{1+\beta}{1-\beta}$
Mass density	$\frac{\rho}{\rho'} = \frac{1-\beta}{1+\beta}$
Kinetic energy density	$\frac{1}{2}\rho' \, c^2 \frac{1-\beta}{1+\beta} \beta^2$



Figures 3a-3d: ER's time, distance, mass density, and energy transformations

3. Empirical Tests of the Time Transformation

Empirical testing of the time transformation derived above is crucial for testing ER theory. Because eq. (4) contradicts the Lorentz invariance (LI) principle, any test that includes both departure *and* arrival of moving bodies, from and to frames of reference should be conclusive in deciding between ER (which abandons LI), and SR (which rests completely on it). To clarify, any test with the above characteristic qualifies as a stringent comparative test between the two theories, such that if one theory is confirmed, the second is automatically falsified. Two types of experiments that I will address hereafter possess such a qualification: the linear Sagnac effect experiments [15, 16] and the neutrino velocity experiments conducted at CERN and elsewhere [17-22]. I will show that all the investigated experiments confirm ER, yielding accurate predictions of the reported results. I will also show the theory predicts, to the same degree of success, the results of the Michelson-Morley experiment [23] and the muon time dilation reported by Frisch & Smith [24].

3.1 Linear Sagnac Effect

First, a brief introduction: The Sagnac effect, named after its discoverer in 1913 [25], has been replicated in many experiments (for reviews, see [26-30]). The Sagnac effect has well-known and crucial applications in navigation [26] and in fiber-optic gyroscopes (FOGs) [31-35]. In the Sagnac effect, two light beams, sent clockwise and counterclockwise around a closed path on a rotating disk, take different time intervals to travel the path. For a circular path of radius *R*, the time difference can be represented as $\Delta t = \frac{2\,v\,l}{c^2}$, where $v=\omega R$ and l is the circumference of the circle ($l=2\pi R$). Today, FOGs have become highly sensitive detectors measuring rotational motion in navigation. In the GPS system, the speed of light relative to a rotating frame is corrected by $\pm\ \omega\ R$, where ω is the radial velocity of the rotating frame and *R* is the rotation radius. A plus/minus signs is used depending on whether the rotating frame is approaching the light source or departing from it, respectively.

Many physicists claim that because the Sagnac effect involved a radial motion, it does not contradict SR and that it should be treated in the framework of general relativity [36, 37].

However, Wang at al. [15, 16] strongly refute this claim in two well-designed experiments that show unambiguously that an identical Sagnac effect appearing in uniform radial motion occurs in linear inertial motion. For example, Wang et al. [15] tested the travel-time difference between two counter-propagating light beams in uniformly moving fiber. Contrary to the LI principle and to the prediction of SR, their findings revealed a travel-time difference of $\frac{2v\,\Delta l}{c^2}$, where Δl is the length of the fiber segment moving with the source and detector at a v, whether the segment was moving uniformly or circularly. This finding in itself should have raised serious questions about the validity of the LI principle and SR. If the Sagnac effect can be produced in linear uniform motion, then the claim that it is a characteristic of radial motion is simply incorrect. Because the rules SR apply to linear uniform motion, the only conclusion is that SR is incorrect. Strikingly, the unrefuted detection of a linear Sagnac effect and its diametrical contradiction with SR has hardly been debated.

Applying ER to the linear Sagnac experiment yields the following difference between the arrival times of the two light beams:

$$\Delta t = \frac{\Delta l}{c-v} - \frac{\Delta l}{c+v} = \frac{2v\,\Delta l}{(c-v)(c+v)} = \frac{2v\,\Delta l}{c^2 - v^2} \approx \frac{2v\,\Delta l}{c^2}\ , \qquad \ldots (6)$$

which is in agreement with the analysis and results reported in [15].

3.2 Predictions of neutrino velocities

First, a brief introduction: In 2011, the OPERA collaboration at CERN announced that neutrinos had travelled faster than light [38]. The reported anticipation time was 60.7 ±6.9 (stat.)± 7.4 (sys.) ns, and the relative neutrino velocity was $\frac{v-c}{c}$ = (5.1 ±2.9) ×10^{-5}. The excitement that swept physicists and laymen concerning the possibility that a new era was "knocking on physics doors" waned a few months later, after OPERA reported the discovery of hardware malfunctions in the GPS system, which resulted in a critical measurement error. After accounting for the error, the anticipation time was only $(2.7 \pm 3.1\ (\text{stat.}) + {}^{+3.8}_{-2.8}$ (sys.)) ×10^{-6}, with corresponding $\frac{v-c}{c}$ = 2.67 x 10^{-6} [17]. Since then, the OPERA and several collaborations, including ICARUS, LVD, and Borexino, have replicated the "null" result [18-21]. The only "faster-than-light" result of which I am aware was reported in 2007 by the MINOS collaboration [22], who reported an early anticipation time of 126 ± 32 (stat.) ± 64 (sys.) ns (C.L. = 68%), with corresponding $\frac{v-c}{c}$ = 5.1±2.9 (stat.+sys.)×10^{-5}. However, the high statistical and system errors reported by MINOS impede the validity of the above quoted result.

Surprisingly, despite the vast body of theoretical research on the topic [e.g., 39-46], no one has attempted to apply SR to deriving point predictions of the $\frac{v-c}{c}$ results reported by OPERA and other collaborations that replicated the "null" result. I will demonstrate that ER precisely predicts six experimental results reported by OPERA, MINOS, ICARUS, LVD, and Borexino collaborations (see Table 2). Given the stark contradiction between the time transformations of ER and SR, one must expect that any attempt to test SR's predictions for the above-mentioned experiments will fail colossally.

