Taming the Probability Amplitude

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Abstract

We show that the complex number structure of the probability allows to express explicitly the relationship between the energy function H and the Laplace principle of equal ignorance (LPEI). This nonlinear relationship reflecting the measurement properties of the considered systems, together with the principle of causality and Newton principle separating the dynamics from initial conditions, lead to the linear Schrodinger equation with the Max Born interpretation, for micro and macro systems!

Contents

1 Introduction

Let us imagine a system consisting of elements about whom full information is unavailable, or moving in a non-smooth manner (everything is quantized), or one for which it does not make sense to talk about motion. For such systems, it is still possible to talk about continuous probabilities of finding the system in certain 'spatial configurations'. Examples of such systems are: a set of microparticles, a set of decisions, or a set of polls.

Descriptions of systems can depend on a chosen precision. If, for example, for a set of particles, the description is such that at any time we simultaneously know its position and momenta changing in a continuous way, then we are dealing with a classical description. If, for any time, we only know simultaneously the probability of positions and momenta, we have a **classical stochastic description**. However, if we only know either the probability of positions or the momenta but not both, and additionally, probability has the complex number structure, it is a **quantum-like description**; Such a description can also be used for systems to which the classical concept of motion is not applied.

In our opinion, the most basic idea introduced into science by QM is the **probability amplitude**, A, of a certain event or process. By means of this, the corresponding probability, P, is obtained using the two operations: conjugation, $A \rightarrow A^{\star}$ and multiplication:

$$P = A^{\star} \cdot A = |A|^2 \tag{1}$$

In this sense, we're talking that probabilities P, which are numbers belonging to the interval $<0,1>$, have a complex structure. It is also called the **Born probability rule.** The structure (1) allows us to relate certain physical functions to probabilities for which we can apply the **Laplace's Principle of Equal Ignorance (LPEI), [18],** which is usually used in such cases as coin tossing or dice, [12]. We can simply choose

$$A \sim e^{iH(p,x)} \tag{2}$$

with a constant module $|A|$, for H, a Hamiltonian of the system with 3n dimensional vectors p, x ,or, H may even be given by a completely different function depending on certain physical variables $\tilde{X}$ and certain hidden variables $\tilde{P}$. We now see that the P (probability) does not depend on H but via (2) we see a relation between P and H. In fact, later we will choose the modified dimensionless characteristic of the system, namely: $\frac{t}{\hbar}H(\tilde{P}, \tilde{X})$. In this case, we get:

$$A \sim e^{i\frac{t}{\hbar}H(\tilde{P},\tilde{X})} \tag{3}$$

In the paper, the LPEI in which a probability has structure (1) and the amplidude A is related to some physical function will be called the **Generalized Laplace Principle of Equal Ignorance (GLPEI).** By GLPEI we will also understand situation in which the amplitude A is an operator $\hat{A}$ expressed by formula:

$$\hat{A} = e^{-i\frac{t}{\hbar}\hat{H})} \tag{4}$$

where $\hat{H} = \hat{H}^{\dagger}$is the Hermitian operator. The *operator-valued probability amplitude* is related to probability in a more indirect and global way, see Sec.4. In fact, the above relations express a connection of measurements with the LPEI and, I think, this is a reason why QM is considering a theory with explicitly incorporated measurements of certain quantities like energy and momentum, see also 46.

As far as the **principle of superposition** is concerned, this is not a vital feature that distinguishes quantum from classical systems, because linear equations are used to describe various systems, particularly in situations of sensitivity to small changes in the initial and boundary conditions. Furthermore, a superposition of the two solutions with a probabilistic interpretation leads to a solution for which the probabilistic interpretation requires some renormalization, just like in statistical physics. In other words, the principle of superposition with the necessity for primitive renormalization is generally associated with theories in which a detailed or complete description of the components of the system is not taken into account.

In fact, the aim of the paper is to show which rules relating to the sum and the product of events enforces the probability structure (1), see Secs 2 and 3. We also want to show a role played by GLPEI in derivation of the Schrodinger equations.

2 Relationship of probability amplitudes with unitary space.

Unitary space is a vector space over the field $\mathcal{C}$ of complex numbers, on which there is a given scalar product (inner product) of vectors. In fact, we resigne from rigorous compliance of the last demand.

