Lunar Drift Explains Lunar Eccentricity Rate

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Abstract Herein, we argue that the observed $+38\,\mathrm{mm/yr}$ secular Lunar drift from the Earth does – to an admirable degree of agreement between theory and observations; explain the observed secular increase in the Lunar eccentricity. At present, the recession of the Moon from the Earth is not any more considered as an anomaly as this is believed to be well explained by conventional physics of Lunar-Earth tides. However, the same is not true when it come to the observed increase in the Lunar eccentricity which is considered to be an anomaly requiring an explanation as to what is the cause behind this phenomenon. We not only demonstrate an intimate connection between these two seemingly unrelated phenomenon, but show that the intimate relationship that we deduce fits so well with observations to a point that – logic dictates that, the Lunar drift must surely be the cause of the secular increase in the Lunar eccentricity.

Keywords astrometry, celestial mechanics, ephemerides, planetary recession

1 Introduction

The most recent analysis (Williams et al. 2014) of Lunar Laser Ranging (LLR) data records panning about $43\,\mathrm{yr}$ revealed – at a 3σ-level of statistical significance; an anomalous increase in the mean Earth-Moon distance of about $+38.10 \pm 0.20\,\mathrm{mm/yr}$. This secular increase is a phenomenon that has been well verified over the years (Dickey et al. 1994; Williams et al. 2001, 2004; Williams and Boggs 2009*b*; Williams et al. 2008, 2013, 2014) and the existence of this phenomena, has been known (*cf.* Stephenson and Morrison 1995) since the times[1] of the famous English astronomer, geophysicist, mathematician, meteorologist, and physicist – Edmund Harley $(1656 - 1742)$. As to its cause, not only is this secular increase attributed to Lunar-Earth tides, it is consistent with tidal theory (Williams and Boggs 2009*a*; Williams et al. 2014), the meaning of which is that it is caused by Earth-Lunar tides.

As determined by the analysis of the aforesaid $43\,\mathrm{year}$ span of LLR data, it has been found that tides induce a semi-major axis rate of $+38.10 \pm 0.20\,\mathrm{mm/yr}$, corresponding to a centennial acceleration rate of the Moon's orbital mean longitude of about $-25.80 \pm 0.10''\mathrm{cy}^{-2}$ (Williams and Boggs 2009*a*; Williams et al. 2014). This modern result is in excellent agreement with analysis made by Stephenson and Morrison (1995) of eclipse data record of the past 2700 years or so; this analysis gave a centennial acceleration rate of the Moon of $-25.80 \pm 0.03''\mathrm{cy}^{-2}$. Further, this LLR result is consistent with analysis made with different data spans, different analysis techniques, analysis of optical observations, and independent knowledge of tides (Williams and Boggs 2009*a*; Williams et al. 2014). Therefore, the $\sim +38\,\mathrm{mm/yr}$ Lunar recession is not an anomaly but a well understood phenomenon resulting from Lunar-Earth tides.

Apart from the secular increase in the mean Earth-Moon distance, the Lunar eccentricity ϵ_{moon} has been found to be undergoing a secular increase and this is considered an anomaly that needs an explanation (cf. Williams et al. 2001; Williams and Dickey 2009; Anderson and Nieto 2009; Iorio 2011*a*,*b*, 2014*a*,*b*). Based on a meticulous analysis of LLR data records spanning $38.7\,\mathrm{yr}$ (*i.e.*, from 1970 March 16 to 2008 November 22: DE421 ephemerides) and using an accurate model that takes into account all known and relevant Newtonian and Einsteinian effects including tidal dissipation in the interiors of both the Earth and the Moon, Williams et al. (2001) deduced an annual lunar eccentricity rate of:

$$\dot{\epsilon}_{\mathrm{moon}} = +(9.00 \pm 2.00) \times 10^{-12}\,\mathrm{yr}^{-1}. \quad (1)$$

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[1]According to Stephenson and Morrison (1995), Edmond Halley (1665) was the first to suggest that the mean motion of the Moon was apparently getting faster, and this suggestion he made after an analysis ancient Solar eclipse records.