To derive the term $\frac{v-c}{c}$ for a typical neutrino-velocity experiment, consider a neutrino that travels a distance *d* from a source (e.g., at CERN) and arrives at a detector (e.g., at Gran Sasso). According to ER, such an experiment includes *three* frames: the neutrino frame *F*, the source frame F', and the detector frame F''. *F* is departing from F' with velocity *v* and approaching F'' with velocity – *v*. F' and $F''$are at rest relative to each other. Using eq. (3), we can write

$$\Delta t_S = \frac{\Delta t}{1-\frac{v}{c}}, \qquad \text{…… (7)}$$

and

$$\Delta t_D = \frac{\Delta t}{1-\frac{-v}{c}} = \frac{\Delta t}{1+\frac{v}{c}}, \qquad \text{…… (8)}$$

where *v* is the neutrino velocity, *c* is the velocity of light. Δt, Δt_S, and Δt_D are the times, as measured in frames *F* (neutrino rest-frame), F' (source), and F'' (detector), respectively.

The neutrino time of flight tof_v (=, where d is the travel distance) is equal to difference between the times as measured in the detector and the source, or

$$tof_v = \frac{d}{v} = \frac{\Delta t}{1+\frac{v}{c}} - \frac{\Delta t}{1-\frac{v}{c}} = - \frac{2\frac{v}{c}}{1-(\frac{v}{c})^2}. \qquad \text{…. (9)}$$

For an *early* neutrino arrival time, *δt*, with respect to the velocity of light, we can write

$$\frac{d}{c} - \delta t = tof_v = - \frac{2\frac{v}{c}}{1-(\frac{v}{c})^2}\frac{d}{v}. \qquad \text{….. (10)}$$

Solving for $\frac{v}{c}$ yields

$$\frac{v}{c} = \left(\frac{2}{1-\frac{c\,\delta t}{d}} - 1\right)^{\frac{1}{2}}, \qquad \text{…. (11)}$$

or

$$\frac{v-c}{c} = \sqrt[2]{\frac{2}{1-\frac{c\,\delta t}{d}} - 1} - 1. \qquad \text{…. (12)}$$

To demonstrate, for the OPERA-corrected result [17], d = 730.085 km and δt = (6.5 ± 7.4 (stat.) $\pm^{+9.2}_{-6.8}$ $(sys.)$) ns. Substituting in eq. (12), we get

$$\frac{v-c}{c} = \left(\frac{2}{1-\frac{299792.458\ x\ 6.5\ x\ 10^{-9}}{730.085}} - 1\right)^{\frac{1}{2}} - 1 \approx -2.67 \text{ x } 10^{-6}, \qquad \text{.... (13)}$$

which is almost identical to the reported result of $\frac{v-c}{c}$ $(Exp.)$ = (2.7 ± 3.1 $(stat.)$ $\pm^{+3.8}_{-2.8}$ (sys.))×10^{-6}. Applying eq. (9) to five others experiments, conducted by MINOS, OPERA, ICARUS, LVD, and Borixeno collaborations, yields the results summarized in Table 2. As the table shows, the mode yields precise predictions for all the tested experiments.

Table 2

Predictions of ER for six neutrino-velocity experiments

Experiment	**Experimental $\frac{v-c}{c}$**	**Predicted $\frac{v-c}{c}$**
MINOS 2007 [22]	**(5.1±2.9))(stat) ×10^{-5}**	**5.14 10^{-5}**
OPERA 2012 (corrected result) [17]	**(2.7 ± 3.1 $(\mathbf{stat.})$ + $^{+3.8}_{-2.8}$ (sys.)) ×10^{-6}**	**2.67 x 10 $^{-6}$**
OPERA 2013 [18]	**(- 0.7 ± 0.5 $(\mathbf{stat.})$ +$^{+2.5}_{-1.5}$ (sys.)) ×10^{-6}**	**- 0.66 x 10^{-6}**
ICARUS 2012 [19]	**(0.4 ± 2.8(stat.) ± 9.8 (sys.)) ×10^{-7}**	**0.41 x 10^{-7}**
LVD [20]	**(1.2 ± 2.5(stat.) ± 13.2 (sys.)) ×10^{-7}**	**1.23 x 10^{-7}**
Borexino [21]	**(3.3 ± 2.9(stat.) ± 11.9 (sys.)) ×10^{-7}**	**3.28 x 10^{-7}**

3.3 Prediction of the time dilation of decaying muons

In muon-decay experiment, muons are generated when cosmic rays strike the upper levels of the Earth's atmosphere. They are unstable, with a life time of τ = 2.2 μ s. With counters that count muons traveling within a velocity of 0.99450**c** to 0.9954**c**, comparing their flux density at both the top and bottom of a mountain gives the rate of their decay. In the most famous muon-decay experiment [24], assuming a velocity of 0.992c of muons in air, researchers found that the percentage of the surviving muons descending from the top of

Mt. Washington to the sea level (d ≈ 1907 m.) was (72.2 ± 2.1) %, considerably higher than 36.79%, the expected percentage resulting from non-relativistic calculation.

To calculate the relativistic muon decay, denote the times at Earth and at a moun's frame by t and t', respectively. Without loss of generality, assume that at the mountain's level, t = t' = 0. For any time t' ($0 \leq t' \leq t'_B$), where t'_B is the muon's time arrival at the bottom, the flux density N(t') could be expressed as

$$N(t') = N(0)\ e^{-\frac{t'}{\tau}}, \qquad \text{..... (14)}$$

where N(0) is the count at the mountain's level. Substituting the value of t' from eq. (4), we get

$$N(t)_{CR} = N(0)\, e^{-\frac{(1-\beta)t}{\tau}}\ . \qquad \text{..... (15)}$$

A similar analysis based on SR yields

$$N(t)_{SR} = N(0)\, e^{-\frac{\sqrt{1-\beta^2}\ t}{\tau}}. \qquad \text{..... (16)}$$

For β =0.992, Figure 4 depicts the rates of decay predicted by ER, SR, and a nonrelativistic calculation. For an ascending time of $\delta t = \frac{d}{v} = \frac{1907\ m.}{2.998\text{x } 10^8} \approx$ 6.36 μs., the predictions of ER and SR are, respectively, $\frac{N(t=6.36)_{CR}}{N(0)}$ x 100= $e^{-\frac{(1-0.992)x\ 6.36}{2.2}}$ x 100 ≈ 97.7% and $\frac{N(t=6.36)_{SR}}{N(0)}$ x 100= $e^{-\frac{\sqrt{1-0.992^2}\ x\ 6.36}{2.2}}$ x 100 ≈ 69.42%. By contrast, according to nonrelativistic considerations, the expected percentage of surviving muons is only $\frac{N(t=6.36)_{NR}}{N(0)}$ x 100= $e^{-\frac{6.36}{2.2}}$ x 100 ≈ 5.55%. Comparison with the observed percentage of 72.2% strongly indicates that a classical analysis fails to account for the observed phenomenon, whereas the two relativistic approaches succeed in achieving that. Note that the predicted values of both theories are not precise, given the fact that the theoretical calculations ignore several factors affecting the flight of descending particles[47].