Using Dirac's notation, for any vector $|\Psi>$, we have:

$$|\Psi>=\sum c_j|\psi_j> \tag{5}$$

where $c_j \in \mathcal{C}$ and the scalar products of any orthonormal base vectors $|\psi_j>$satisfy the following relations:

$$<\psi_i|\psi_i>=\delta_{ji} \equiv Kronecker's\, \delta \tag{6}$$

Hence,

$$<\Psi|\Psi>=\sum_{j=1}^{n\, or\, \infty} |c_j|^2 \tag{7}$$

and

$$<\psi_i|\Psi>=c_i \tag{8}$$

For the *normalized vector* $|\Psi>$, $<\Psi|\Psi>=1$, the following Born probabilistic interpretation is possible:

c_j - can be interpreted as an amplitude of probability for occurrence of the event denoted by j, if appropriate actions (measurements) are carried out,

$|\Psi>$- can be called a *generating vector* of all these amplitudes.

Eq.8 shows the relationship of amplitudes to the scalar product $< | >$ of vectors in the considered linear space (unitary space).

If the vector $|\Psi>$ generates all possible information about the system, at a given time t, then we call it a *state vector.* Then, the formula 5 should contain the dependence on time, and in a complete base $\{|\psi_j>\}_{j\in N}$ we can write:

$$|\Psi(t)>=\sum c_j(t)|\psi_j> \tag{9}$$

where we chose here the time independent orthonormal base vectors$\{|\psi_j>\}_{j\in N}$.

We obtain an important formula

$$<\psi_i|\Psi(t)>=c_i(t) \tag{10}$$

that c**an be interpreted** in the following way: This is a probability amplitude, or probability, of the j-th random event represented by the vector $|\psi_j>$, e.g. getting j on the toss of a dice, or getting a particular measurement result for a system in the state $|\Psi(t)>$. By a *state of the system represented by the vector* $|\Psi(t)>$ one can understand a dice's velocity and its location at the moment t , or, in the case of random variables - the whole set of velocities and locations related to the ensemble of identical dices located at the time t with the same conditions. A choice of different bases can be associated with different descriptions of individual results.

We have, of course, if the above sum exhausts all possibilities indexed by j:

$$<\Psi(t)|\Psi(t)>=\sum_j |c_j(t)|^2=1 \tag{11}$$

In the operator language:

$$|\Psi(t)>=U(t)|\Psi(0)>;\ U^*(t)U(t)=I \tag{12}$$

where an *one side 'unitary' evolution* was used.

Usually, it is assumed that $|\Psi>\in H$, where H is a Hilbert space. It is interesting that the base vectors are largely arbitrary and do not necessarily belong to the space H. In QM they usually are eigenvectors of the considered system having a specific physical interpretation related to specific measurements but this does not mean that among base vectors there are vectors describing, for example, the living and the dead cat from the famous Schrodinger's cat paradox! So that was, if it existed a system and appropriate the measuring instrument with such results. We do not think that a cat inside the box and a cat with open box correspond to this situation:-). Perhaps, incidentally, this point of view was adopted by Bohr, who disagreed with Schrodinger on this point.

We assume a linear space with a scalar product (**unitary space**) and various **not necessarily orthogonal bases**, for $t>0$. This will allow us to say something about the dynamics of systems. Let us choose the base vectors $\{|\varphi_j(t)>\}$ in such a way that the components $c_j(t)$ of vector $|\Psi(t)>$ are constant, for all t:

$$|\Psi(t)>=\sum c_j|\varphi_j(t)> \tag{13}$$

Taking scalar products of the vector $|\Psi>$ with vectors of the previous base, we get:

$$<\psi_i|\Psi(t)>=\sum_j c_j<\psi_i|\varphi_j(t)> \tag{14}$$

Choosing base vectors fulfilling:

$$|\varphi_j(t)>_{t=0}=|\psi_j> \tag{15}$$

for j=1,..., we see that the initial vector $|\Psi(0)>$ is completely described by the coefficients c_j, see (6). The assumption that, for **certain** bases at least :

$$|<\psi_i|\varphi_j(t)>|^2=constant\ in\ t \tag{16}$$

would mean that the absolute value of all elemens which describe the dynamics of the system do not depend on the time. Did not resemble it the independence on time of certain forces in inertial frames? Further, the dependence on i, j might be due to the influence of the initial conditions on these terms.

