Purely Mechanical Memristors and the Missing Memristor

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This work is suppressed since 2010 and led to established scientists to be banned on the arXiv. We thank VIXRA for making critical work like this still available to relatively uncorrupted parts of the scientific community. We define a mechanical analog to the electrical basic circuit element $M = \mathrm{d}\varphi/\mathrm{d}Q$, the ideal mechanical memristance $M = \mathrm{d}p/\mathrm{d}x$, p being momentum. A never before described mechanical memory resistor $M(x)$ is independent of velocity v and has a pinched hysteretic loop that collapses at high frequency in the v versus p plot: a perfect memristor. However, its memristance does not crucially involve inert mass, and the mechanical system helps clarifying that memristor devices hypothesized on grounds of physical symmetries require more. The missing mechanical perfect memristor needs to be crucially mass-involving (MI) precisely like the 1971 implied memristor device needs magnetism. Discussing novel MI memristive systems clarifies why such perfect MI memristors and EM memristors have not been discovered and may be impossible.

Keywords: Memristive Systems; Memristors; Electromagnetic symmetry

1 Introduction

It is now almost common knowledge that "memristors" have been hypothesized and that various "memristors" were discovered. However, precisely what was predicted when, and what was when discovered is controversial. The issue is made difficult by that "memristor" refers variously to memory resistor, memristive system, perfect memristor, the theoretical 'basic two-terminal circuit element' (BCE), an electromagnetism (EM) involving device, and others. We clarify these issues with mechanical analogs, one being a purely mechanical perfect memristor. This claim depends on the preferred terminology, but the mechanical case is a precise analog of the electrical case, and so the decision is ultimately linked to whether the historically hypothesized "missing memristor" is an EM device which has never been discovered. Our chosen topic requires a detailed introduction of terminology. Given the widespread confusion, we must commit to one

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terminology, and such will always conflict with many readers' preferences. Given the wide audience that memristors attract, "ideal" must be the opposite of "real" for example; it should not sometimes mean "perfect." The 'ideal resistor' *R*, 'ideal capacitor' *C*, 'ideal inductor' *L*, and 'ideal memristor' *M* (also 'resistance,' 'capacitance,' 'inductance,' and 'memristance') are the four BCE of electrical circuit theory. BCE are *basic* because they are *independent* of each other like a *basis* of four linearly independent vectors. One cannot connect ideal resistors together but end up with capacitance. It is therefore that real devices are never ideal. For example, two metal plates make a *real* (non-ideal) capacitor *device*; but it has always some resistance, too. Moreover, the BCE are *passive* and were called "passive circuit elements,"[1] meaning they do not supply any energy. Violating passivity can violate independence. The BCE are *theoretical ideal* entities and strictly speaking all impossible as real devices. Their relevance rests in theoretical modeling and they exist by definition; they do not need to be discovered! In 1971, a real memristor device was hypothesized,[2] and it is called the *missing fourth,*[3] because real resistor, capacitor, and inductor devices were known. The 'perfect memristor'-versus-'memristive system' distinction was defined only in 1976![4] Charge controlled systems $M_{(Q)}$ have been called ideal/real/true/genuine/perfect memristors, because *M* depends not also on current *I* for example. We call them 'perfect memristor' in order to be consistent with the 2008 claimed discovery of "the missing memristor": "… *we present a physical model of a two-terminal electrical device that behaves like a perfect memristor…*" [5] A memristive system can depend also on the current *I* for example, thus $M_{(Q,I)}$. Memristive behavior is known from thin films since before 1971.[6] The 2008 claim showed that films of TiO_2 between metals, well known since the 1960s, can be described as resistors with memory.[7] Nonlinear resistors with memory have been described by Kubo theory in the 1950s.[8] There was immediately controversy around that the devices are neither new nor the 1971 implicated device.[9,10] Very similar devices were discovered in 1995,[11] but their discoverers do not regard them as the missing memristors. Before 2008, nonvolatile memory applications[12] were usually not called memristors.[13,14] The core question is: Did the 1971 hypothesis imply an 'EM memristor,' here defined as one that involves magnetism in a crucial way? Such has certainly not been discovered.