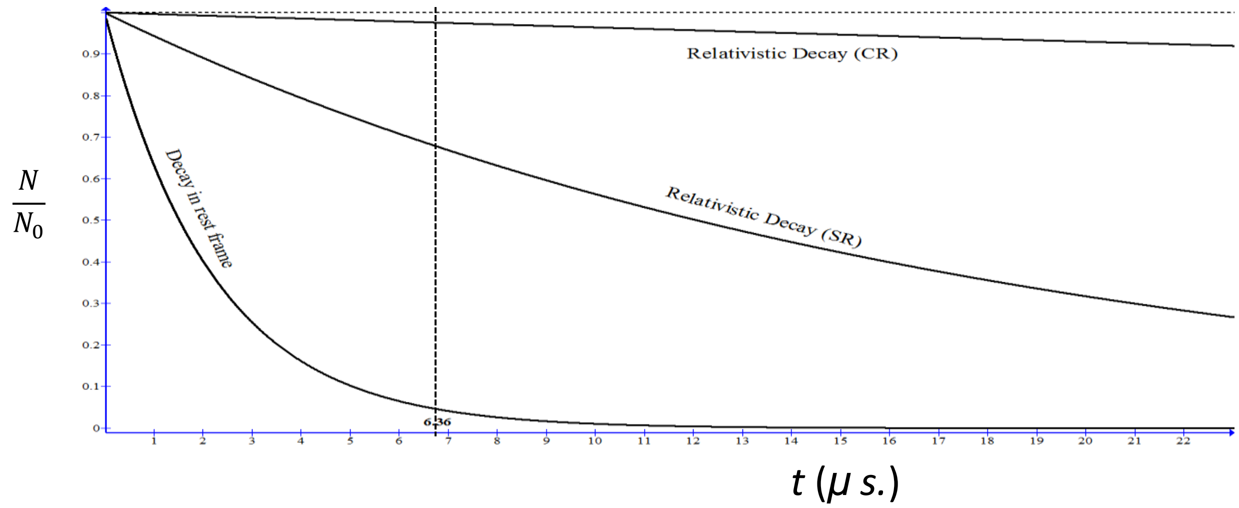


Figure 4: Predicted rates of muon decay

3.4 Prediction of Michelson-Morley's null result

In their seminal paper [23], Michelson and Morley (M&M) reported an experiment set to test the velocity of the motion of Earth in the presumed. M&M analyzed the motion of the parallel and perpendicular waves (with respect to Earth's motion). They found (incorrectly) that the displacement of the interference fringes is given by: $2\ D_0(\frac{v}{c})^2 = 2\ D_0\beta^2$, where $D_0$is the interferometer arm's length at rest. It is well known that the results of the M&M experiment, and many subsequent experiments [e.g., 48-53], were far less than the above prediction. As M&M reported, "Considering the motion of the earth in its orbit only, this displacement should be $2\ D_0(\frac{v}{c})^2 = 2\ D_0$ x 10^{-8}. The distance D was about 11 meters, or 2×10^7 wavelengths of yellow light; hence, the displacement to be expected was 0.4 fringes. The actual displacement was certainly less than the 20th part of this (prediction) and probably less than the 40th part," ([23], p. 341) which is "too small to be detected when masked by experimental errors" ([23], p. 337).

It is well-known that SR was successful in predicting the M&M null result without inclusion of the notion of ether, and that by this, it opened a new era of post-Newtonian physics. Here, I show that the proposed ER performs as well as SR in predicting the null effect.

To account for the relativistic effects on the distance that light travels in the round trip, I replace $2D_0$ by $D_1 + D_2$in the equation derived by M&M, where D_1 and D_2 are the departure and arrival distances, respectively. Using the distance transformation depicted in Table 1, we get

Fringe Shift = (D_1+ D_2) β^2= D_0 ($\frac{1+\beta}{1-\beta} + \frac{1-\beta}{1+\beta}) = D_0 \frac{1+\beta^2}{1-\beta^2}\beta^2$, ….. (17)

where β= $\frac{v}{c}$, c ≈ 299792.458 $km/_s$, and v is the velocity of Earth around the (v ≈ 29.78 $km/_s$). Substituting β = $\frac{29.78\ km/_s}{299792.458\ km/_s}$ ≈ 9.9340 x 10^{-5} and D_0 =11m (the interferometer's arm length in the M&M experiment) in eq. (25), we obtain a predicted fringe shift of approximately 1.09 x 10^{-7}, which is five orders of magnitude smaller than the reported experimental resolution (of ≤ 0.02). The comparable prediction made by SR is $2\ D_0\ \beta^2 = 2\sqrt{1-\beta^2}\ \beta^2$, which after substitution yields ≈ 1.97x 10^{-8}. Given the resolution in the M&M experiment, the difference between the two predictions (≈ 8.9 x 10^{-8}) is negligible. Table 1 summarizes similar calculations performed for several M&M type experiments, while contrasting them with the respective predictions of SR.

