

An analysis of four Smarandache concatenated sequences using the notion of cm-integers

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Abstract. In this paper I show that Smarandache concatenated sequences presented here (*i.e.* The consecutive numbers sequence, The concatenated odd sequence, The concatenated even sequence, The concatenated prime sequence), sequences well known for the common feature that contain very few terms which are primes, *per contra*, contain very many terms which are c-primes, m-primes, c-reached primes and m-reached primes (notions presented in my previous papers, see "Conjecture that states that any Carmichael number is cm-composite" and "A property of repdigit numbers and the notion of cm-integer").

Note:

The Smarandache concatenated sequences are well known for sharing a common feature: they all contain a small number of prime terms. Interesting is that, *per contra*, they seem to contain a large number of c-primes and m-primes. More than that, applying different operations on terms, like the sum of two consecutive terms or partial sums, we obtain again a large number of c-primes and m-primes respectively of c-reached primes and m-reached primes.

Note:

In the following analysis I will not show how I calculated the c-reached primes and the m-reached primes, see for that my paper "The notions of c-reached prime and m-reached prime".

Verifying the observation for the following Smarandache concatenated sequences:

- (1) The Smarandache consecutive numbers sequence

S_n is defined as the sequence obtained through the concatenation of the first n positive integers. The first ten terms of the sequence (A007908 in OEIS) are 12, 123, 1234, 12345, 123456, 1234567, 12345678, 123456789, 12345678910.

This sequence seems to have the property that the value of the sum of two consecutive terms is often (I conjecture that always) a cm-integer.

The first few such values are:

- : $12 + 123 = 135 = 3^3 \cdot 5$. This number is cm-composite, having three c-reached primes, 7, 23, 43, and three m-reached primes, 23, 31, 47;
- : $123 + 1234 = 1357 = 23 \cdot 59$. This number is cm-prime, having the c-reached prime equal to 37 and the m-reached prime equal to 1;
- : $1234 + 12345 = 13579 = 37 \cdot 367$. This number is cm-prime, having the c-reached prime equal to 331 and the m-reached prime equal to 43;
- : $12345 + 123456 = 135801 = 3^2 \cdot 79 \cdot 191$. This number is cm-composite, having a c-reached prime, 521, and a m-reached prime, 601;
- : $123456 + 1234567 = 1358023 = 67 \cdot 20269$. This number is c-prime, having the c-reached prime equal to 139;
- : $1234567 + 12345678 = 13580245 = 5 \cdot 7 \cdot 587 \cdot 661$. This number is cm-composite, having three c-reached primes, 1693, 22549 and 387973, and two m-reached primes, 7561 and 1940041.

(2) The Smarandache concatenated odd sequence

S_n is defined as the sequence obtained through the concatenation of the first n odd numbers (the n -th term of the sequence is formed through the concatenation of the odd numbers from 1 to $2 \cdot n - 1$). The first ten terms of the sequence (A019519 in OEIS) are 1, 13, 135, 1357, 13579, 1357911, 135791113, 13579111315, 1357911131517, 135791113151719.

This sequence seems to have the property that the value of the terms is often (I conjecture that always) a cm-integer.

The first few such values are:

- : 13. This number is prime, so cm-prime by definition;
- : $135 = 3^3 \cdot 5$. This number is cm-composite, having four c-reached primes, 5, 7, 23 and 43, and three m-reached primes, 23, 31 and 47;
- : $1357 = 23 \cdot 59$. This number is c-prime, having the c-reached prime equal to 47;
- : $13579 = 37 \cdot 367$. This number is cm-prime, having the c-reached prime equal to 331 and the m-reached prime equal to 403;

- : $1357911 = 3^3 \cdot 19 \cdot 2647$. This number is cm-composite, having a c-reached prime equal to 23767 and two m-reached primes equal to 8111 and 23879;
- : $135791113 = 11617 \cdot 11689$. This number is c-prime, having the c-reached prime equal to 73.

(3) The Smarandache concatenated even sequence

S_n is defined as the sequence obtained through the concatenation of the first n even numbers (the n -th term of the sequence is formed through the concatenation of the even numbers from 1 to $2 \cdot n$). The first ten terms of the sequence (A019520 in OEIS) are 2, 24, 246, 2468, 246810, 24681012, 2468101214, 246810121416, 24681012141618, 2468101214161820.

This sequence seems to have the property that the value of the numbers $(S - 1)$, where S are the partial sums, is often (I conjecture that always) a cm-integer.

The first few such values are:

- : $2 + 24 - 1 = 25 = 5 \cdot 5$. This number is cm-prime, having the c-reached prime equal to 1 and the m-reached prime equal to 5;
- : $2 + 24 + 246 - 1 = 271$. This number is prime, so cm-prime by definition;
- : $2 + 24 + 246 + 2468 - 1 = 2739 = 3 \cdot 11 \cdot 83$. This number is c-composite, having two c-reached primes equal to 239 and 911;
- : $2 + 24 + 246 + 2468 + 246810 - 1 = 249549 = 3 \cdot 193 \cdot 431$. This number is cm-composite, having a c-reached prime equal to 149 and a m-reache primes equal to 8111 and 1009;
- : $2 + 24 + 246 + 2468 + 246810 + 24681012 - 1 = 24930561 = 3 \cdot 1187 \cdot 7001$. This number is m-reached composite, having a m-reached prime equal to 22189.

This sequence seems also to have the property that the value of the numbers $(S - 1)$, where S is the sum of two consecutive terms, is often a cm-integer.

The first few such values are:

- : $2 + 24 - 1 = 25 = 5 \cdot 5$. This number is cm-prime, having the c-reached prime equal to 1 and the m-reached prime equal to 5;
- : $24 + 246 - 1 = 269$. This number is prime, so cm-prime by definition;
- : $246 + 2468 - 1 = 2713$. This number is prime, so cm-prime by definition;

: $2468 + 246810 - 1 = 249277 = 7 \cdot 149 \cdot 249$. This number is m-composite, having a reached m-prime equal to 35617.

(4) The concatenated prime sequence

S_n is defined as the sequence obtained through the concatenation of the first n primes. The first ten terms of the sequence (A019518 in OEIS) are 2, 23, 235, 2357, 235711, 23571113, 2357111317, 235711131719, 23571113171923, 2357111317192329.

This sequence seems to have the property that the value of the numbers $a(n) - a(n-1) - 1$ is often a cm-integer.

The first few such values are:

: $235 - 23 - 1 = 211$. This number is prime, so cm-prime by definition;

: $2357 - 235 - 1 = 2121 = 3 \cdot 7 \cdot 101$. This number is m-composite, having two m-reached primes, 107 and 709.

: $235711 - 2357 - 1 = 233353$. This number is prime, so cm-prime by definition;

: $23571113 - 235711 - 1 = 23335401$. I haven't completely analyzed the number, but is at least m-composite having a m-reached prime 804697;

: $2357111317 - 23571113 - 1 = 2333540203 = 541 \cdot 4313383$. This number is c-prime (because $4313383 - 541 + 1 = 4312843 = 389 \cdot 11087$ and $11087 - 389 + 1 = 10699 = 13 \cdot 823$ and $823 - 13 + 1 = 811$, which is prime) having the c-reached prime equal to 811;

: $235711131719 - 2357111317 = 233354020401 = 3^2 \cdot 25928224489$. This number is m-composite (because $3 \cdot 25928224489 + 3 - 1 = 77784673469$) having the m-reached prime equal to 77784673469.