Subsequent analysis by Williams and Dickey (2009) – an analysis that relays upon the initial work of Williams et al. (2001); yielded a much larger anomalous eccentricity rate of:

$$\dot{\epsilon}_{\rm moon} \;=\; +\,(16.00\,\pm\,5.00)\,\times\,10^{-12}\,{\rm yr}^{-1}. \tag{2}$$

Now, in their latest work where they used improved tidal models, Williams et al. (2014) extended their analysis of the LLR data by using the new DE430 ephemerides (Folkner et al. 2014). In this latest analysis, the anomalous eccentricity rate of the Lunar orbit did not vanish but lingered on – *albeit*, significantly reduced and now amounting to:

$$\dot{\epsilon}_{\rm moon} \;=\; +\,(5.00\,\pm\,2.00)\,\times\,10^{-12}\,{\rm yr}^{-1}. \tag{3}$$

In this reading, we shall adopt (3) as a measure of the eccentricity rate of the Lunar orbit. What we shall do is to demonstrate that a link between the eccentricity rate of the Lunar orbit and its annual secular drift of $\sim\,+38\,{\rm mm/yr}$, these can intimately be related to one another. What is striking is that the deduced relationship between the eccentricity rate and the lunar drift agree so well that, seductively, one is compelled some how to consider this relation to be authentic so much that, if the Lunar drift is considered as not being an astrometric anomaly requiring new physics, on the same footing, the eccentricity rate of the Lunar orbit is also to be considered not as being an anomaly requiring new physics to explain it.

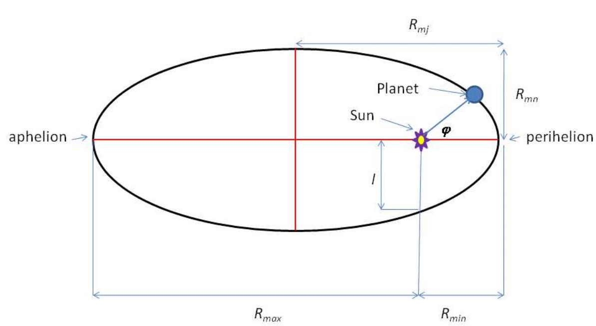

Fig. 1 This diagram gives a *birds-eye-view* of the planetary orbit. The orbit is an ellipse with the Sun at one of foci. The minor and major axis are represented by $\mathcal{R}_{\rm mn}$ and $\mathcal{R}_{\rm mj}$ respectively. The minimum and maximum distance of the planet from the Sun are $\mathcal{R}_{\rm min}$ and $\mathcal{R}_{\rm max}$ respectively. The distance $l = (1-\epsilon^2)\mathcal{R}_{\rm min}$ is the distance of the planet away from the Sun when $\varphi = 90^{\rm o}$: ϵ is the eccentricity of the orbit.

2 Theorem (Relation of Eccentricity Rate and Drift)

The eccentricity ϵ of a closed orbit – such as those for planets in our Solar system; is defined such that:

$$\epsilon = \frac{\mathcal{R}_{\rm max}-\mathcal{R}_{\rm min}}{\mathcal{R}_{\rm max}+\mathcal{R}_{\rm min}}, \tag{4}$$

where $\mathcal{R}_{\rm max}$ and $\mathcal{R}_{\rm min}$ aphelion and perihelion distances of the orbiting test body, and these terms $(\mathcal{R}_{\rm max}, \mathcal{R}_{\rm min})$ are defined in Figure (1), then:

$$\frac{\dot{\epsilon}}{\epsilon} = -\frac{\dot{\mathcal{R}}_{\rm mj}}{\mathcal{R}_{\rm mj}}. \tag{5}$$

2.1 Proof

In the present subsection, we shall take the former definition of ϵ *i.e.* ϵ in-terms $\mathcal{R}_{\rm max}$ and $\mathcal{R}_{\rm min}$ $[\epsilon = \epsilon(\mathcal{R}_{\rm max}, \mathcal{R}_{\rm min})]$. Differentiating (4) with respect to time and then dividing the resultant equation by ϵ, one obtains:

$$\frac{\dot{\epsilon}}{\epsilon} = \frac{\dot{\mathcal{R}}_{\rm max}-\dot{\mathcal{R}}_{\rm min}}{\mathcal{R}_{\rm max}+\mathcal{R}_{\rm min}} - \frac{\dot{\mathcal{R}}_{\rm max}+\dot{\mathcal{R}}_{\rm min}}{\mathcal{R}_{\rm max}+\mathcal{R}_{\rm min}}. \tag{6}$$

