Fitting Galaxy Rotation Curves Without Dark Matter

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Abstract

The notion is presented that fitting galaxy rotation curves is possible without the invocation of dark matter. Equations of motion similar to those used to describe the internal motions of terrestrial weather systems are applied to the rotation curves of galaxies. However, due to the four-dimensional nature of the equations, additional terms arise. One of these additional terms is particularly useful in matching the rotation curves of galaxies. The extra term presumably describes internal properties of the central galactic black hole. It also determines, in part, the galactic rotation curve, just as the motion of winds in a terrestrial weather system are determined, in part, by the rotation of the Earth.

1 Introduction

The first indication that something was lacking in galactic theory occurred when it was noticed that stars at the outer regions of galaxies rotated more quickly than expected. The rotational speeds of these stars was greater than the speeds determined theoretically from the amount of luminous matter in the galaxy. It was concluded, therefore, that there must be some kind of unobserved (or dark) matter present in the galaxy in order to account for the extra speed.

Alternative theories have been proposed in an effort to explain this discrepancy. One such theory, Milgrom's "Modified Newtonian Dynamics" (MOND) [1], introduces an additional acceleration into Newton's gravitation theory that results in very nice fits to a large number of rotation curves. However, it contains the unacceptable (to me) prediction that rotation speeds remain constant to infinity. Another theory which gives rotation curves comparable to Milgrom's is Moffat's "Scalar-Tensor-Vector Gravity Theory" (STVG or MOG) [2], which unlike Milgrom's theory, leads to a falling off of rotation speeds outside of galactic boundaries. Moffat's theory, unfortunately, requires a variable Newtonian gravitational constant G and speed of light c.

Here I introduce a model that was inspired by terrestrial atmospheric dynamics. The motion of air in these systems is well described by Newtonian equations of motion. By extending these three-dimensional equations to four-dimensions, extra terms appear which turn out to be useful in describing the motions of the contents of galaxies.

2 Equations of Motion

I will present the equations of motion using a somewhat unorthodox method starting with the presentation of two four-vector products [3]. The results of these products are then compared to the Newtonian equations of motion.

The first of the four-vector products referred to is

$$\vec{\Omega}\,\vec{X} = \vec{\Omega}\cdot\vec{X} + \vec{\Omega}\times\vec{X} + \vec{\Omega} : \vec{X} \tag{1}$$

where $\vec{\Omega} = (\Omega_1, \Omega_2, \Omega_3, \Omega_4)$ is the *angular velocity (or frequency) four-vector* and $\vec{X} = (X_1, X_2, X_3, X_4)$ is the *position four-vector*. The components 1, 2, and 3, here, are spatial and the component 4 is temporal.

The second four-vector product is

$$\begin{aligned}
\vec{\Omega}\,(\vec{\Omega}\,\vec{X}) &= \vec{\Omega}\cdot(\vec{\Omega}\,\vec{X}) + \vec{\Omega}\times(\vec{\Omega}\,\vec{X}) + \vec{\Omega} : (\vec{\Omega}\,\vec{X}) \\
\\
&= \vec{\Omega}\cdot(\vec{\Omega}\cdot\vec{X} + \vec{\Omega}\times\vec{X} + \vec{\Omega} : \vec{X}) \\
&+ \vec{\Omega}\times(\vec{\Omega}\cdot\vec{X} + \vec{\Omega}\times\vec{X} + \vec{\Omega} : \vec{X}) \\
&+ \vec{\Omega} : (\vec{\Omega}\cdot\vec{X} + \vec{\Omega}\times\vec{X} + \vec{\Omega} : \vec{X}) \\
\\
&= \vec{\Omega}\cdot(\vec{\Omega}\cdot\vec{X}) + \vec{\Omega}\cdot(\vec{\Omega}\times\vec{X}) + \vec{\Omega}\cdot(\vec{\Omega} : \vec{X}) \\
&+ \vec{\Omega}\times(\vec{\Omega}\cdot\vec{X}) + \vec{\Omega}\times(\vec{\Omega}\times\vec{X}) + \vec{\Omega}\times(\vec{\Omega} : \vec{X}) \\
&+ \vec{\Omega} : (\vec{\Omega}\cdot\vec{X}) + \vec{\Omega} : (\vec{\Omega}\times\vec{X}) + \vec{\Omega} : (\vec{\Omega} : \vec{X})
\end{aligned} \tag{2}$$

Equations (1) and (2), I propose, are the four-dimensional generalized equations of velocity and acceleration, respectively.