2 Circuit Theory without Magnetism and without Mass

The *current-charge relation* $I = \mathrm{d}Q/\mathrm{d}t$ is the *first fundamental relation* (FR1) of circuit theory. It simply defines the current I as a time derivative of the charge Q, and it can be defined for mass flows for example. In our mechanical analog, the "charge" is position x, which flows past a moving body and is conserved behind it ('conserved charge'). FR1 is velocity $v = \mathrm{d}x/\mathrm{d}t$. Electrical charges have force fields between them. Hence, charge storage will store a corresponding energy which can also be dissipated in the charge flow. In the electrical case, this is described by voltages U. Charge storage leads to our first BCE, $C = \mathrm{d}Q/\mathrm{d}U_{\mathrm{C}}$. The mechanical analog of a real capacitor device is to "store" x by the displacement of a spring with Hooke's spring stiffness k. The BCE is therefore $1/k = \mathrm{d}x/\mathrm{d}F_{\mathrm{k}}$. Dissipation is modeled by $R = \mathrm{d}U_{\mathrm{R}}/\mathrm{d}I$. Our mechanical resistor device is a light hollow sphere submerged in oil in an orbiting satellite (no gravity, no buoyancy). If the oil is sufficiently viscous and speeds low, the oil's flow around the sphere will be laminar. This makes the friction force F_{f} proportional to v. The drag coefficient $c = \mathrm{d}F_{\mathrm{f}}/\mathrm{d}v$ is the BCE. The circuit couples these forces, for example if the body in oil is dragged by the spring.

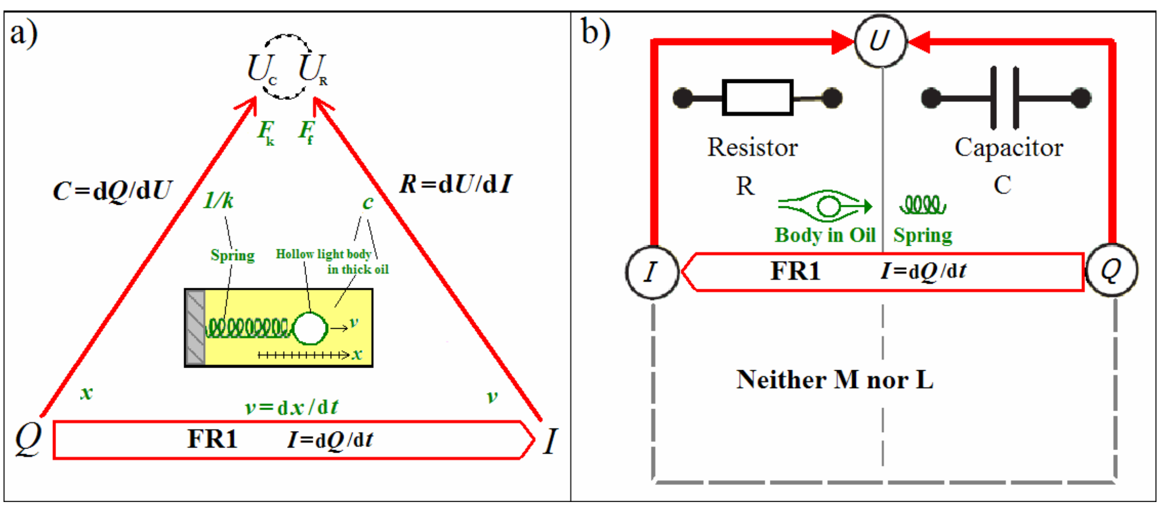

Figure 1: The symmetry of the three fundamental circuit variables Q, I, and U; (a) the chains indicate that the voltages U_{C} and U_{R} are coupled by the circuit; the mechanical analog of a light hollow sphere in oil is inset; its variables are shown in green. The mass of sphere and spring are negligible, so the system is strongly over-damped and cannot oscillate. (b) M and L are on the same footing and *both* still equally absent. The arrows indicate that physical charge is prior to the definition of current, and the general force terms U and F are discovered via the devices.

A triangular symmetry (Fig. 1a) thus connects the three fundamental circuit variables *Q*, *I*, and *U* (*x*, *v*, and *F*, respectively), and the BCE are more generally written as $C_{(U,Q)} = \mathrm{d}Q/\mathrm{d}U$, $R_{(U,I)} = \mathrm{d}U/\mathrm{d}I$, etc.