We know that:

$$\mathcal{R}_{\rm mj} = \frac{1}{2}\left(\mathcal{R}_{\rm max}+\mathcal{R}_{\rm min}\right), \tag{7}$$

$$\mathcal{R}_{\rm max} = (1+\epsilon)\,\mathcal{R}_{\rm mj}, \tag{8}$$

$$\mathcal{R}_{\rm min} = (1-\epsilon)\,\mathcal{R}_{\rm mj}. \tag{9}$$

From (7), we will have:

$$\frac{\dot{\mathcal{R}}_{\rm max}+\dot{\mathcal{R}}_{\rm min}}{\mathcal{R}_{\rm max}+\mathcal{R}_{\rm min}} = \frac{2\dot{\mathcal{R}}_{\rm mj}}{2\mathcal{R}_{\rm mj}} = \frac{\dot{\mathcal{R}}_{\rm mj}}{\mathcal{R}_{\rm mj}}. \tag{10}$$

From (8) and (9), we have that the subtraction of $\mathcal{R}_{\rm min}$ from $\mathcal{R}_{\rm max}$ is such that $(\mathcal{R}_{\rm max}-\mathcal{R}_{\rm min} = 2\epsilon\mathcal{R}_{\rm mj})$, so that $(\dot{\mathcal{R}}_{\rm max}-\dot{\mathcal{R}}_{\rm min} = 2\dot{\epsilon}\mathcal{R}_{\rm mj}+2\epsilon\dot{\mathcal{R}}_{\rm mj})$. From this result and as-well the definition of $\mathcal{R}_{\rm mj}$ as given in (7), it follows that:

$$\frac{\dot{\mathcal{R}}_{\rm max}-\dot{\mathcal{R}}_{\rm min}}{\mathcal{R}_{\rm max}+\mathcal{R}_{\rm min}} = \frac{2\dot{\epsilon}\mathcal{R}_{\rm mj}+2\epsilon\dot{\mathcal{R}}_{\rm mj}}{2\mathcal{R}_{\rm mj}} = \dot{\epsilon}+\epsilon\frac{\dot{\mathcal{R}}_{\rm mj}}{\mathcal{R}_{\rm mj}}. \tag{11}$$

Now, substituting (10) and (11) into (6), we will have:

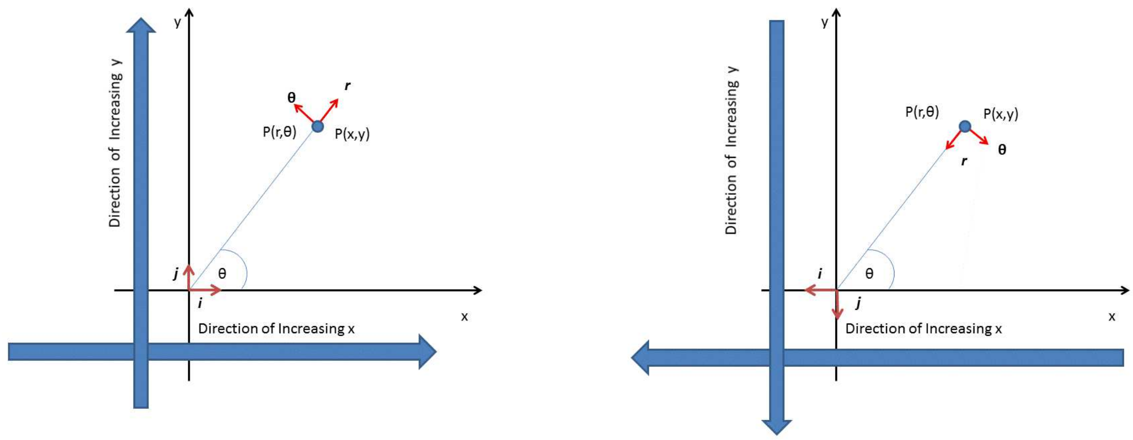

Fig. 2 In the left panel is a diagram of the usual coordinate system that we are used to *i.e.* the Right Handed Coordinate (RHC) system. To the right panel is the parity transformed coordinate system of the RHC-system, *i.e.*, the Left Handed Coordinate (LHC) system. In the RHC-system, a drift is when $(\dot{r} > 0)$ and in the LHC-system, a drift will occur when $(\dot{r} < 0)$.