Table 3

Predictions of findings reported by classical Michelson-Morley type experiments

Experiment	Arm length (meters)	Expected Fringe shift	Measured Fringe shift	Experimental Resolution	ER prediction	SR prediction
Michelson and Morley [23]	11.0	0.4	< 0.02 or ≤ 0,01	0,01	≈ 4.34 x 10^{-7}	≈ 4.34 x 10^{-7}
Miller [48]	32.0	1.12	≤ 0.03	0.03	≈ 1.27 x 10^{-6}	≈ 1.26 x 10^{-6}
Tomaschek [49] (star light)	8.6	0.3	≤ 0.02	0.02	≈ 3.40 x 10^{-7}	≈ 3.40 x 10^{-7}
Illingworth [50]	2.0	0.07	≤ 0.0004	0.0004	≈ 7.89 x 10^{-8}	≈ 7.90 x 10^{-8}
Piccard & Stahel [51]	2.8	0.13	≤ 0.0003	0.0007	≈ 1.11 x 10^{-7}	≈1.11 x 10^{-7}
Michelson et al. [52]	25.9	0.9	≤ 0.01	0.01	≈ 1.02 x 10^{-6}	≈ 1.02 x 10^{-6}
Joos [53]	21.0	0.75	≤ 0.002	0.002	≈ 8.30 x 10^{-7}	≈ 8.30 x 10^{-7}

As the table shows, both theories predict the null results. Moreover, the differences between the predictions of ER and SR are either zero or in the order of magnitude of 10^{-10}.

4. ER's Solution to the Twin Paradox

The twin paradox is undoubtedly the most famous thought experiment in physics. The enormous literature about it renders any attempt to review it almost impossible. In the twin paradox, one of two twins stays on Earth while the other travels near the speed of light to a distant star and returns to Earth. According to SR, the "traveling" twin returns truly and verifiably younger than the "staying" twin. Because all Earth's inhabitants share the same frame with the "staying" twin, one may conclude that the "traveling" twin returns to the future. Albert Einstein proposed this solution in his famous 1905 paper [12]. Although he called SR's answer a "peculiar consequence" (*eigent ümliche Konsequenz*), Einstein stated that the traveling brother is the one who becomes younger. According to Einstein, this solution is independent of whether the travel path is composed of straight lines or a closed curve of any shape. In Einstein's words: "If there are two synchronous clocks at A, and one of them is moved along a closed curve with constant velocity [v] until it has returned to A, which takes, say t seconds, then this clock will lag on its arrival at A by seconds behind the clock that has not been moved" [12].

Other attempts to solve the twin paradox evoke the relativity of accelerating frames. After developing general relativity, Einstein resorted to this explanation in 1918, when he argued that because one of the clocks is in an accelerated frame of reference, the postulates of the SR do not apply to it, and thus "no contradictions in the foundations of the theory can be construed" [54, 55]. More recent attempts that evoke general relativity are plentiful [e.g., 56-59], but the acceleration argument could be easily dismissed by making the distance between Earth and the remote star long enough to render the acceleration effect arbitrarily small [60].

To solve the twin paradox in the framework of ER, consider the example in Figure 5, in which one twin (Joe) stays on Earth (the "staying twin) and the other twin (Jane, the "traveling" twin) travels to a distant star and returns back to Earth. Assume the travel start times, relative to Earth (F) and to the spaceship (F'), are synchronized such that $\overrightarrow{t_1} = \overrightarrow{t_1{}'}$. Furthermore, assume that upon the arrival of Jane at the distant star, a signal is sent from the star to Joe's station on Earth, indicating Jane's arrival to the star. To solve the paradox, I treat the paths Earth → Star and Star → Earth, each in turn.

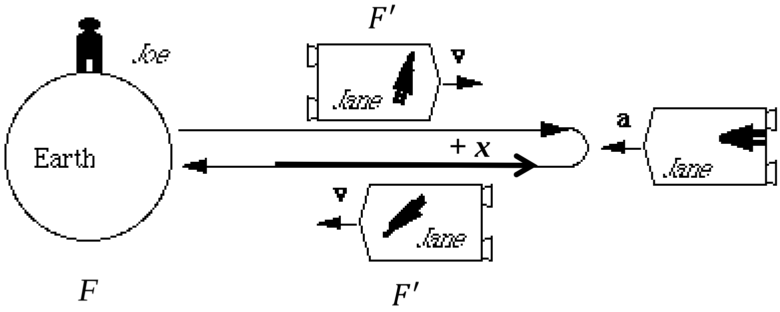


Figure 5. Twin Paradox

a. *Earth → Star*

The signal indicating Jane's arrival of Jane to the star will arrive to Earth with a delay of $\frac{d}{c}$ s., where d is the distance between Earth and the star, and c is the velocity of light (both measured at the Earth's frame).

Denote by $\overrightarrow{t_2}$ and $\overrightarrow{t_2{}'}$ Jane's arrival times to the star, as measured by the "staying" and the "travelling" twins, respectively. We can write $\overrightarrow{t_2} = \overrightarrow{t_2{}'} + \frac{d}{c}$, or

$$\overrightarrow{t_2{}'} = \overrightarrow{t_2} - \frac{d}{c}. \qquad \text{..... (18)}$$

We also have

$$\overrightarrow{t_1} = \overrightarrow{t_1{}'} \qquad \text{..... (19)}$$

b. *Star → Earth.*

The "staying" twin receives the signal indicating the "travelling" twin has departed from the distant star with a delay of $\frac{d}{c}$. This signal leads him to conclude his "travelling" twin has departed from the star $\frac{d}{c}$ s later than the time measured by the travelling twin. Denote the return trip's start time as measured by the "staying" and the "travelling" twins by $\overleftarrow{t_3}$ and $\overleftarrow{t_3'}$, respectively, and the respective arrival times to Earth by $\overleftarrow{t_4}$ and $\overleftarrow{t_4'}$. We can write $\overleftarrow{t_3} = \overleftarrow{t_3'} + \frac{d}{c}$, or

$$\overleftarrow{t_3'} = \overleftarrow{t_3} - \frac{d}{c}. \qquad \text{..... (20)}$$