2.1 Absence of magnetism

Maxwell's equations (ME) come in two pairs. The first pair (MEP1) relates the free charge and current densities: { $\nabla \cdot \vec{D} = \rho_{\mathrm{free}}$; $\nabla \times \vec{H} = \vec{j}_{\mathrm{free}} + \mathrm{d}\vec{D}/\mathrm{d}t$ }. The second pair is the first pair's magnetic twin, but there are no magnetic charges because magnetic fields are a relativistic correction: { $\nabla \cdot \vec{B} = 0$; $\nabla \times \vec{E} = -\mathrm{d}\vec{B}/\mathrm{d}t$ } (MEP2). Magnetism is due to Lorentz-Fitzgerald contraction and time dilatation on moving charge distributions. The second equation of MEP1 can be written $\nabla \times \vec{B} = \mu_0 \vec{j}_{\mathrm{free}} + \varepsilon_0 \mu_0 \mathrm{d}\vec{\mathrm{E}}/\mathrm{d}t$; and $c_0 = (\varepsilon_0 \mu_0)^{-1/2}$ is the velocity of light. If μ_0 were much smaller, we would not know magnetism. In the mechanical analog, the thick oil renders the mass of the body unnoticed. Neglecting magnetism is equivalent to concentrating on the non-relativistic limit $c_0 \to \infty$ as done in much of mechanics, classical and quantum. These are not philosophical thought experiments imagining perhaps impossible worlds. Instead, such clarifies the nature of the involved symmetries by showing for example where true magnetism is necessary. Moreover, neglecting magnetism is what the concept of independent BCE is partially about! There are always magnetic fields with any current, but circuit theory models RC-circuits usually without mentioning *L*. Differently put, especially circuit theory finds neglecting magnetism unproblematic. For now, with *B* and *H* negligible, only parts of MEP1 remain. Integration of ρ_{free} and j_{free} results in charge *Q* and current *I* and thus FR1. In other words, also non-magnetic electrical circuit theory does derive from Maxwell theory, but MEP2 is not involved. The circuit theory discussed here does not know about magnetism.

2.2 Flux, *L*, and *M* without magnetism and a mechanical massless memristor

Defining a "flux" $\varphi = \int U \mathrm{d}t$ provides a so called second fundamental relation, $U = \mathrm{d}\varphi/\mathrm{d}t$ (FR2). Unlike the very similar equation further below, it does *not* derive from MEP2. It is defined this way because φ is thereby a canonically dual charge, because the force term U, which relates to energy, is the dual current (compare FR2 to FR1). Current is a time derivative, and energy E and its canonical dual time t are the main players in dynamical physical theories. Circuit theory rests on this because energy conservation and charge conservation lead to Kirchhoff's loop and node rule, respectively. The mechanical equivalent is therefore the canonical dual to x, namely momentum $p = \int \mathrm{F d}t$ leading to $F = \mathrm{d}p/\mathrm{d}t$ as the FR2. Apart from the new U-to-φ edge (Fig. 2), there are thus again two more edges, almost as if we introduced another charge with a force field: φ-to-I and φ-to-Q. These correspond to two further binary relations, and thus two more BCE can be defined: $L_{(\varphi,I)} = \mathrm{d}\varphi/\mathrm{d}I$, and $M_{(\varphi,Q)} = \mathrm{d}\varphi/\mathrm{d}Q$ relates the dual charges. Such a tetrahedral construct provides complete circuit theory in the sense of that the four BCE can model all potentially non-linear circuit behaviors of the fundamental circuit variables.