$$\frac{\dot{\epsilon}}{\epsilon} = \dot{\epsilon} + \epsilon \frac{\dot{\mathcal{R}}_{\mathrm{mj}}}{\mathcal{R}_{\mathrm{mj}}} - \frac{\dot{\mathcal{R}}_{\mathrm{mj}}}{\mathcal{R}_{\mathrm{mj}}}, \tag{12}$$

and now taking the term $\dot{\epsilon}$ to the left hand-side of the equality sign and thereafter rearranging, we will have:

$$(1-\epsilon)\frac{\dot{\epsilon}}{\epsilon} = -(1-\epsilon)\frac{\dot{\mathcal{R}}_{\mathrm{mj}}}{\mathcal{R}_{\mathrm{mj}}}. \tag{13}$$

From the above, it follows that:

$$\frac{\dot{\epsilon}}{\epsilon} = -\frac{\dot{\mathcal{R}}_{\mathrm{mj}}}{\mathcal{R}_{\mathrm{mj}}}. \tag{14}$$

Hence result. This result is independent of the mechanism that is the cause of the drift or the eccentricity rate.

3 Drifts and the Type of Coordinate System Used

Now, what (14) implies is that an increasing Lunar eccentricity rate must result from a Moon that instead of receding from the Earth, it approaches the Earth. Thus, at a *prima face* level, it would appear as though (14) can not explain the observed increase in the Lunar eccentricity rate. As we shall demonstrate shortly, this results from the choice of the coordinate system that one employs *i.e.* whether this coordinate system is left or right handed. For the purposes of this reading, the Left Handed Coordinate (LHC) system and the Right Handed Coordinate (RHC) systems are defined in Figure (2). For simplicity, we have in Figure (2) considered the two dimensional case $[(x, y); (r, \theta)]$, this can be extended to three dimensions and the result obtained is not changed.

The RHC-system is the coordinate system that we are used to, while the LHC-system is the mirror image of the RHC-system. In the RHC-system, a drift will occur when $(\dot{r} > 0)$ and in the LHC-system, a drift will occur when $(\dot{r} < 0)$. What this implies is that the sign of $\dot{r}$ depends on the choice of the coordinate system. In-order to in-cooperate this into our result (14), there is need to introduce a parity term δ_{RL}, which is such that $(\delta_{\mathrm{RL}} = -1)$ if the coordinate system is right handed and $(\delta_{\mathrm{RL}} = +1)$ if the coordinate system is left handed, that is to say:

$$\frac{\dot{\epsilon}}{\epsilon} = -\delta_{\mathrm{RL}}\frac{\dot{\mathcal{R}}_{\mathrm{mj}}}{\mathcal{R}_{\mathrm{mj}}}. \tag{15}$$

Now, from §(15), in-order that we are in tandem with physical and natural reality as revealed by observations, it follows that for the Moon, we must have $(\delta_{\mathrm{RL}} = -1)$ since $(\dot{\epsilon}_{\mathrm{moon}}/\epsilon_{\mathrm{moon}} > 0)$ and $(\dot{\mathcal{R}}^{\mathrm{moon}}_{\mathrm{mj}}/\mathcal{R}^{\mathrm{moon}}_{\mathrm{mj}} > 0)$, where $\mathcal{R}^{\mathrm{moon}}_{\mathrm{mj}}$ is the Moon's semi-major axis. What this all implies is that *Nature* may very well employ a LHC-system instead of the RHC-system that we employ.

4 Comparison with Observations

In-order that we evaluate (15) for the Lunar eccentricity rate, we need to compute $\dot{\mathcal{R}}^{\mathrm{moon}}_{\mathrm{mj}}/\mathcal{R}^{\mathrm{moon}}_{\mathrm{mj}}$. For the Earth-Moon system, the Earth is the central massive body and the Moon is the orbiting test body. At apogee of the Moon, the centres of mass of the two systems are $4.055 \times 10^{8}\,\mathrm{km}$ while at perigee, they are $3.633 \times 10^{8}\,\mathrm{km}$ apart. This means the mean distance of the Earth-Moon system is $(3.80 \pm 0.20) \times$

$10^8\,\mathrm{km}$. The "error" $\pm 0.20\times 10^8\,\mathrm{km}$ is not an error bar but a "bar" expressing the range between the maximum and minimum distance. From this, it follows that:

$$\frac{\dot{\mathcal{R}}_{\mathrm{mj}}^{\mathrm{moon}}}{\mathcal{R}_{\mathrm{mj}}^{\mathrm{moon}}} = (100.00\pm 5.00)\times 10^{-12}\,\mathrm{yr}^{-1}. \tag{16}$$