3 Classical and QM descriptions, Laplace's Principle of Equal Ignorance (LPEI) and arbitrariness of phases

In classical mechanics (CM), the state of thrown dice or coins can be described, based on the known laws of motion and known or unknown initial conditions. But such a description is very complicated and unnecessary, and in these cases, by assuming reliability of the objects used (e.g. a dice), we limit ourselves to stating that all **possible outcomes are equally likely.** This is famous *Laplace principle of equal ignorance* (LPEI). We can express this, for example, using the following ***generating vector***:

$$|\Psi>=\frac{1}{n}\sum_{i-1}^{n}|i> \tag{17}$$

with **orthogonal vectors** $<i|j>=\delta_{ij}$, and with interpretation of the coefficients of the above expansion as a probability of the $|i>$result. There is good reason for using QM language here. In this language the probability of getting the classical $|j>$ result is given by

$$<j|\Psi>=\frac{1}{n} \tag{18}$$

In QM, $<j|\Psi>$, is 'only' a probability amplitude! This means that $|<j|\Psi>|^2=\frac{1}{n}$ is a probability. This opens up the interesting possibility of expressing the **LPEI** in many different ways. We can now relate the probability amplitude $< j|\Psi >$ to various complex valued functions with constant modules:

$$< j|\Psi >= \frac{1}{\sqrt{n}} expi\psi_j \tag{19}$$

with an arbitrary phase ψ_j which can even depend in an arbitrary way on the time t:

$$\psi_j = \psi_j(t) \tag{20}$$

In this case, the generating vector

$$|\Psi(t) >= \sum_{j}^{n} |j >< j|\Psi(t) > \tag{21}$$

in spite of its dependence on the time t, generates equal probabilities $| < j|\Psi(t) > |^2$ which do not depend on the time, for all possible $|j >$. We also have:

$$< \Psi(t)|\Psi(t) >= \sum_{j=1}^{n} | < j|\Psi(t) > |^2 = 1 \tag{22}$$

as the probability of occurrence of any event.

In a more general case of probability amplitude:

$$< j|\Psi(t) >= c_j expi\psi_j(t);\ \psi_j(t) = \psi_j(t)*, \tag{23}$$

$$< \Psi(t)|\Psi(t) >= \sum_{j=1}^{n} |c_j|^2 = 1 \tag{24}$$

In this case the generating vector $|\Psi(t) >$(state vector) depends on the time t, even though the square of its length does not depend on the time and a **probabilistic interpretation of $|c_j|^2$ and their sum is also possible**. The same is possible, for

$$|\Psi(t) >= \sum_j |j > c_j(t) exp\{i\psi_j(t)\};\ \psi_j = \psi_j^* \tag{25}$$

where * means complex conjugation, if extra condition on the time depended coefficients $c_j(t)$:

$$< \mathit{\Psi}(t)|\Psi(t) >= \sum_j c_j^\star(t) c_j(t) = 1 \tag{26}$$

is imposed. We can treat expressions $c_j(t)exp\{i\psi_j(t)\}$ as probability amplitudes of becoming (obtaining due to measurement) of event j . The fact that there are no restrictions on the phase functions $\psi_j(t)$, comes from the vectors orthonormality $|j >$. Of course, not everyone orthonormal set of orthonormal vectors is appropriate, which can be seen in the case of cat paradox: let us take a state which is a superposition of states: the cat dead and cat alive and other states. We immediately become confused if we assume that these radically different states of cat correspond to 'simultaneous reality', instead of to different possibilities - related to or identified with our knowledge about the system.

The above arbitrariness of the phase functions can be used for incorporation in theory the fundamental property of physical systems which is responsible for unique or probabilistic predictions, namely the causality condition, see below.

