**High-resolution Frequency Measurements of 0S0**

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**Abstract:** Precision measurements of the 0S0 frequency after eleven earthquakes in the years 2001 to 2011 by fifteen superconducting gravimeters show that this resonance frequency of the Earth has temporarily changed greatly. In addition to a slow drift there are also wide frequency jumps.

**Introduction**

After earthquakes, the Earth vibrates like a bell at different frequencies and the whole set of different natural frequencies is recorded by various instruments. Some spectral lines as 0S0 are weak attenuated and can be measured for months. This very long period of time allows very accurate frequency measurements in different time periods. First measurements[1] of the three strongest earthquakes showed that - contrary to previous assumptions - the frequency changes over time.

To check this surprising finding, the 0S0 frequency was measured after eleven strong earthquakes. The zero-beat method is - in contrast to FFT - hardly affected by signal interference and therefore allows the analysis of much weaker earthquakes than the event 2004-12-26.

The striking frequency jumps in the years 2006 and 2007 and the systematic frequency shift during the 2010 earthquake are amazing. These results are not "outliers" of single SG and they are not caused by faulty data.

In all records, only rough data errors and data gaps were replaced by zero. In the data flow, there are only two frequency filters: The selective reduction of the strong tide-signals near 22 μHz and a broadband low-pass and after the frequency mixing. The bandwidths were chosen so that the desired frequency range around 814 μHz is not impaired. Calculation errors may be excluded.

The underlying data of this examination were measured by a net of about twenty SG distributed over all continents, the data are collected in the Global Geodynamic Project[2].

**The waterfall spectrum**

The two waterfall spectra shown in [1] contain no reference frequency to allow an evaluation of the accuracy of the frequency determination. Following a suggestion by D. Crossley, a weak sine signal of 814.9500 μHz has been added as a marker to the data taken by the station MB. The amplitude of this auxiliary signal corresponds to the weakest detectable earthquake the entire period. This injection frequency appears near the right edge of the picture below. Ideally this should be a vertical, very thin line. The tremulous course and the considerable width show that the FFT method can measure frequencies only imprecisely and is influenced by the amplitude of the adjacent 0S0 frequency. FFT (with a sequence length 2200) is obviously unsuitable to measure the 0S0 frequency more accurately than ±0.03 μHz. Extending the sequence length improves the frequency resolution but smears the picture in vertical direction. That would be no problem if the frequency of the oscillation would be precisely defined and the vibration would begin with correct phase after each earthquake. Both assumptions are unrealistic.

The picture also shows that the actual frequency fluctuations of 0S0 are much larger than the resolution of the FFT.

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The Preparation of the data

All available CORMIN-data of SG-stations were bundled into separate two-year-clusters like CB1011 or ST1011. To prevent intermodulation by numeric overload of the mathematical coprocessor inside the computer, the very strong spectral lines around 22 µHz were attenuated by narrow notch filters[3]. After decimating the sampling rate to 1/360 s, the signal was multiplied by a sine wave of selectable frequency (f_{BFO} ≈ 814.66 µHz) and phase. Finally, the data was low-pass filtered with a bandwidth of 80 µHz.

This zero-beat method shifts the “high frequency” information around f_{BFO} to much lower frequencies, where they can be analyzed more precisely. The only variable parameter is the length of the interval. The adjacent channel interference by S5 about 20 µHz higher does not disturb because mean value of this damped sine wave is zero.

Since the initial region of the integration period (IP) may be disturbed[4], the frequency measurement must not start earlier than 200 hours after the earthquake. The maximum length of the IP was arbitrarily set at 1100 hours to ensure a good signal-to-noise ratio. To measure the frequency several times with different conditions, the IP was decreased in 30 steps down to the minimum length of 520 hours. In undisturbed data, all results should match and show no systematic drift.

The only exception is the earthquake 2007-08-15 with a maximum IP of 640 hours because a stronger earthquake (with different phase and frequency) follows. In order to still allow 30 different measurements, the step size was reduced to 10 hours. It is inevitable that during these truncated IPs, data glitches gain greater influence and worsen the results.
Calculating the Frequency

The measurement of frequency with the zero-beat method is based on minimizing the average and slope of the product \((Signal) \cdot \cos(2\pi ft + \phi)\) in a predetermined interval, called integration period (IP). For each SG station thirty different IPs were set. For each IP the frequency \(f\) (start value \(= 814.66\ \mu\text{Hz}\)) and the phase \(\phi\) are iterated until the frequency change of successive steps is smaller than \(10^{-7}\ \mu\text{Hz}\). Finally, the results of all stations are connected to a single chain and the average frequency and error bars are calculated with the jackknife method, based on a minimum of 360 single calculations. Normally the chain of frequencies resembles the blue curve in the figure below.