We also have

$$\overleftarrow{t_4} = \overleftarrow{t_4'}. \qquad \text{..... (21)}$$

c. *Earth → Star → Earth*

The total time measured by the "staying" twin is

$(\overrightarrow{t_2} - \overrightarrow{t_1}) + (\overleftarrow{t_4} - \overleftarrow{t_3}),$ ….. (22)

whereas the total time measured by the "travelling" twin is

$(\overrightarrow{t_2'} - \overrightarrow{t_1'}) + (\overleftarrow{t_4'} - \overleftarrow{t_3'}).$ .... (23)

Substituting the values of $\overrightarrow{t_1'}$, $\overrightarrow{t_2'}$, $\overleftarrow{t_3'}$, and $\overleftarrow{t_4'}$ from eqs. 18-21 in eq. 23, we get

$(\overrightarrow{t_2'} - \overrightarrow{t_1'}) + (\overleftarrow{t_4'} - \overleftarrow{t_3'}) = ((\overrightarrow{t_2} - \frac{d}{c}) - \overrightarrow{t_1}) + (\overleftarrow{t_4} - (\overleftarrow{t_3} - \frac{d}{c}))$

$= (\overrightarrow{t_2} - \overrightarrow{t_1}) + (\overleftarrow{t_4} - \overleftarrow{t_3}) - \frac{d}{c} + \frac{d}{c} = (\overrightarrow{t_2} - \overrightarrow{t_1}) + (\overleftarrow{t_4} - \overleftarrow{t_3}).$ ….(24)

Thus, the twins age equally. The above solution obliterates the possibility of return to the future, at least not in the manner prescribed by SR.

5. Kinetic Energy

As the table (see also appendix A2) shows, the kinetic energy density for a body of rest mass density $\rho' = \rho_0$ is given by

$e = \frac{1}{2}\rho_0 c^2 \frac{(1-\beta)}{(1+\beta)}\beta^2 = e_0 \frac{(1-\beta)}{(1+\beta)}\beta^2,$ ….. (25)

where $\beta = \frac{v}{c}$ and $e_0 = \frac{1}{2}\rho_0 c^2$.

Figure 6 depicts $\frac{e}{e_0}$ as a function of velocity *β.* Strikingly, for departing bodies (positive β values), the kinetic energy density displays a non-monotonic behavior. It increases with β up to a maximum at velocity $\beta = \beta_{cr}$, and then decreases to zero at β = 1. To calculate β_{cr}, I derive $\boldsymbol{\frac{e}{e_0}}$ with respect to β and equate the result to zero. The resulting relationship (see Appendix A2) is**:**

$\beta^2 + \beta - 1 = 0,$ ....(26)

which solve for positive β at:

$\beta_{cr} = \frac{\sqrt{5}-1}{2} = \Phi \approx 0.618,$ ….(27)

where Φ is the famous Golden Ratio best known for its aesthetic properties [61, 62]. Substituting $\boldsymbol{\beta_{cr}}$ in the energy-density expression (see Appendix A2) yields:

$$e_{max} = \Phi^5 e_0 \approx 0.09016994\, e_0 . \quad \text{.... (28)}$$

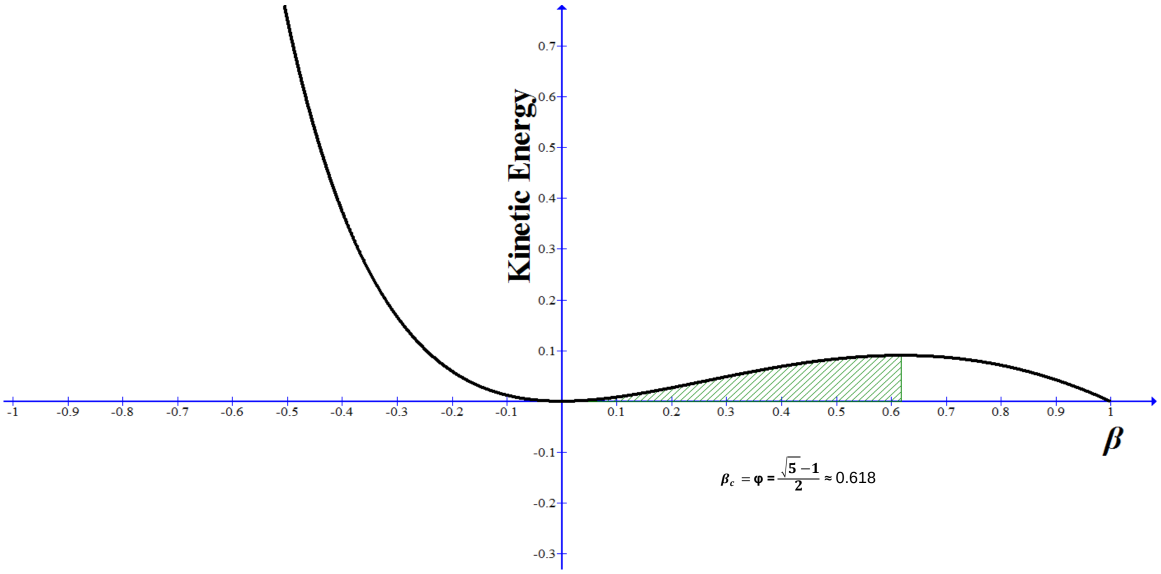


Figure 6. Kinetic energy density as a function of velocity

For an approaching body, the predicted kinetic energy density increases sharply with an increase in velocity. Strikingly, the energy density for β = - Φ (≈ - 0.618) is precisely 1+ Φ (≈ 1.618). To show this result, substituting the exact value of Φ in eq. 25 and using the relationship $1 + \Phi = \frac{1}{\Phi}$ yields

$$e = \frac{(1+\Phi)}{(1-\Phi)}(\Phi)^2 e_0 = \frac{(1+\Phi)\Phi}{\frac{(1-\Phi)}{\Phi}} e_0 = \frac{(1+\Phi)\Phi}{\frac{1}{\Phi}-1} e_0 = \frac{(1+\Phi)\Phi}{1+\Phi-1} e_0 = (1+\Phi) \approx 1.618\, e_{0.} \quad \text{.. (29)}$$

6. Summary and Concluding Remarks

Einstein's relativity theory dictates, as a *force majeure*, an ontic view of the world, according to which relativity is a true state of nature. Here, I took a fundamentally different approach that adopts an epistemic view, according to which relativity results from a difference in knowledge about Nature between observers who are in relative motion with respect to each other. Postulation that the laws of physics are the same in all inertial frames of reference (SR's first axiom), and specifying that information translated from one frame of reference to another is carried by light or another wave of equal velocity, I calculated the modulations in information about measurements of time, distance, mass, and kinetic energy (summarized in Table 1).