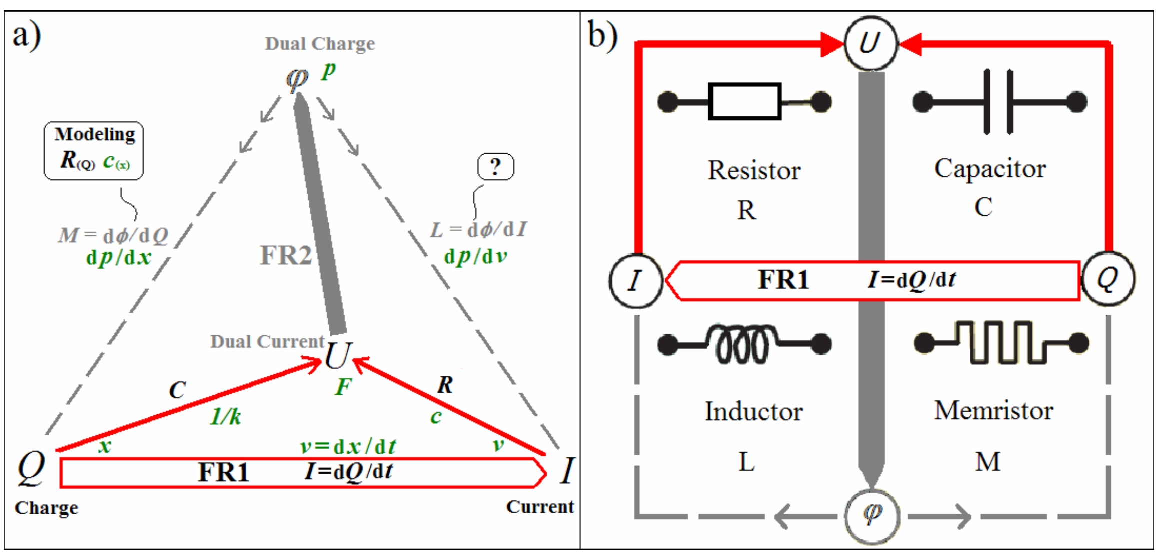

Figure 2: Illustration of the extended symmetry: (a) FR2 with the fourth variable φ (p in the mechanical system) erect a tetrahedron on top of the previous triangle. L and M label two new edges which correspond to two new BCE. The mechanical system's $L = \mathrm{d}p/\mathrm{d}v$ has units of kg, but the system's sphere does not have an inert mass yet (one would use different units when growing up in that universe). (b) The usually given table is now complete, but without magnetism, the "inductor" cannot be the EM inductor device. M and L help modeling but suggest no new devices.

Because of $\mathrm{d}\varphi/\mathrm{d}Q = (\mathrm{d}\varphi/\mathrm{d}t)/(\mathrm{d}Q/\mathrm{d}t) = \mathrm{d}U/\mathrm{d}I$, memristance is a resistance with standard units of Ohm, $[R] = \Omega$, and a linear memristor $M = \varphi/Q$ is a constant Ohmic resistor. Independence between BCE therefore requires M to be non-linear, as all the BCE can generally be. The mechanical analog is now obvious, and we hereby describe a new, mechanical ideal memristor BCE with M being a non-linear $\mathrm{d}p/\mathrm{d}x$ having the units of drag resistance, $[c] = \mathrm{kg/s}$. M is a resistance that depends on $Q_{(t)}$; it memorizes the charge that has flown through it; hence "memristance." It facilitates modeling charge dependent resistors $R_{(Q)}$. The mechanical analog of this is $c_{(x)}$, for example if the oil's dynamic viscosity η depends on x.