First surprise

The red curve in the picture above shows an obvious correlation between the IP - length and the frequency, valid only for the \(S_0\) oscillation, which was excited by the earthquake on 2010-02-27. This unique feature was observed after no other earthquake. Twelve of fourteen records show the same characteristic change of frequency over time. For IP = 1,100 hours, the frequency is about 0.003 \(\mu\text{Hz}\) higher than for IP = 520 hours. In between, the frequency decreases approximately linearly. Since all IPs start 200 hours past the earthquake, this observation indicates a significant increase in frequency by about 0.006 \(\mu\text{Hz}\) (up-chirp). This peculiarity can not be observed in the very noisy records of stations MC and PE. Due to numerous data glitches, the average frequency is significantly higher.

Omitting those two records and superimposing the remaining twelve, the systematic and slow frequency shift of the \(S_0\) oscillation is even more striking. First signs of a frequency modulation were already discovered in [1].
Second surprise

The frequency measurements of the $O_{S0}$-oscillation after eleven strong earthquakes between the years 2001 to 2011 show two abnormalities:

- In the years 2007/08, there is a striking accumulation of six moderate earthquakes. The amplitudes are much smaller than in 2004, but they clearly exceed the background noise.
- During this period, unexpectedly large frequency jumps occur. The jump distance exceeds $3\sigma$ and is highly significant. What has happened in the earth during those years?

It is unclear whether the two phenomena are correlated. The following table summarizes the results.

<table>
<thead>
<tr>
<th>Date of EQ</th>
<th>Mag.</th>
<th>Frequency (µHz)</th>
<th>Error bar (µHz)</th>
<th>Number of Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001-06-23</td>
<td>M=8.4</td>
<td>814.65445636</td>
<td>±0.000363</td>
<td>12*30=360</td>
</tr>
<tr>
<td>2004-12-26</td>
<td>M=9.1</td>
<td>814.6572308</td>
<td>±7.27000e-05</td>
<td>16*30=480</td>
</tr>
<tr>
<td>2005-03-28</td>
<td>M=8.6</td>
<td>814.6584254661</td>
<td>±0.000240485</td>
<td>17*30=510</td>
</tr>
<tr>
<td>2006-11-15</td>
<td>M=8.3</td>
<td>814.6573838539</td>
<td>±0.000496113</td>
<td>12*30=360</td>
</tr>
<tr>
<td>2007-01-13</td>
<td>M=8.1</td>
<td>814.6445123656</td>
<td>±0.000744101</td>
<td>13*30=390</td>
</tr>
<tr>
<td>2007-04-01</td>
<td>M=8.1</td>
<td>814.6603787153</td>
<td>±0.000496498</td>
<td>13*30=390</td>
</tr>
<tr>
<td>2007-08-15</td>
<td>M=8.0</td>
<td>814.666360224</td>
<td>±0.000998587</td>
<td>12*30=360, max. IP=640 hours</td>
</tr>
<tr>
<td>2007-09-12</td>
<td>M=8.5</td>
<td>814.6574586452</td>
<td>±0.000331269</td>
<td>13*30=390</td>
</tr>
<tr>
<td>2008-05-12</td>
<td>M=7.9</td>
<td>814.664021907</td>
<td>±0.001190231</td>
<td>13*30=390</td>
</tr>
<tr>
<td>2010-02-27</td>
<td>M=8.8</td>
<td>814.6582445488</td>
<td>±9.52000e-05</td>
<td>14*30=420</td>
</tr>
<tr>
<td>2011-03-11</td>
<td>M=9.0</td>
<td>814.6565997235</td>
<td>±3.82000e-05</td>
<td>12*30=360</td>
</tr>
</tbody>
</table>

All error bounds are much narrower than that in [5] and [6].
**Analyzed records**

2001-06-23: CB, H2, M1, M2, MA, MB, MC, S1, S2, ST, W1, W2.
2004-12-26: CB, H1,H2, KA, M1, M2, MA, MB, MC, ME, S1, S2, ST, VI, W1, W2.
2005-03-28: CB, H1,H2, KA, M1, M2, MA, MB, MC, ME, S1, S2, ST, TC, VI, W1, W2.

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