ER has some nice properties.:(1) It is very simple. (2) It satisfies the Einstein-Podolsky-Rosen's necessary condition for a theory's *completeness*, in the sense that "every element of the physical reality must have a counter part in the physical theory" (see [9], p. 777). In fact, all the variables in the theory are observable by human senses or are directly measurable by human-made devices. (3) The theory applies, without alterations or the addition of free parameters, to describing the dynamics of very small and very large bodies.

Applying the theory's time transformation to the twin paradox yields a commonsense solution, according to which the twins reunite after aging equally. This solution does not require an arbitrary designation of Earth as the “preferred” frame of reference, which stands in opposition to the mere idea of relativity. For the domain of small particle physics, ER succeeds like SR in explaining the null result of the famous Michelson-Morley experiment [23], and the time dilation of decaying muons [24]. More important, ER accounts for the well-known Sagnac Effect and for the results of six neutrino-velocity experiments [17-22], whereas SR fails to do so. As mentioned above, both the linear Sagnac effect [15, 16] and the neutrino-velocity experiments (although the second was not intended to do so) constitute stringent tests for the LI principle, and thus qualify for pitting ER against SR, such that if one is confirmed the second is automatically refuted. This state of affairs should lead an unbiased scientist to accept ER and reject SR. The argument often raised by proponents of SR that hundreds of experiments have confirmed SR is simply nonscientific because it ignores the stringent condition for theory testing, expressed in Carl Popper's falsification principle [67, 68]. As Albert Einstein himself, in reflecting about this important issue, pointed out, “If an experiment agrees with a theory it means ‘perhaps’ for the latter; if it does not agree, it means ‘no’” (quoted in ref. 68, p. 203).

Of note is that application of ER's transformations derived here to cosmology and astrophysics, in complementarity with the classical Doppler effect, proves impressively successful in suggesting plausible answers to key cosmological questions, including the inflationary expansion of the universe at very high redshifts [69], the nature of dark matter and dark energy [69, 70], the evolutionary timeline of chemical elements, the nucleosynthesis [69], and the dynamics of massive black holes at the center of galaxies [71]. Moreover, the derived expressions for (Ω_{matter},Ω_{Λ}) fit nicely with the findings of several observations based on ΛCDM cosmologies, conducted at various redshift ranges [69].

Undoubtedly, the most astonishing results of ER are the ones that emerged unexpectedly from the investigation of the kinetic energy term. For departing bodies, ER predicts a complete breakdown of classical physics at β =Golden Ratio ≈ 0.618, beyond which higher velocities are associated with lower kinetic energy density. This result echoes a recent finding [63] demonstrating that the quantum criticality of cobalt niobate atoms' energy exhibit a Golden Ratio symmetry. No less striking are the results indicating that the predicted maximal energy density at β=Golden Ratio is precisely equal to Hardy's probability of entanglement (0.09016994), and that the kinetic energy density of approaching bodies at velocity β=Golden Ratio is precisely 1+Golden Ratio ≈ 1.618 times the classical term $e_0 = \frac{1}{2}\rho_0 \ c^2$ (see eq. 23). The emergence of these quantum numbers from a deterministic relativity theory based on SR's first axiom plus an axiom specifying light as an information carrier is puzzling. One possible explanation is to attribute their emergence to mere coincidence. However, given the many proven results of the theory, and the rarity of such a coincidence actually happening, this explanation is highly improbable. Another possibility, which deserves further investigation, is that *ER* reveals more than one genuine thread for a possible connection between an epistemic view of relativity and quantum mechanics, with the Golden Ratio symmetry playing a key role.

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Appendix A

A1. ER's distance transformation

Consider the two frames of reference F and F'shown in Figure 3. Assume the two frames are moving away from each other at a constant velocity *v*. Assume further that at time t_1 in F (and t_1' in F'), a body starts moving in the +*x* direction from point x_1 (x_1' in F') to point x_2 (x_2' in F'), and that its arrival is signaled by a light pulse that emits exactly when the body arrives at its destination. Denote the internal framework of the emitted light by F_0. Without loss of generality, assume $t_1 = t_1' = 0$, $x_1 = x_1' = 0$. Also denote $t_2 = ,\ t_2' = t'$, $x_2 = x,\ x_2' = x'$.

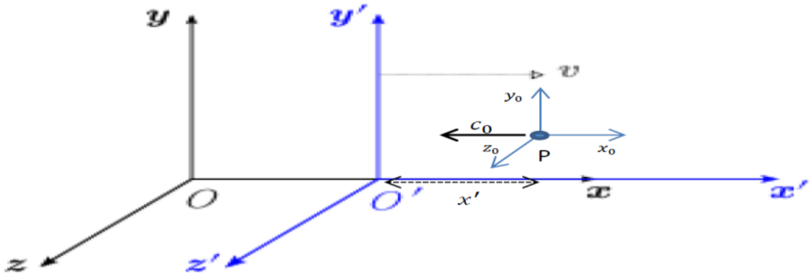

Figure 1a: Two observers in two reference frames, moving with velocity v with respect to each other

From eq. (4), the time t_p in F_0 that the light photon takes to reach an observer in F' equals

$$t_p = (1 - \left(-\frac{v}{c}\right)\ ) \, t' = (1 + \beta) \, t', \qquad \text{..... (1a)}$$

where t' is the corresponding time in F' and c is the velocity of light in the internal frame. Because F' is moving away from F with velocity *v*, the corresponding time that the light photon takes to reach F is equal to

$$t = t_p + \frac{vt}{c} = t_p + \beta \, t. \qquad \text{...... (2a)}$$

Substituting t_p from eq. (1a) in eq. (2a) yields

$$t = (1 + \beta) \, t' + \beta \, t \, ,$$

or

$$\frac{t}{t\prime} = \frac{(1+\beta)}{(1-\beta)} . \qquad \text{...... (3a)}$$

But $x = c\,t$ and $x' = \mathrm{c}\,t'$. Thus, we can write

$$\frac{x}{x\prime} = \frac{t}{t\prime} = \frac{(1+\beta)}{(1-\beta)} . \qquad \text{...... (4a)}$$

The relative distance $\frac{x}{x'}$ as a function of β, together with the respective relative distance according to SR (in dashed black), are shown in Figure 2a. Whereas SR prescribes that irrespective of direction, objects moving relative to an internal frame will contract, ER predicts that a moving object will contract or expand depending on whether it approaches the internal frame or departs from it.