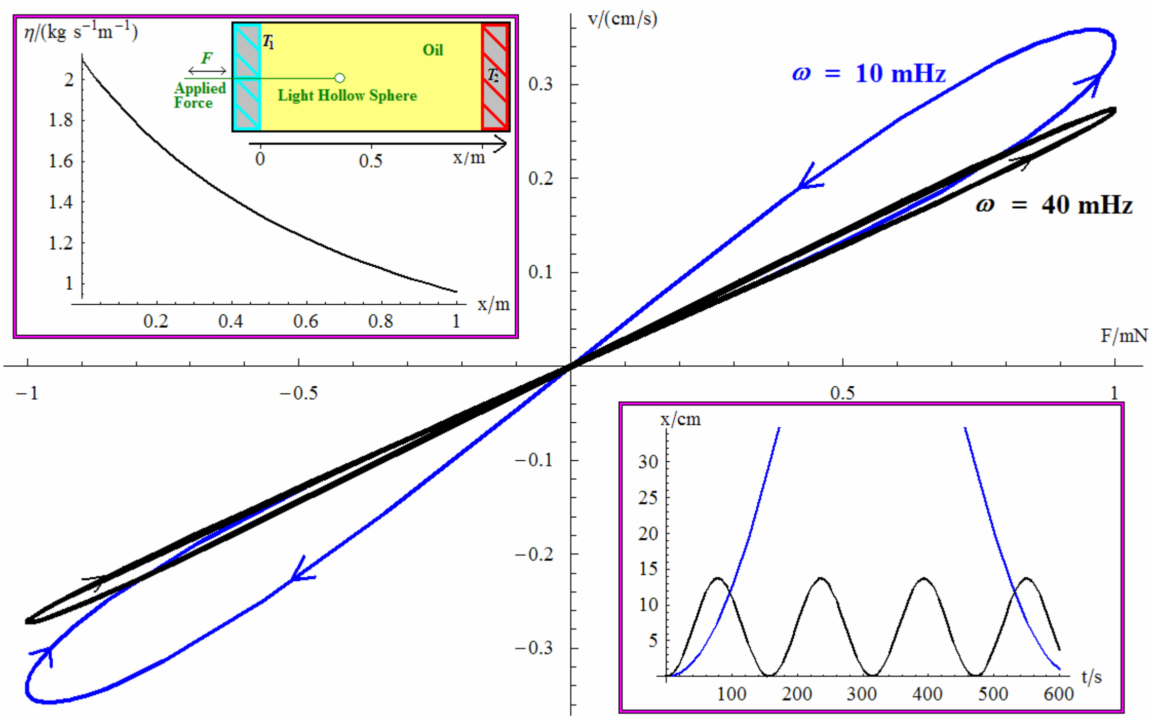

Figure 3: Simulation of the purely mechanical perfect memristor with the numerical NDSolve function of Mathematica5®. During the ten simulated minutes, the blue loop is just about almost completed while the black loop is almost passed through four times. The lower inset shows position x versus time; the blue curve peaks at 70 cm. The upper inset illustrates the system schematically and shows the oil's viscosity η as it depends on x.

Viscosity is modeled with the Andrade equation $\eta = A\, e^{B/T}$.[15,16] A heavy fuel oil (HFO) such as HFO-380 is for our purposes sufficiently well modeled with $A = 0.2$ kg m^{-1} s^{-1} and $B = 47°\mathrm{C}$. If the $x = 0$ end of the oil bath is held at 20°C while $x_{\max} = 1$ m is held at

30°C, the gradient is approximately $T = 20°\mathrm{C} + (10°\mathrm{C/m})x$. The resulting $\eta_{(x)}$ is plotted in Fig. 3. The body is spherical, thus $c = 6\pi\eta r$. The radius is taken to be $r = 1$ cm. If applied forces stay below $F_{\max} = 1$ mN, speed $v_{\max} = F_{\max} / c_{(1\mathrm{m})}$ cannot exceed 5 mm/s. The oil's density $\rho = 0.9$ g cm^{-3} changes comparatively little with *T*. The Reynolds number $\mathrm{Re} = \rho v(2r)/\eta$ is thus below 0.1 and the oil flow always laminar. Many researchers call any system "memristor" if it has a so called "pinched hysteretic loop" in the *I*-vs-*U* plot. The mechanical system shows a pinched hysteretic loop in the corresponding *v*-vs-*F* plot if the sphere starts at the rest position at $x = 0$ and a force $F_{\max} \sin(\omega t)$ with frequency $\omega = 10$ mHz is applied (Fig. 3). At 40 mHz, the loop is much narrower. It becomes a linear resistor at high frequency, like memristors should. Some may want to call it a force-controlled mechanical memristive system analogous to voltage controlled memristive systems[17] (compare Figure 2 of that reference or this one[5]). If all "*Resistance switching memories are memristors,*"[18] it is certainly a memristor. The $c_{(x)}$ does not need to be written as $c_{(x,v)}$, so it is a *perfect* memristor!

2.3 No missing real devices are suggested without magnetism or mass

We described not just electrical circuit theory without (noticeable) magnetism but also an equivalent mechanical circuit theory without (noticeable) mass. What real device does *L* correspond to? There is no such thing yet! *L* will become something physical below, but whether such exists and in which way, depends on the details of the system, the universe we (or the hollow sphere) are in. The grounds on which the real memristor device was proposed is still absent. A real EM inductor device (or crucially mass-involving (MI) inductor device) *cannot* be known yet, but this *third* device is vital to the originally predicted *missing fourth.*[3] The 1971 proposal cited Mendeleev's 1870 prediction as a relevant precedent, because that hypothesis rested on empty cells in the periodic table of the elements[19] just like the missing memristor device was a vacant cell in the table Fig. 2b. In hindsight, some may like to object that our purely mechanical perfect memristor is a third device and that an inductor device can now already be hypothesized. However, one would *not* regard such memristors as a new kind of device, because they are no more than nonlinear, charge dependent resistors. *L* and *C* are also opposing edges in the tetrahedron just like *M* and *R*, and an inductor is in a sense a dual-capacitor. Nevertheless, MI or EM inductors are clearly not just nonlinear, somehow merely mathematically dual-charge dependent capacitors! The memristor can only be hypothesized as an interesting (potentially different kind of) device because the three devices, namely resistor, capacitor, and *EM* (or *MI*) inductor are obviously very different things. Mass is fundamentally inertia like magnetism is EM inductance. These are clearly new kinds of phenomena, for example magnetism involving an *induced* voltage, a force that would not be without relativistic effects. Moreover, these phenomena could be conceivably different. Our magnetism, that relativistic effect we happen to observe instead of magnetic monopoles for example, turns out to supply an EM-dual magnetic charge that behaves like the canonically dual charge. This is a coincidence for all circuit theory knows.