Now, from (16), given that[2], $\epsilon_{\mathrm{moon}} = 0.0549$, it follows that:

$$\frac{\dot{\epsilon}_{\mathrm{moon}}/\epsilon_{\mathrm{moon}}}{\dot{\mathcal{R}}_{\mathrm{mj}}^{\mathrm{moon}}/\mathcal{R}_{\mathrm{mj}}^{\mathrm{moon}}} = \frac{+(90.00\pm 40.00)\times 10^{-12}\,\mathrm{yr}^{-1}}{+(100.00\pm 5.00)\times 10^{-12}\,\mathrm{yr}^{-1}}, \tag{17}$$

so that:

$$\frac{\dot{\epsilon}_{\mathrm{moon}}/\epsilon_{\mathrm{moon}}}{\dot{\mathcal{R}}_{\mathrm{mj}}^{\mathrm{moon}}/\mathcal{R}_{\mathrm{mj}}^{\mathrm{moon}}} = +0.90\pm 0.50. \tag{18}$$

If the Lunar eccentricity rate is due to the observed Lunar drift, then, according to (15), the magnitude of the ratio $[(\dot{\epsilon}_{\mathrm{moon}}/\epsilon_{\mathrm{moon}})/(\dot{\mathcal{R}}_{\mathrm{mj}}^{\mathrm{moon}}/\mathcal{R}_{\mathrm{mj}}^{\mathrm{moon}})]$ is – from theory; expected to be identically equal to unity. Clearly, the above result (18), is clear testimony that the recession of the Moon is most certainly the cause of the increase in the Lunar eccentricity given the excellent agreement between the two measurements and what is expected from theory.

5 Discussion

In our view, the agreement between theory and observations as revealed by (18), is so good that, one is compelled by the dictates of logic to conceive in their mind that the observed Lunar drift is most certainly the cause of the Lunar eccentricity rate. Surely, the result of the present letter is important in that, if it is accepted as being correct or plausible, then, the observed Lunar eccentricity rate seizes forthwith to be an anomalous phenomenon requiring an as to its cause. If the Lunar drift is as a result of Earth-Lunar tides, so is the Lunar eccentricity rate. These two phenomenon are – according to (15); intimately tied together into an intricate and inseparable Gordian knot.

Amongst others, this agreement between theory and observations is a clear endorsement of the models that have been used to deduce the values $\dot{\epsilon}_{\mathrm{moon}}$ and $\dot{\mathcal{R}}_{\mathrm{moon}}$, for if these models where somehow not correct, the agreement between theory and observations as revealed by (18) would not hold somehow. An agreement — as that revealed by (18); using an incorrect model would only occur by an extremely very rare fortuitous chance and this is so because, the theoretical result (15) is independent of any model used; the meaning of which is that – any demonstration that a given model agrees with this result is nothing short of an endorsement of the validity of this model.

The issue of the secular Lunar eccentricity rate and the Lunar drift has attracted the attention of a significant number of researchers with some seeking an explanation from conversational physics (*e.g.* Williams et al. 2004; Acedo 2013*a*,*b*; Nyambuya 2014, amongst others); others appeal to cosmology (*e.g.* Iorio 2011*a*,*b*, 2014*b*, amongst others); while others seek an explanation from exotic ideas (*e.g.* Xin 2011; Riofrio 2012; Ziefle 2013; Williams et al. 2014, amongst others). Given the attention this phenomenon has attracted, the present letter may be important in that it now ties these two phenomenon into an intricate and inseparable phenomenon, thereby reducing the avenues which this problem may be tackled.