4 Schrodinger equations, Newton's principle, unitarity and causality conditions

Of course, we could derive the Schrodinger equation by the identification of vectors $|j>$with eigenvectors of the Hamilton operator $\hat{H}$. Then, for constant coefficients c_j, and for

$$\psi_j(t) = -\frac{t}{\hbar}E_j \tag{27}$$

where E_j are eigenvalues related to eigenvectors $|j>$,we would obtain the Schrodinger equation satisfied by the state vector $|\Psi(t)>$. But here, we would like to derive this equation, insofar as this is possible, based on an analogy with the classical approach in which the averaging is done with respect to the initial positions and the momenta of the system. But in the case of QM, the dynamic equations, (Newtonian equations), will not be used directly as in the case of classical statistical mechanics. Adapting the formula (25) to the continuous case and using the function notation for vectors, e.g. $|j> \rightarrow f(x,j)$, we propose the following formula:

$$\Psi(t,x) = \int dpe^{ixp}exp\{-\frac{i}{\hbar}[t\cdot H(p,x) + G(t;p,x)]\}c(p) \tag{28}$$

for

$$G(0;p,x) = \dot{G}(0;p,x) = 0 \tag{29}$$

where dot over G means the time derivative. Here, $H(p,x)$ is a Hamiltonian function, i.e. the energy of the classical system expressed by means of momenta and positions of the system elements. Products of three functions occurring in the above integral correspond to the 'conjunction' of **three properties**, from which the first two are expressed by GLPEI and third is affiliated with other properties as the conditions of causality and/or probabilistic interpretation of the theory.

Assuming that the formula (28) satifies the causality condition (35), the restriction (29) guarantees that the Schrodinger equation

$$i\hbar\frac{\partial}{\partial t}\Psi = \hat{H}\Psi \tag{30}$$

is satified with the Hamilton operator $\hat{H}$ which is expressed by the classical Hamilton function with non-commuting variables, see also below.

Let us consider briefly where the naturalness of the proposed formula lies. Please note that

$$\Psi(0,x) = \int dp e^{ixp} c(p) \tag{31}$$

and this means that if Ψ **satisfies a first order equation of evolution**, then the formula (28) satisfies 'any' (broad) initial condition for this equation. The first order evolution equation is a characteristic feature of the one-time probabilistic approch to classical systems.

Further, the occurrence of the $\frac{t}{\hbar}H(p,x)$ is also quite natural in view of the dimensinlessness of such combination of three dimensional quantities: $t, \hbar, H(p,x)$. The **function** $A = exp\{-i\frac{t}{\hbar}H(p,x)\}$ expresses the GLPEI. Moreover, if we agree that this function expresses the dynamics of the system, then apart from LPEI (Sec.3), the equation:

$$A^{\star}A = 1 \tag{32}$$

very likely expresses also the **Newton's principle** of independence of system dynamics from the initial variables (initial conditions). Moreover, because of the conservation of energy, these principles can be valid for any instant of time. Of course, we could say the same thing about the L instead of H. But history of QM teaches us that the extension of the space system at the primary level and the introduction of phase space rather than configuration space, shows us the heuristic validity of this step.

Now, calculating the time derivative of the expression (28), at $t = 0$, we get:

$$\frac{\partial}{\partial t}\Psi(t,x)_{t=0} = operator \cdot \Psi(0,x) \tag{33}$$

If the operator appearing in the equation does not depend explicitly on the time, and this is the case for the energy integral, it can be assumed that the above equation holds for all times, see 38, and this is the Schrodinger equation in which, for systems not composed of 'atoms', constant $\hbar$ may be replaced by another constant.

The above reasoning can be justified as follows:

$$\Psi(t,x) \Longleftrightarrow |\Psi(t) >= \hat{U}(t)|\Psi(0) > \tag{34}$$

where $U(t)$ is an operator transforming the initial state of the system represented by the vector $|\Psi(0) >$into the vector $|\Psi(t) >$representing the actual state. The ***causality condition*** means that, for an arbitrary $t > 0$ and an **arbitray** increment Δt ,

$$\hat{U}(\Delta t + t) = \hat{U}(\Delta t)\hat{U}(t) \tag{35}$$

For $\Delta t \simeq 0$,

$$\hat{U}(\Delta t) = (I + \Delta t\dot{\hat{U}}(0) + o(\Delta t))\hat{U}(t) \tag{36}$$

where the dot over a symbol indicates the time derivative at t=0. Hence,

$$\hat{U}(\Delta t + t)|\Psi(0)> = (I + \Delta t\dot{\hat{U}}(0) + o(\Delta t))\hat{U}(t)|\Psi(0)> \tag{37}$$

and, finally:

$$|\dot{\Psi}(t)> = \dot{\hat{U}}(0)|\Psi(t)> \tag{38}$$

T*he* ***causality condition*** tells us that an infinitesimal increment of the given evolution, $(\Psi(\Delta t + t) - \Psi(t))$, at any time t, does depend in a linear way on the present state of the system where the operator coefficient describing the dynamics of the evolution is constant in time. In fact, in the general case, the causality condition 35 is the simplest relation of the linearity to nonlinearity, which means that these notions complement one another, see also [19]. Due to this relation, the initial lost of information is transfered to an arbitrary moment of the system evolution, see (35). This is a Marcov process.