To show the correspondence between (1) and (2), and Newton's equations of motion, we need to ignore the temporal terms, since Newton's equations contain only spatial terms. Consequently, we first make the change $\vec{X} = (\vec{r}, 0)$, where $\vec{r} = (X_1, X_2, X_3)$. For now, we will keep all of the components of $\vec{\Omega}$, since the time component appears in the spatial products $\vec{\Omega} : (\vec{\Omega}\times\vec{X})$ and $\vec{\Omega} : (\vec{\Omega} : \vec{X})$. After rearranging terms, (1) now becomes

$$\vec{\Omega}\,\vec{r} = \vec{\Omega} : \vec{r} + \vec{\Omega}\times\vec{r} \tag{3}$$

Now compare (3) with the Newtonian equation

$$\vec{v} = \dot{\vec{r}} + \vec{\Omega}\times\vec{r} \tag{4}$$

where the dot over $\vec{r}$ indicates the time derivative of $\vec{r}$.

Comparing (3) and (4), it would seem that $\vec{\Omega}\,\vec{r} = \vec{v}$ and

$$\vec{\Omega} : = \frac{d}{dt} \tag{5}$$

when the vector being operated on, that is, the vector to the right of the " : " symbol, is spatial (thus making the component of the vector to the left of the symbol temporal). Consequently, the time component of $\vec{\Omega}$ appears to be interchangeable with the time derivative. The validity of this claim will become more apparent shortly.

Using (5), we find that (3) becomes

$$\vec{v} = \dot{\vec{r}} + \vec{\Omega} \times \vec{r} \tag{6}$$

which is the same as (4). Note that, here, $\dot{\vec{r}}$ is in the direction of r, since the term $\vec{\Omega} : \vec{r}$ is in the direction of r. Thus, (6) may be thought of as the vector sum of the time rate of change of $\vec{r}$ in the direction of $\vec{r}$, or $\dot{\vec{r}}_{\parallel}$, and the time rate of change of $\vec{r}$ perpendicular to $\vec{r}$ and $\vec{\Omega}$, or $\dot{\vec{r}}_{\perp}$. In other words, $\vec{v} = \vec{v}_{\parallel} + \vec{v}_{\perp}$, where $\vec{v}_{\parallel} = \dot{\vec{r}}_{\parallel} = \dot{\vec{r}}$ and $\vec{v}_{\perp} = \dot{\vec{r}}_{\perp} = \vec{\Omega} \times \vec{r}$.

We now alter (2) in the same way we altered (1), that is, we start by ignoring the (first three) temporal terms. We then drop the term $\vec{\Omega} \times (\vec{\Omega} \cdot \vec{X})$ since $\vec{\Omega} \cdot \vec{X}$ is temporal and, therefore, incompatible with the cross product. However, we keep the term $\vec{\Omega} : (\vec{\Omega} \cdot \vec{X})$, since it is spatial, despite $\vec{\Omega} \cdot \vec{X}$ being temporal. After substituting (5) and $\vec{X} = (\vec{r}, 0)$ into (2), we end up with

$$\vec{\Omega}\,(\vec{\Omega}\,\vec{r}\,) = \vec{\Omega} \times (\,\vec{\Omega} \times \vec{r}\,) + \vec{\Omega} \times \dot{\vec{r}} + \vec{\Omega} : (\,\vec{\Omega} \cdot \vec{r}\,) + \frac{d}{dt}(\,\vec{\Omega} \times \vec{r}\,) + \frac{d}{dt}\left(\frac{d}{dt}\,\vec{r}\right) \tag{7}$$