3 With magnetism: Real EM inductor device suggests a further device

In order to relate to the EM inductors that are *a third kind* of real device, flux must derive from the 'magnetic pair' of Maxwell theory. The electric field of MEP2 integrates to the magnetically *induced* voltage U_{in}, and the integration of the magnetic field B results in the *magnetic* flux $\varphi_{\text{m}} = -\int_{\text{Area}} \mathrm{d}\vec{f} \cdot \vec{B}$. This "flux-*linkage*" links the magnetic field to the induced voltage. The resulting *voltage-flux* relation relates U_{in} to the time derivative of the magnetic flux: $U_{\text{in}} = \mathrm{d}\varphi_{\text{m}} / \mathrm{d}t$. The magnetic field adds φ_{m} as a fourth corner that erects a tetrahedron on top of the triangle (Fig. 4a), much like introducing electric forces erected that triangle.

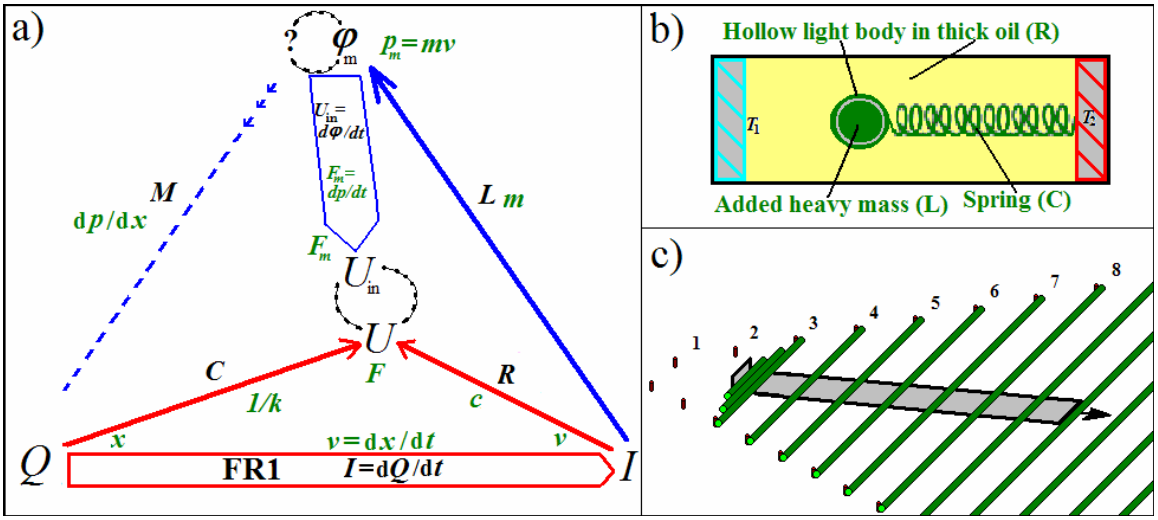

Figure 4: (a) The symmetry that suggest an EM memristor device. (b) The sphere has now a heavy mass *m*. The spring-system can therefore oscillate like LRC-circuits. (c) A system picks up massive rods (green), depicted when adding the third rod, taking it away from its holding pins (red). d*m*/d*x* is proportional to x^{α} with suitable lengths and weights of rods. The rods stick sufficiently to add inertia also on the way back, but the sudden impact with their holding pins removes them again from the pile.