The ***unitarity condition***

$$\hat{U}(t)\hat{U}^{+}(t) = \hat{U}^{+}(t)\hat{U}(t) = \hat{I} \tag{39}$$

imposed on an evolution of the system provides the probabilistic interpretation of the state (wave) function $\Psi(t, x)$ and its coefficients. If the state function (state vector) is expanded on an appropriate, physically motivated, orthogonal base, e.g., we are using the eigenvectors of certain observables, then appropriate physical interpretation can be attached to the coefficients of the expansion. In fact, to probabilistic interpretation is needed only one of the above equations (***one-side unitarity***):

$$\hat{U}^{+}(t)\hat{U}(t) = \hat{I} \tag{40}$$

which could be used in the case of the non-Hermitian Hamilton operators!

The general form of formula 28 can also be justified by taking into account the operator solution of the causality and unitarity conditions:

$$\hat{U}(t) = e^{t\cdot\hat{B}},\ \hat{B} = -\hat{B}^{\dagger} \tag{41}$$

where †means Hermitian conjugation of the **operator** $\hat{B}$. From Schrodinger equation 30, we know that

$$\hat{B} = -\frac{it}{\hbar}\hat{H} \tag{42}$$

$\hat{H}$ is Hermitian Hamilton operator. Hence, using Dirac notation, we can write:

$$|\Psi(t)> = e^{-\frac{i}{\hbar}t\hat{H}}|\Psi(0)> \tag{43}$$

This evolution equation can be rewriten in a function form as follows:

$$< x|\Psi(t) >= \int dp < x|e^{-\frac{i}{\hbar}t\hat{H}}|p >< p|\Psi(0) > \tag{44}$$

where $< x|\Psi(t) >$ is interpreted as the probability amplitude of finding the system in the state $|\Psi >$ at the configuration x.

The unit operator $\hat{I}_P$, in the space spanned by the complete set of orthogonal vectors $|p >$, is represented by

$$\hat{I}_P = \int dp|p >< p| \tag{45}$$

The vectors $|x >\in R^{3N}$, refering to the x-configuration of the system through the amplitude (44), are connected with the vectors $|p >$through the equation:

$$< x|p >= e^{ixp} \tag{46}$$

which in the simplest way expresses the GLPEI relating to measurements of the particle's position or momentum. On the other hand, the above equality simply represents the relationship between the QM bases from which one describes the particle(s) positions and the second its momenta.

We also need the assumptions:

$$< x|\hat{p}^2|p >= p^2 e^{ixp},\ < x|U(\hat{x})|p >= U(x)e^{ixp} \tag{47}$$

Hence, we get

$$< x|\hat{H}|p >= H(p,x)e^{ixp} \tag{48}$$

where $H(p,x)$ is a classical Hamiltonian.

At these assumptions, taking from 44 the time derivative at $t = 0$, one can get the Schrodinger equation at zero time. From the causality condition (35), the Schrodinger equation at any time t has the same form.

5 Physical and mathematical principles of superposition and GLPEI

What conclusions can be drawn from the foregoing? First of all, in the case of incomplete information about elements of the systems, both classical and quantum systems can and should be described by linear equations. There is however one essential difference between the linearity of equations which result from the incomplete knowledge about the system and the linearity of Maxwell's equations, for example. In the first case, any linear combination of the two solutions is not physical and we call the *mathematical principle of superposition*, and in the second it is, and we call the *physical principle of superposition.* In other words, what we considered to be a fundamental feature of quantum systems is not a fundamental feature but rather is a result of the impossibility of a detailed description of the system. We hypothesize here that the classical and quantum systems are more similar to each other, [4], than it is generally thought to be the case, see however: [12]. We believe that this conviction is also expressed in the GLPEI (generalized Laplace Principle of Equal Ignorance), which, due to the structure 1 of the probability, is able to combine the Principle of Equal Ignorance with classical dynamical functions. The LPEI expresses our belief that **if we do not know anything about a set of random events, the best thing is to accept equal probabilities for different events**. The GLPEI relates this belief to certain functions depending on these events.