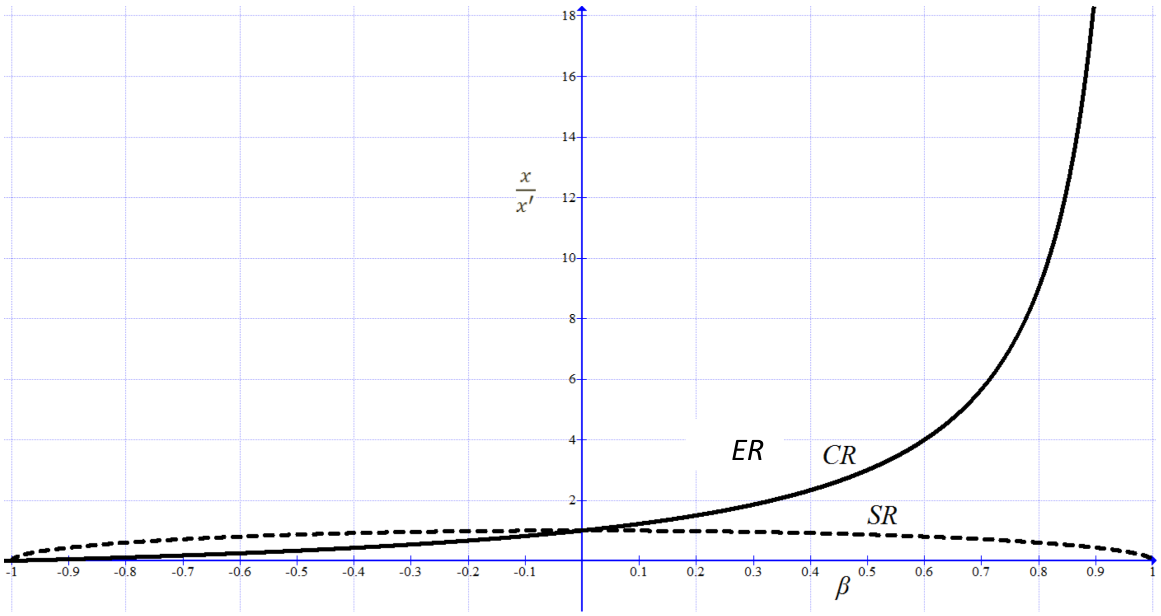

Figure 2a. Distance transformation for the one-way trip. The dashed line depicts the corresponding prediction of SR.

A2. Mass and energy transformations

Consider the two frames of reference F and F'shown in Figure 3a. Suppose the two frames are moving relative to each other at a constant velocity v. Consider a uniform cylindrical body of mass m_0 and length l_0 placed in F' along its travel direction. Suppose that at time t_1, the body leaves point x_1 ($x_1{}'$ in F') and moves with constant velocity v in the +x direction until it reaches point x_2 ($x_2{}'$ in F') in time t_1 ($x_2{}'$ in F'). The body's density in the internal frame F' is given by $\rho' = \frac{m_0}{A\, l_0}$, where A is the area of the body's cross section, perpendicular to the direction of movement. In F, the density is given by $\rho = \frac{m_0}{Al}$, where l is the object's length in F. Using the distance transformation (eq. 4a), l could be written as $l = \frac{1+\beta}{1-\beta}\, l_0$. Thus, we can write

$$\rho = \frac{m_0}{Al} = \frac{m_0}{A\ l_0\ (\frac{1+\beta}{1-\beta})} = \rho'\ (\frac{1-\beta}{1+\beta}), \qquad \text{..... (5a)}$$

or

$$\frac{\rho}{\rho_0} = \frac{1+\beta}{1-\beta}\ . \qquad \text{..... (6a)}$$

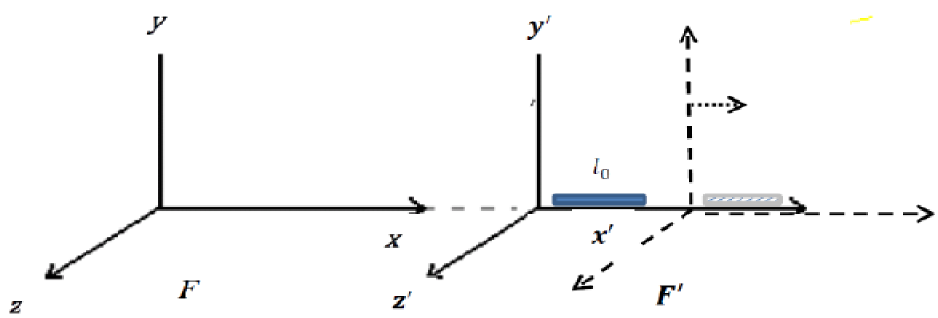

Figure 3a. Two observers in two reference frames, moving with constant velocity v with respect to each other.

The kinetic energy of a *unit of volume* is given by:

$$e = \frac{1}{2} \rho\, v^2 = \frac{1}{2}\, \rho_0\ \frac{(1-\beta)}{(1+\beta)}\ v^2, \qquad \text{..... (7a)}$$

or

$$e = \frac{1}{2}\, \rho_0\ c^2\ \frac{(1-\beta)}{(1+\beta)}\ \beta^2 \ = \ \frac{(1-\beta)}{(1+\beta)}\ \beta^2\ \ e_{0,} \qquad \text{..... (8a)}$$

where $\beta = \frac{v}{c}$ and $e_0 = \frac{1}{2}\ \rho_0\ c^2$.