φ_{m} is an EM-dual, magnetic charge, as is illustrated by calculating φ_{m} around a Dirac monopole $\vec{B} \propto \vec{r}/(r^2|\vec{r}|)$ and comparing the result $B = \varphi_{\mathrm{m}}/(4\pi r^2)$ with the electron's $E = e/(4\pi r^2)$. However, it is the known device, here the EM inductor, which led us to construct the tetrahedral symmetry in the first place (also historically). The symmetry *together with the real inductor* may now suggest that *M* perhaps also corresponds to a real device. However, this hypothesized device, suggested on grounds of the EM inductor, is an EM memristor in the sense that it must also be absent without magnetism. It must be absent without magnetism, because its existence would otherwise suggest the inductor device (precisely in the same way as the memristor was suggested to be the missing fourth next to the known third device, the inductor). Without magnetism however, a suggested inductor device cannot be the EM inductor, even if something somehow similarly behaving is found. Moreover, the circuits couple the devices; U_{C} gives rise to U_{R} falling along a de-charging resistor. Q is made from the same charges on the capacitor as it is then in the resistor while de-charging. This suggests a more direct coupling between the hypothesized memristor's flux and the inductor's flux as indicated in Fig. 4a. It suggests that they should be fundamentally the same magnetic flux. Also the original 1971 hypothesis demanded an EM memristor: "… *the physical mechanism characterizing a memristor device must come from the instantaneous (memoryless) interaction between the first-order electric field and the first-order magnetic field…*"[2] This is today controversial, although it was precisely this which made the hypothesized memristor interesting to many. The implied EM memristor promised to be a new, *fourth kind* of device that moreover corresponds to the EM inductor like a sort of complementary EM symmetry counterpart, much like electrons suggest magnetic monopoles.

In the mechanical analog, we can switch on inertia, or if the sphere's mass m was merely not noticeable in the thick oil, add a heavy mass m (Fig. 4b), which is equivalent to an LRC-circuit's L.[1] $\varphi_{\mathrm{m}} = L\, I$ is here mass momentum $p_{\mathrm{m}} = m\, v$. The new force $\mathrm{d}p_{\mathrm{m}} / \mathrm{d}t = m\, a = F_{\mathrm{m}}$ derives from the new momentum like U_{in} from φ_{m}. A real "dual-charge capacitor" device corresponds to m, namely *inert* mass. 93.9g of iridium fit into the sphere to keep it noticeably moving after suddenly switching off $F_{\max}$. Given that three kinds of real devices now correspond to all three out of four mechanical BCE, we hereby hypothesize a missing fourth, precisely as done in 1971. It should be a MI memristor that cannot exist in a world without (observable) inert mass. The position dependence of c due to T-gradients is insufficient, because that mechanical memory resistor is already known; it does not require inert mass; it was not postulated as a fourth missing device; and its $\mathrm{d}p = 6\pi r \eta_{(x)} \mathrm{d}x$ is not inertia carrying mass momentum. Its $\mathrm{d}p$ couples to p_{m} via the circuit (forces F), for instance if we attach a heavy mass at the free, left end of the forcing-lever in Fig. 3. This coupling is even more direct if m is inside the hollow sphere, but such is simply adding a MI inductor into a memory resistor; the mass is not crucially involved in the memristance but merely turns our perfect memristor into a memristive system.

3.1 Can EM and MI memristors be discovered?

Especially in the EM case one should not expect a new set of devices as if a new independent field is introduced. The symmetry would be richer with true magnetic charges. Magnetic monopoles would allow magnetic capacitor devices, but our magnetism is a relativistic effect. Loosely speaking, it should not surprise if only half of the naively expected two new devices exist, namely EM inductors but not EM memristors. Momentum p_{m} is a new charge with its force $F_{\mathrm{m}} = m\,a$, not a relativistic effect. Contraptions that pick up mass along x (Fig. 4c) yield MI memristive systems, because $m_{(x)}$ makes mass crucial, and the memristance is effectively a $c_{(x,v)}$ as well as nonlinear, e.g. $\left(\mathrm{d}p/\mathrm{d}x\right)\big|_{\mathrm{v}} = v\left(\mathrm{d}m/\mathrm{d}x\right) \propto v x^{\alpha}$; $\alpha \neq 0$. However, for a perfect $c_{(x)}$, mass must depend on v. This is generally possible, for example in special relativity theory, but the mass must depend in just the right way and yet still be inert mass with momentum even at zero velocity although there is no mass momentum without velocity. EM memristors may perhaps have φ_{m} also at zero current if magnetic fields are sustained as EM fields via electro-optics. MI memristors may need nanotechnology, but EM memristors are rightly expected in optics or similar; so "*those interested in memristive devices were searching in the wrong places*"[5] misunderstands.