Noting that

$$\frac{d}{dt}(\,\vec{\Omega} \times \vec{r}\,) = \dot{\vec{\Omega}} \times \vec{r} + \vec{\Omega} \times \dot{\vec{r}} \tag{8}$$

and

$$\frac{d}{dt}\left(\frac{d}{dt}\,\vec{r}\right) = \ddot{\vec{r}} \tag{9}$$

we insert (8) and (9) into (7) which, after rearranging and combining terms, becomes

$$\vec{\Omega}\,(\vec{\Omega}\,\vec{r}\,) = \ddot{\vec{r}} + \dot{\vec{\Omega}} \times \vec{r} + 2\,\vec{\Omega} \times \dot{\vec{r}} + \vec{\Omega} \times (\,\vec{\Omega} \times \vec{r}\,) + \vec{\Omega} : (\,\vec{\Omega} \cdot \vec{r}\,) \tag{10}$$

Newton's equation for the acceleration $\vec{a}$ is

$$\vec{a} = \ddot{\vec{r}} + \dot{\vec{\Omega}} \times \vec{r} + 2\,\vec{\Omega} \times \dot{\vec{r}} + \vec{\Omega} \times (\,\vec{\Omega} \times \vec{r}\,) \tag{11}$$

We notice that all the terms on the right-hand side of (10), except the last one, appear on the right-hand side of (11), providing further reason to accept the validity of (5). We see that this extra term, $\vec{\Omega} : (\,\vec{\Omega} \cdot \vec{r}\,)$ is in the direction of the spatial part of Ω, since the term in parentheses is temporal. If we make the substitution $\vec{a} = \vec{\Omega}\,(\vec{\Omega}\,\vec{r}\,)$, we can write (10) as

$$\vec{a} = \ddot{\vec{r}} + \dot{\vec{\Omega}} \times \vec{r} + 2\,\vec{\Omega} \times \dot{\vec{r}} + \vec{\Omega} \times (\,\vec{\Omega} \times \vec{r}\,) + \vec{\Omega} : (\,\vec{\Omega} \cdot \vec{r}\,) \tag{12}$$

I will adopt (12) as my equation of motion in the attempt to fit the galaxy rotation curves.

3 The Data

The galaxy rotation curve data used here is from Sanders & McGaugh (2002) [4], as trimmed for quality by McGaugh (2005) [5]. The actual rotation curve data used in the figures is retrievable elsewhere.[1]

[1] please see http://astroweb.case.edu/ssm/data/galaxy_massmodels.dat

4 Determination of Radial Mass

It will be assumed that test particles follow concentric circular paths in a common plane in a static spherically symmetric gravitational field. To find the radial mass distribution, I started with the Newtonian equation of motion for circular orbits

$$\frac{v_{Newton}^2}{r} = \frac{GM}{r^2} \tag{13}$$

where G is the Newtonian gravitational constant, v_{Newton} is the magnitude of the Newtonian rotational velocity and $M = M(r)$ is the total mass within r.[2] I then solved (13) for M, resulting in

$$M = \frac{v_{Newton}^2 r}{G} \tag{14}$$

Since the total mass within r is the linear sum of the stellar and gaseous masses $M = M_{stars} + M_{gas}$ and $M_{stars} = v_{stars}^2 r/G$ and $M_{gas} = v_{gas}^2 r/G$, the Newtonian velocity is obtained by summing stellar and gaseous velocities in quadrature,

$$v_{Newton}^2 = v_{stars}^2 + v_{gas}^2 \tag{15}$$

After substituting (15) into (14), we get the total mass within r. However, the resulting values for M, since they were derived from the velocity data points, are also in the form of data points. Therefore, in order to use these values for M in the fitting process, it was necessary to convert the data points to a smooth curve. To accomplish this, I fit a seventh-degree polynomial to the data points for M. The resulting polynomial for $M = M(r)$ was then used to represent the radial mass distribution of each galaxy.

5 Fitting the Rotation Curves

The rotational velocity $v(r)$ of a test particle is determined classically, as usual, from

$$\frac{v^2(r)}{r} = a(r) \tag{16}$$

where $a(r)$ is the radial acceleration of the test particle. The Newtonian gravitational acceleration is

$$a(r) = \frac{GM}{r^2} \tag{17}$$

If we insert (17) into (16), we get the Newtonian equation of motion (13). However, this equation has been shown to be inadequate for describing the rotational velocities of stars in galaxies, outside the inner regions. Let us, instead, try (12) as a starting point to find a replacement for (17).