For $\beta \rightarrow 0$ (or v << c), eq. 6a reduces to $\rho = \rho_0$, and the kinetic energy density expression (Eq. 7a) reduces to Newton's expression $e = \frac{1}{2}\rho_0 v^2$. Figures 4a and 5a, respectively, depict the relativistic mass density and energy as a function of β**.**

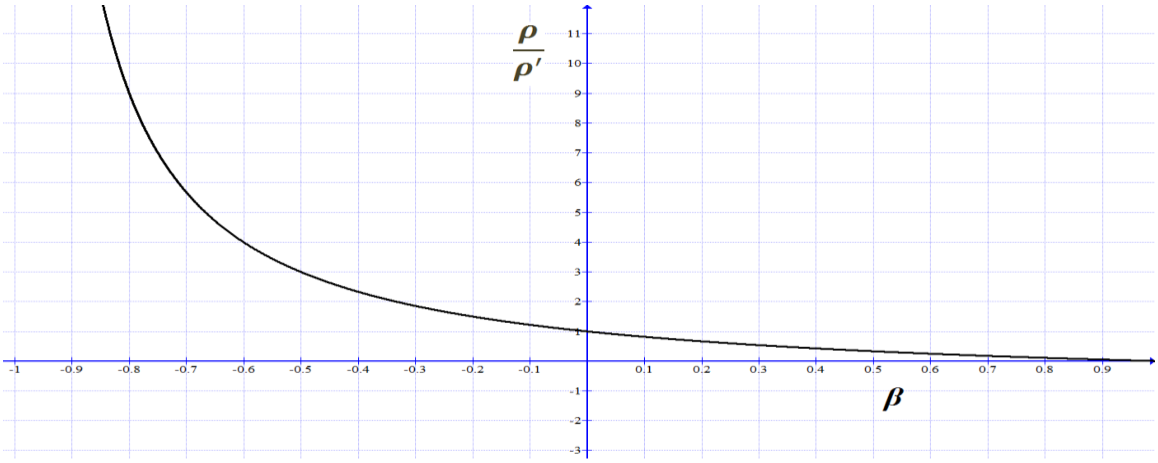

Figure 4a. Mass density as a function of velocity

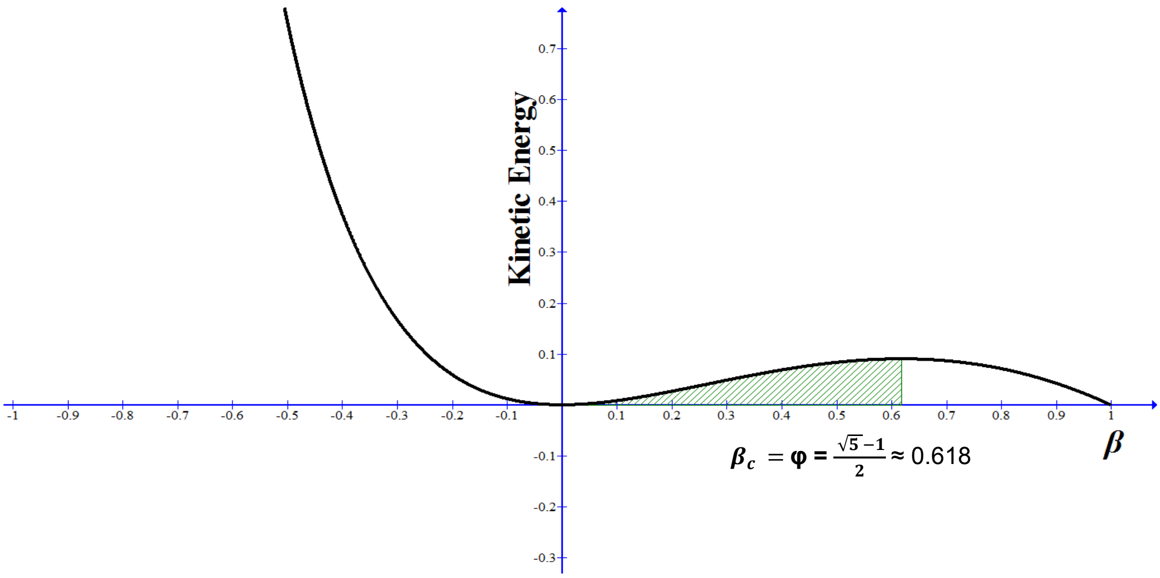

Figure 5a. Kinetic energy density as a function of velocity

As shown Figure 4a shows, the density of departing bodies relative to an observer in *F* is predicted to decrease with β, approaching zero as $\beta \rightarrow 1$, whereas the density in F for approaching bodies is predicted to increase with β up to infinitely higher values as $\beta \rightarrow -1$.

Strikingly, for departing bodies, the kinetic energy density displays a non-monotonic behavior. It increases with β up to a maximum at velocity $\beta = \beta_{cr}$, and then decreases to zero at β = 1. Calculating β_{cr} is obtained by deriving eq. 8a and equating the result to zero:

$$\frac{d}{d\beta}\left(\beta^2 \frac{(1-\beta)}{(1+\beta)}\right) = 2\beta \frac{(1-\beta)}{(1+\beta)} + \beta^2 \frac{[(1+\beta)(-1)-(1-\beta)(1)]}{(1+\beta)^2} = 2\beta \frac{(1-\beta^2-\beta)}{(1+\beta)^2} = 0. \quad \text{.... (9a)}$$

For $\beta \neq 0$, we get

$$\beta^2 + \beta - 1 = 0, \qquad \text{...(10a)}$$

which solves for positive β at

$$\beta_{cr} = \frac{\sqrt{5}-1}{2} = \Phi \text{ (Golden Ratio)} \approx 0.618. \qquad \text{....(11a)}$$