

**Conjecture which states that there exist an infinity
of squares of primes of the form $109+420k$**

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Abstract. In this paper I conjecture that there exist an infinity of squares of primes of the form $109 + 420*k$, also an infinity of primes of this form and an infinity of semiprimes $p*q$ of this form such that $q - p = 60$.

Conjecture:

There exist an infinity of squares of primes of the form $p^2 = 109 + 420*k$, where k positive integer.

The first eight terms of this set:

$p^2 = 529(=23^2), 1369(=37^2), 2209(=47^2),$
 $10609(=103^2), 11449(=107^2), 26569(=163^2),$
 $29929(=173^2), 54289(=233^2) (\dots),$ obtained for $k = 1,$
 $3, 5, 25, 27, 63, 71, 129 (\dots)$

Conjecture:

There exist an infinity of primes of the form $p = 109 + 420*k$, where k positive integer.

The first twenty terms of this set:

$p = 109, 1789, 3049, 3469, 3889, 4729, 5569, 6829, 7669,$
 $8089, 8929, 9349, 9769, 12289, 14389, 15649, 16069,$
 $17749, 18169, 19009 (\dots),$ obtained for $k = 0, 4, 7, 8,$
 $9, 11, 13, 16, 18, 19, 21, 22, 23, 29, 34, 37, 38, 42,$
 $43, 45 (\dots)$

Note that, for k from 55 to 60, the formula creates a chain of six consecutive primes (23209, 23629, 24049, 24469, 24889, 25309).

Conjecture:

There exist an infinity of semiprimes of the form $p*q = 109 + 420*k$, where k positive integer, such that $q - p = 60$.

The first six terms of this set:

$p*q = 60483(=13*73), 5989(=53*113), 8509(67*127),$
 $15229(=97*157), 21509(=137*197), 37909(=167*227) (\dots),$
obtained for $k = 2, 14, 20, 36, 64, 90 (\dots)$

Comment:

The conjectures above inspired me a way to find larger primes when you know two primes p, q such that $q - p = 60$, both primes of the form $10*k + 3$ or of the form $10*k + 7$. There are almost sure easy to find primes between the numbers of the form $p*q - 210*k$, where k positive integer.

Examples:

- : $m = 13*73 - 210*k$ is prime for $k = 1$ ($m = 739$);
- : $m = 23*83 - 210*k$ is prime for $k = 1$ ($m = 1699$);
- : $m = 37*97 - 210*k$ is prime for $k = 2$ ($m = 3169$);
- : $m = 43*103 - 210*k$ is prime for $k = 1$ ($m = 4219$);

- : $m = 104123*104183 - 210*k$ is prime for $k = 7$ ($m = 10847845039$);
- : $m = 104183*104243 - 210*k$ is prime for $k = 1$ ($m = 10860348259$);
- : $m = 104323*104383 - 210*k$ is prime for $k = 3$ ($m = 10889547079$);
- : $m = 104537*104597 - 210*k$ is prime for $k = 6$ ($m = 10934255329$);
- : $m = 104623*104683 - 210*k$ is prime for $k = 1$ ($m = 10952249299$).

Note that the formula $p*q + 210*k$ (under the given conditions) seems also to conduct pretty soon to primes; for m from the last five examples above we have:

- : $104123*104183 + 210*3 = 10847847139$, prime;
- : $104183*104243 + 210*9 = 10860350359$, prime;
- : $104323*104383 + 210*2 = 10889548129$, prime;
- : $104537*104597 + 210*4 = 10934257429$, prime;
- : $104623*104683 + 210*1 = 10952249719$, prime.