4 Concluding remarks

The mechanical analog illuminated a core problem: L and M are on the same, symmetrical footing in circuit theory, but the discovered memristors (also our own perfect mechanical one) cannot deliver the grounds of hypothesizing inductors, neither EM/MI inductors *nor lesser ones*, as was explained. Disagreeing with this either implicitly claims that after finding perfect memristors, and even without ever finding any inductors, mere circuit theory predicts inert mass and relativistic magnetism. Or otherwise, and this is the apparent consensus today, one claims that the whole issue was never more than mere circuit theory, and magnetism and mass are merely interesting ways of realizing inductors. This is clearly not true, neither historically, nor do we have rigorous arguments for why the ratio $M = dQ_{Dual}/dQ$ does not allow an EM memristor, an interesting *fourth kind* of device that is not just a complicated resistor. Science must keep searching or disproving. The widespread opinion about that the missing memristor has been discovered is detrimental toward that endeavor.

References

[1] Feynman, R., Leighton, R. B. & Sands, M. L. The Feynman lectures on physics vol. 1. Pearson Education Inc. chapter 23-3, page 23-5 to 23-6, (1965).

[2] Chua, L. O. Memristor – the missing circuit element. *IEEE Trans. On Circuit Theory* **CT-18**, 507-519 (1971).

[3] Tour, J. M. & He, T. The fourth element. *Nature* **453**, 42-43 (2008).

[4] Chua, L. O. & Kang, S. M. Memristive Devices and Systems. *Proceedings of the IEEE* **64**(2), 209-223 (1976).

[5] Strukov, D. B., Snider, G. S., Stewart, D. R. & Williams, R. S. The missing memristor found. *Nature* **453**, 80-83 (2008).

[6] Argall, F. Switching Phenomena in Titanium Oxide Thin Films. *Solid-State Electronics* **11**(5), 535-541 (1967).

[7] Pershin, Y. V. & Di Ventra, M. Memory effects in complex materials and nanoscale systems. *Adv. in Phys.* **60**(2), 145-227 (2011).

[8] Di Ventra, M. & Pershin, Y. V. On the physical properties of memristive, memcapacitive and meminductive systems. *Nanotechnology* **24**(25), 255201 (2013).

[9] Chandra, S. On the Discovery of a Polarity-Dependent Memory Switch and/or Memristor (Memory Resistor). *IETE Technical Review* **27**, 179-180 (2010).

[10] Vongehr, S. Missing the Memristor. *Adv. Sci. Lett.* **17**, 285-290 (2012).

[11] Upadhyaya, H. M. & Chandra, S. Polarity dependent memory switching behavior in Ti/Cd Pb S/Ag system. *Semiconductor Science and Technology* **10**, 332-338 (1995).

[12] Waser, R. & Aono, M. Nanoionics-based resistive switching memories. *Nature Materials* **6**, 833-840 (2007).

[13] Lau, C. N., Stewart, D. R., Williams, R. S. & Bockrath, M. Direct Observation of Nanoscale Switching Centers in Metal/Molecule/Metal Structures. *Nano Letters* **4**(4), 569-572 (2004).

[14] Wu, X., Zhou, P., Li, J., Chen, L. Y., Lu, H. B., Lin, Y. Y. & Tang, T. A. Reproducible unipolar resistance switching in stoichiometric ZrO2 films. *Applied Physics Letters* **90**, 183507 (2007).

[15] Y. Rodenbush CM, Hsieh FH, Viswanath DS. Density and viscosity of vegetable oils. *J. Am. Oil. Chem. Soc.* **76**, 1415e9 (1999).

[16] Esteban, B., Riba, J-R., Baquero, G., Rius, A. & Puig, R. Temperature dependence of density and viscosity of vegetable oils. *Biomass and Bioenergy* **42**, 164-171 (2012).

[17] Di Ventra, M., Pershin, Y. V. & Chua, L. O. Circuit elements with memory: memristors, memcapacitors and meminductors. *Proceedings of the IEEE* **97**, 1717 (2009).

[18] Chua, L. O. Resistance switching memories are memristors. *Appl. Phys. A* **102**(4), 765-783 (2011).

[19] van Spronsen, J. W. The Periodic System of Chemical Elements. New York: Elsevier (1969).