A simple algorithm to express any odd composite number that is a product of k-primes not necessarily distinct as a sum of exactly k unequal terms

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Abstract: If N is an odd composite number that can be written as a product of k-primes not necessarily distinct, then we have devised a simple algorithm that would allow us to express N as the sum of exactly k terms all distinct derived using its prime factors.

Results:

Let N be an odd composite number that is a product of exactly k-primes
\( p_1, p_2, p_3, \ldots, p_{k-2}, p_{k-1}, p_k \).

Therefore \( N = p_1 p_2 p_3 \ldots p_k \)

Consider a circle.

If in step 1, we cut we cut it at \( p_1 \) positions it would result in \( p_1 \) arcs.

(Note that we have used a total of \( p_1 \) cuts to the circle until now to yield \( p_1 \) arcs)

If in step 2, we cut within each arc from step 1 at \( p_2-1 \) positions, then we would end up with \( p_1 p_2 \) arcs and in this step alone we have used \( p_1(p_2-1) \) cuts.

(Note that we have used a total of \( p_1 + p_1(p_2-1) \) cuts to the circle from the beginning to yield \( p_1 p_2 \) arcs at end of step 2)

If in step 3, we cut within each of the arcs at end of step 2 at \( p_3-1 \) positions, then we would end up with \( p_1 p_2 p_3 \) arcs and in this step alone we have used \( p_1 p_2 (p_3-1) \) cuts.

(Note that we have used a total of \( p_1 + p_1(p_2-1) + p_1 p_2 (p_3-1) \) cuts to the circle from the beginning to yield \( p_1 p_2 p_3 \) arcs at end of step 3)

Using the same strategy at the end of k-steps we would end up with N arcs which is \( p_1 p_2 p_3 \ldots p_k \) and we have used \( p_1 p_2 p_3 \ldots (p_k-1) \) cuts in the kth step.

(Note that we have used a total of
\( p_1 + p_1(p_2-1) + p_1 p_2 (p_3-1) + \ldots + p_1 p_2 p_3 \ldots (p_k-1) \) cuts to the circle from the beginning to yield \( N = p_1 p_2 p_3 \ldots p_k \) arcs at end of kth step)

Conclusions:

If \( N = p_1 p_2 p_3 \ldots p_k \) where \( p_1, p_2, p_3, \ldots, p_{k-2}, p_{k-1}, p_k \) are k-primes not necessarily distinct then we can express N as the sum of k distinct terms as follows
\[ N = p_1 + p_1(p_2-1) + p_1p_2(p_3-1) + \ldots + p_1p_2p_3\ldots(p_{k-2}-1) + p_1p_2p_3\ldots(p_{k-1}-1) + p_1p_2p_3\ldots(p_{k-1}) \]

Since we are dealing with an odd composite number \( N \), none of the \( k \) prime factors of \( N \) is equal to the even prime 2 and therefore all the \( k \)-terms in the sum partition derived using the above algorithm are unequal.