In fact, in QM we see the composition of three things:

$$QM \Longleftrightarrow \{GLPEI + classical\, dynmics\; + linear\, evolution\} \tag{49}$$

This allows us to derive Schrodinger equations in quantum as well as in classical cases. In the first case, it is related to the fact that the measurement of positions randomly changes the momenta of a system, and in the second case, that the scale of the phenomenon and its dynamics - invalidates the importance of specific values of variables, for example, the momenta. The assumption about linear evolution of a system is taken from observation that a less precise description of any system can be described by linear equations for averaged or smoothed solutions and their correlations, see e.g. [3]. The linear evolution contains automatically the causality condition.

Does it make sense to consider the wave function for the entire universe? On this question we will try to give an answer in terms of the classic model: We assume a classical description of the universe in which the initial conditions are random variables. In this case, the theory provides only average results but observations of the universe provide, at best, only one result. But this universe is not measured with absolute accuracy and this may be an argument for the use of mean values.

See also [5], where QFT is used to explain the measurement problem.

It seems to me that the considerations presented here suggest that the amplitude of the wave function is a carrier of information, and not material representative of any particle or particles related to a given point in the 3D space. This belief is particularly clear in the case of the N particles, for N$>$ 1, because otherwise it would mean that this material representative 'lives' in the n-D space, where n is an arbitrary integer, but we should have more respect to Ockham's razor principle.

A similar issues concerning complex probability structure can be found in [8] and in the literature given there. See also Feynman's negative probabilities and what he says about the mystery of QM. Personally I think that the structure 1 is also an expression of the ***relativity of information***: Information exists only in a relative sense - that is, in relation to some other information, [14].

Finally, we could sum up this article as follows: micro-particles are classical objects with specified momenta and positions, but the inability to determine them exactly forces us to use 'quantum' description which differs from the classical canonical description in that it uses a complex structure of the probability, 1, which allows to connect the total energy of the system with the Laplace

Principle of Equal Ignorance (LPEI). To such conclusion, I think, allows us to come this article and a surprising analogy of classical and quantum descriptions in the case of incomplete information about a system, see [7, 3]. It should be added that after the discovery of the Schrodinger equation additional dicovery was important about the meaning of eigenvectors and eigenvalues of certain operators which describe the system after a given measurement when repeated measurements no longer brings anything new in classical as well as in quantum case.

We must also add that the structure 1 must be further generalized to take into account relativistcally invariant theories.

We can also say that not taking into account a detailed description of the components of the system it relates to **nonlocality**, [3]. This inherently involves the absence of a sharp definition of certain notions and variables, expressed by randomness, vagueness or ambiguity, see [2]. But this is connected with information about the system and I think again that nonlocality of quantum description is inherently related to information and not to any physical interactions. Here for information we rather understand knowledge, which does not exclude the immediate transfer of the measurement results between distant points.

One more: Determination of initial and/or the boundary conditions is associated with the measurement. If the measurement of the part of variables in a particular moment of the time is such that the values of the remaining variables are completely disrupted, then we are dealing with quantum mechanical description. In this sense, we can say that properties of the measuring instruments are included in the theoretical description.

6 Probability amplitudes in classical physics

Let us assume that all possible states of a system are described by the function called also the field

$$\varphi = \varphi[\tilde{x}; \alpha] \tag{50}$$

where $\tilde{x}$ desribes different points in the space-time and different components of the field φ. It is also the functional, if we take into account its dependence on the initial and/or boundary conditions imposed on the system and denoted by α. In the case of 'wild' solutions, different averaging or smoothing procedures are used (n-point information (n-pi)), which satisfy the linear equation, see [3]; App.1. In such cases, the probability density or the weight density, due to the linearity of equations for n-pi, can be substituted by the corresponding **density of probability amplitude**. Then, to obtain a physically verified formula, we have to take into account either only real or only imaginary parts of the corresponding n-point functions (n-pi). But I hope that this work suggests that the probability amplitude should also be used in situations where it makes no sense to talk about the differentiable trajectories of the system and the only guarantee of any rationality is the Laplace Principle of Equal Ignorance or its generalization ((G)LPEI).