Before we close, it is perhaps important that we point out a related and interesting investigation that we have made on the observed $+38\,\mathrm{mm/yr}$ Lunar drift. Apart from the Lunar drift, analysis of ephemerides of the Earth obtained from spacecraft data (Krasinsky and Brumberg 2004; Standish 2005; Pitjeva 2012) has revealed that the Earth-Moon system is undergoing a secular drift of about $+(15.00\ \pm\ 7.00)\,\mathrm{cm/yr}$ (Krasinsky and Brumberg 2004) away from the Sun. Realising this, and knowing that a secular drift amongst others implies a change in the orbital angular momentum $(\boldsymbol{J})$ of the test body, in the reading Nyambuya (2014), we applied the Law of Conversation of total angular momentum $(\boldsymbol{L})$ to the Earth-Moon system (that is, the sum of the orbital angular momentum $\boldsymbol{J}$ and the spin $\boldsymbol{S}$ angular momentum of the test body: $\boldsymbol{L} = \boldsymbol{J} + \boldsymbol{S}$), where upon we where able to explain the observed $+38\,\mathrm{mm/yr}$ Lunar drift as not being a result of tides, but a direct result of the observed annual $+(15.00\ \pm\ 7.00)\,\mathrm{cm/yr}$ drift of the Earth-Moon system.

The Law of Conservation of total angular momentum is independent of the gravitational model or mechanism responsible for the observed drifts. If tides are what is responsible for the observed Lunar drift, then, these same tides are what is responsible for the secular drift of the Earth-Moon system, therefore, the drift of the Earth-Moon will forthwith seize to be an anomalous observation as its explanation will be well within the provinces of the conventional theory of Earth-Moon tides.

At present, we are in the process of building a theory of gravitation that we have coined the Azimuthally Symmetric Theory of Gravitation (Nyambuya 2010, 2015*a*,*b*, abbreviated ASTG-model). This theory is built on the usual Poison-Laplace equation $(\boldsymbol{\nabla}^2\Phi = 4\pi G\varrho)$. Usually, gravitation is assumed to be a spherically symmetric phenomenon and because of this, the gravitational potential is assumed to be

[2]See: http://nssdc.gsfc.nasa.gov/planetary/factsheet/moonfact.html. Visited on this day $14/3/2015@14\mathrm{h}26\ \mathrm{GMT}+2$.

dependent only on the radial coordinate r *i.e.* $\Phi = \Phi(r)$. In the ASTG-model, we extended the gravitational potential's dependence on the coordinates so that $\Phi = \Phi(r, \theta)$, and the angular dependence (θ) is attributed to the spin of the gravitating body. We have shown in the reading Nyambuya (2015*b*) that the ASTG-model, does, to a reasonable extent, provide an alternative explanation of the observed secular recession of the Earth-Moon system. The ASTG-model attributes the secular drift of the Earth-Moon system to the loss of orbital angular momentum which is itself – according to the ASTG-model; caused by the azimuthally dependent gravitational field $[\Phi = \Phi(r, \theta)]$.

Now, as already said, in the reading Nyambuya (2014), we demonstrated that independent of any gravitational model, the Law of Conservation of total angular momentum when applied to the recession (away from the Sun) of the Earth-Moon system, this recession can be shown to be the cause of the observed Lunar drift and not tides. What this implies is that there is a need to introspect the ASTG-model with respect to tidal theory. Tidal theory takes into account first and second order azimuthal gravitational terms that arise due local effects induced by the shape and distribution of mass for a given gravitating object. What tidal theory does not do is to attribute the extra gravitational poles to the spin of the gravitating object but to the shape and distribution of mass for a given gravitating object. Just maybe, it might very be that tidal theory and the ASTG-model share a common ground. Because of this issue of Lunar drift and Lunar eccentricity, we now anticipate that in the very near future, we shall conduct an introspection of the ASTG-model – *i.e.*, a study on how it compares with tidal theory.

6 Conclusion

Assuming the correctness and acceptability of the thesis presented herein, we hereby set-forth the following as our conclusion:

1. If the observed $+38\,\mathrm{mm/yr}$ Lunar drift is as a result of Earth-Lunar tides, so is the Lunar eccentricity rate. Therefore, these two phenomenon are not anomalous in their nature as they are explained by the conventional physics of Earth-Lunar tides.

2. The excellent agreement between theory and observations here witnessed in the present reading is a clear endorsement of the models that have been used to deduce the values $\dot{\epsilon}_{\mathrm{moon}}$ and $\dot{\mathcal{R}}_{\mathrm{moon}}$, for if these models where somehow not correct, the agreement between the independent theoretical result (15) and observations as revealed by (18) would not hold. Apart from the endorsement of the models, it is a glowing testimony to the accuracy of the LLR data and the data reduction methods used.

3. The eccentricity rate (14) suggests that *Nature* may very well employ a LHC-system instead of the RHC-system that we employ.

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