Referring to (12), since the gravitational field is assumed to be static, $\vec{\Omega}$ is constant in time, thus $\dot{\vec{\Omega}} = 0$. Also, I make the assumption here, for phenomenological reasons, that the spatial part of Ω is parallel to r and $\dot{\vec{r}}$. Therefore, $\dot{\vec{\Omega}} \times \vec{r} = \vec{\Omega} \times \dot{\vec{r}} = \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) = 0$ and (12) becomes

$$\vec{a} = \ddot{\vec{r}} + \vec{\Omega} : (\vec{\Omega} \cdot \vec{r}) \tag{18}$$

[2] From here on, to avoid clutter, I will refer to the magnitude of the velocity and the magnitude of the acceleration as the velocity and acceleration, respectively.

Also for phenomenological reasons I set the magnitude of $\vec{\Omega}$ to $|\vec{\Omega}| = \Omega_0 \exp(-r/r_0)$, where Ω_0, the angular velocity (or frequency) at $r = 0$, and r_0, the scale factor, are free parameters.

The magnitude of the term $\ddot{\vec{r}}$ I take to be the Newtonian acceleration $|\ddot{\vec{r}}| = GM/r^2$. Since the spatial part of $\vec{\Omega}$ and $\vec{r}$ are parallel, the magnitude of $\vec{\Omega}:(\vec{\Omega}\cdot\vec{r})$ is $|\vec{\Omega}:(\vec{\Omega}\cdot\vec{r})| = |\vec{\Omega}||\vec{\Omega}||\vec{r}| = \Omega_0^2\, r \exp(-2\,r/r_0)$. Consequently, the radial acceleration, from (18), now becomes

$$a(r) = \frac{GM}{r^2} + \Omega_0^2\, r \exp(-2\,r/r_0) \tag{19}$$

Equation (19) will replace (17). Substituting (19) into (16), we get

$$\frac{v^2(r)}{r} = \frac{GM}{r^2} + \Omega_0^2\, r \exp(-2\,r/r_0) \tag{20}$$

Multiplying both sides of (20) by r and taking the square root, we have

$$v(r) = \sqrt{\frac{GM}{r} + \Omega_0^2\, r^2 \exp(-2\,r/r_0)} \tag{21}$$

Equation (21) will be used to fit the rotation curves. For small r and $r \gg r_0$ (that is, at distances much greater than the galaxy boundary), this reduces to the Newtonian equation.

The fitting of (21) to the rotation curve data sets, with free parameters Ω_0 and r_0, was done using the program Gnuplot. The data sets included in the fitting process were limited to those with 8 or more data points due to the degree of the polynomial used to represent the mass distribution. A good fit to the data was obtained for 32 of the 35 galaxies in the reduced sample. The galaxies for which a less than good fit resulted were NGC 1003, NGC 3726, and UGC 2885. The results are presented in Fig. 1 . The best-fit values for the free parameters Ω_0 and r_0, for each galaxy, are shown in Table 1 .

6 Conclusions

In the introduction I stated that the inspiration for this model came from terrestrial weather systems. It seems that equation (20) is analogous to the gradient flow momentum equations of atmospheric dynamics [6], with GM/r^2 replacing the pressure term and $\Omega_0^2\, r \exp(-2\,r/r_0)$ replacing the Coriolis term. It is important to note, however, that GM/r^2 is not a pressure term and $\Omega_0^2\, r \exp(-2\,r/r_0)$ is not a Coriolis term.

Initially, I was hoping that Ω_0 would be a constant among all galaxies, but it appears to vary from one galaxy to the next. My original intention was to show that the universe is rotating or oscillating at a constant rate Ω_0, contributing to the rotation of galaxies much as the rotation of the earth contributes to the rotation of weather systems. However, since it varies between galaxies, Ω_0 may instead be a characteristic of the central mass or black hole of each galaxy.

The need to invoke dark matter to account for the rotation curves of galaxies is eliminated due to the ability to fit them well with equation (21) alone.

Acknowledgements

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References

[1] http://www.weizmann.ac.il/weizsites/milgrom

[2] http://www.johnwmoffat.com

[3] http://vixra.org/abs/1301.0112

[4] http://arxiv.org/abs/astro-ph/0204521

[5] http://arxiv.org/abs/astro-ph/0506750

[6] http://en.wikipedia.org/wiki/Balanced_flow#Gradient_flow

Table 1: Free Parameters

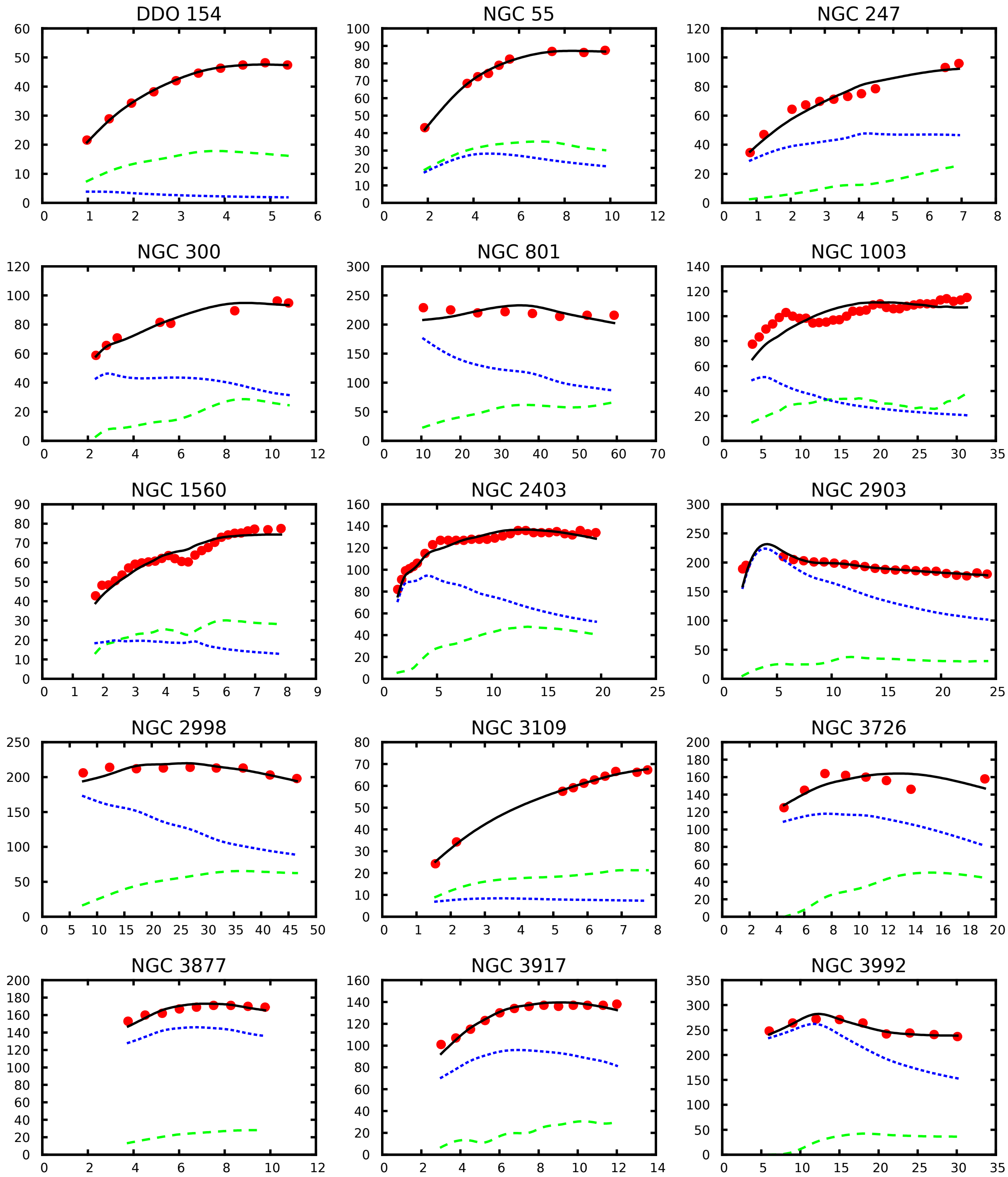
Galaxy (1)	Ω_0 (2)	r_0 (3)
DDO 154	76.24	5.14
NGC 55	69.40	9.97
NGC 247	87.53	7.62
NGC 300	66.20	11.24
NGC 801	44.06	38.20
NGC 1003	40.80	22.48
NGC 1560	72.83	8.19
NGC 2403	61.24	16.14
NGC 2903	52.00	24.19
NGC 2998	47.27	32.35
NGC 3109	54.19	10.94
NGC 3726	63.28	16.12
NGC 3877	95.16	8.42
NGC 3917	85.24	10.46
NGC 3992	34.37	52.41
NGC 4013	42.09	31.96
NGC 4088	60.59	18.40
NGC 4100	64.63	16.40
NGC 4157	52.91	23.34
NGC 4183	70.23	11.90
NGC 4217	60.60	19.20
NGC 5033	52.02	25.71
NGC 5371	49.87	29.00
NGC 5533	39.83	46.40
NGC 5585	76.05	9.45
NGC 5907	41.23	36.95
NGC 6674	37.61	48.84
NGC 6946	58.46	20.56
NGC 7331	35.80	46.80
UGC 1230	40.41	26.56
UGC 2885	31.56	65.49
UGC 5005	56.34	20.01
UGC 6983	84.25	8.98
IC 2574	42.50	15.53
M33	96.96	7.51

Explanation of Columns in Table 1:

(1) Galaxy name.

(2) Frequency/angular speed at $r = 0$ in units of $10^{-17} s^{-1}$.

(3) Scale factor in units of kpc.



DDO 154
NGC 55
NGC 247
NGC 300
NGC 801
NGC 1003
NGC 1560
NGC 2403
NGC 2903
NGC 2998
NGC 3109
NGC 3726
NGC 3877
NGC 3917
NGC 3992

Figure 1:

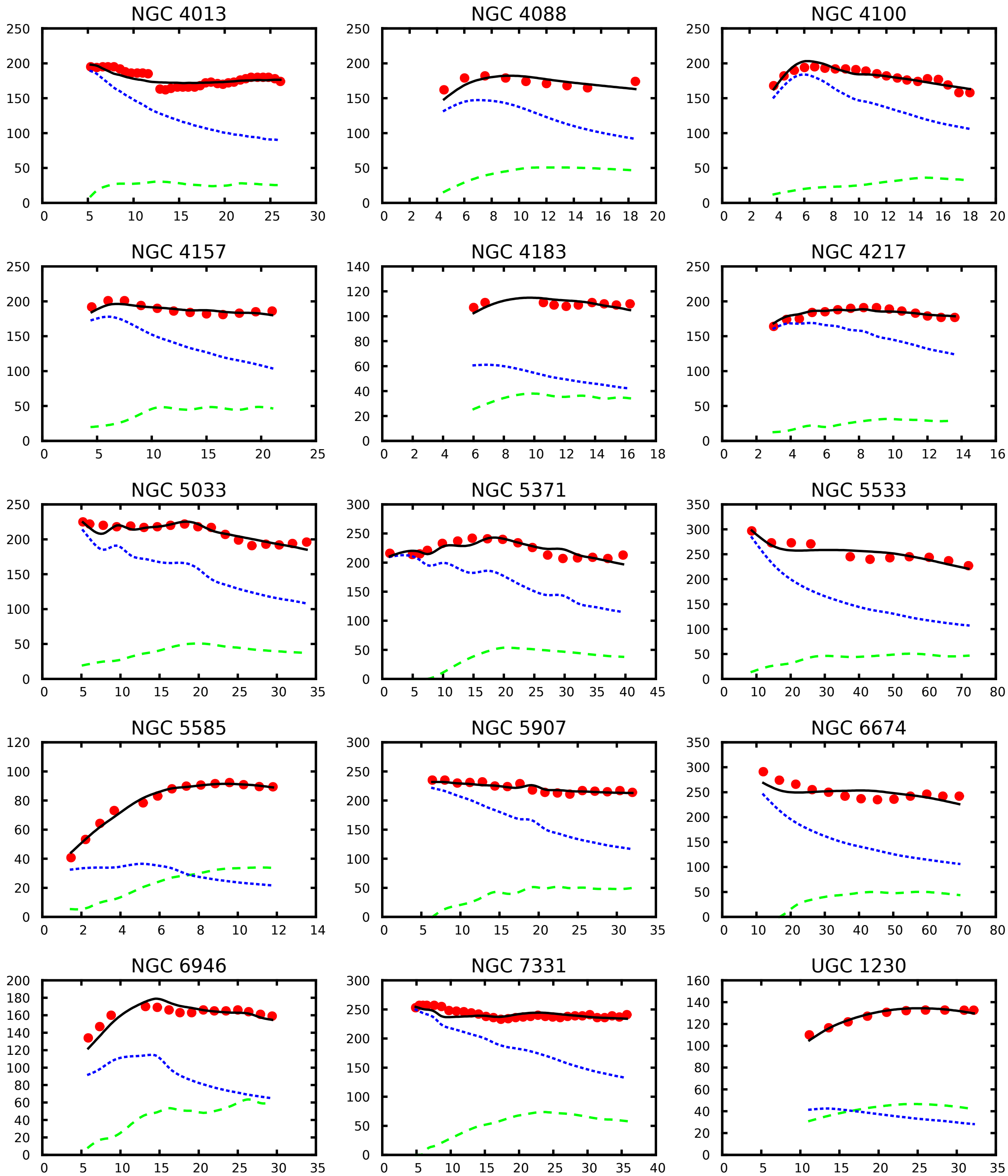


Figure 1: Continued.

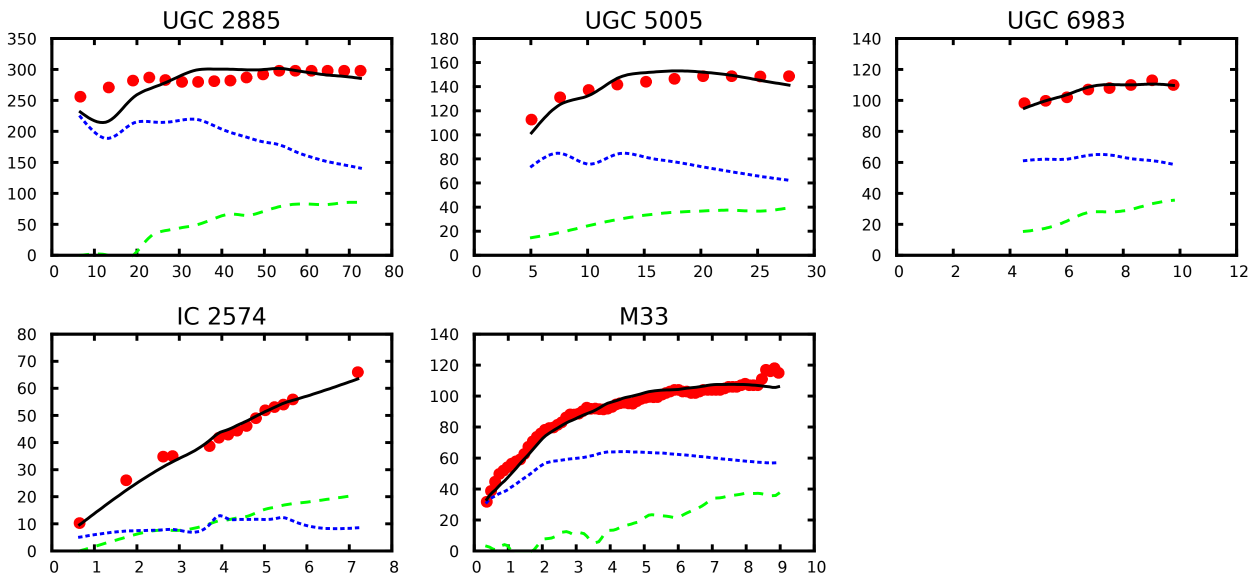


Figure 1: Fits to the observed rotation curve data of Sanders & McGaugh (2002)/McGaugh (2005). The horizontal axis is the radial distance from the center of the galaxy in kiloparsecs. The vertical axis is the rotational velocity in kilometers/sec. The dotted and dashed lines represent the Newtonian stellar disk and gaseous rotation curves, respectively. The red dots show the observed rotation curve data points, while the solid line represents the fit of (21) to the observed rotation curve.