App.1 A proposed experiment

It is not clear, for me, whether the structure of the probability 1 is an expression of the disturbing influence of the measurement process or the results of the wave properties of the system, see, e.g., [9]. Perhaps, with this dilemma can be resolved in the weightlessness conditions by a chaotic stream of macroscopic particles passing through two slits to mimic the lack of knowledge of their momenta. If, by appropriate selection of the parameters of the slits and the duration of the experiment, we were to obtain an 'interference' image or just a picture differing from the cumulative image obtained with a subsequent closed the first and the second slit, this could mean that the wave description may be replaced by a classical statistical description in which the particles do not have a defined momenta and wherein the probability has a structure 1. Perhaps, the experiments done with entities much larger than electrons and photons confirm this possibility (Wikipedia (2013)).

Suggestions of some authors about the possibility of using QM beyond the microscopic scale, see [11], provide some hope of getting positive result.

App.2 Bell's theorem and its consequences

Bell's Theorem says:

'No physical theory of local hidden variables (HV) can ever reproduce all of the predictions of Quantum Mechanics'

A theory of local HV may correspond to a classical description of the system usually reduced to analyzing certain differential equations in which some set of independent (initial and/or boundary) variables are treated as HV. These HV are unambiguously related to possible measurements of dynamical variables (observables).

My comment: (Perhaps the preceding sentence is also my comment?)

So, by taking the n-point information considered e.g. in [7] or in [3], in which the initial momenta or other variables are treated as hidden (random) variable, [10], we should not receive, with the help of such a theory, all the QM predictions. It seems apparent that conclusion, that the loss of information, which carries GLPEI is so radical that the sense of some of the concepts of classical, deterministic description of the system as, for instance, the concept of the differentiable trajectory of each particle of the system, cease to make sense, see again [7, 3].

App.3 Quotes from Streater, [12].

1. We review the mathematical development of probability, enphasising that quantum theory is a generalization (p1)

2. ..., Heisenberg, with help of the Copenhagen interpretation, invented a generalization of the concept of probability, and physicists showed that this was the model of probability chosen by atoms and molecules. (ps22/3)

At this point I would like to thank J. Dwyer for pointing me Streater's paper [12], see (ResearchGate (2014)).

App.4 Top down causation and Quantum Mechanics

Note first that the Hamiltonian, which appears in the QM, is the classical Hamiltonian. This means that it describes the macroscopic objects as planets which to a large scale can be regarded as point objects. These objects are formed by atoms whose structure reiterates planetary systems. Recall the Bohr's planetary model of atoms. Hence, with the help of GLPEI, we derive the Schrodinger's equation. In this sense the OM is an example of the top down causation with a *final structure of the atom* that guarantees its stability. At this point I recomend very interesting discussion on the ResearchGate forum related to the Sumanta Chakradorty's question: 'Does a uniformly accelerated charge radiate?' (2014/15)

App.5 A proper question?

I think that we should not ask ourself how or with what the universe is made, but we should ask ourself how best to describe the universe:-)!

Good illustrations of this philosophy are QM, the introduction in astrophysics of dark energy and matter, and many other scientific papers. See also [17], [3] and any history of science.

App.6 Semiotic exercise

Semiosis=sign activity, Semiotics=the sciences studying sign activity, see [15].

According to Charles Sanders Peirce (1839-1914) the sign can be used as a substitute for information. In fact, Peirce understands a *sign* as a triadic relation connecting the primary sign to its object through the production of an interpretant. "These three instances of the sign relation are connected by a **tripod rather than by a triangle** in order to emphasize the internal logic of the sign relation, which should never be confused with a mere summation of three relations between corners in a triangle" (",'' is a quote from [15]):

```
   A tripod:           |          QM:
Sign    Object              |psi>     an atom
   x........x                  x           x
     x...x                       x       x
       x                             x
       x                             x
Interpretant               Born or other interpretations
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Can we treat QM with its multiplicity of interpretations as an argument that the QM has a lot to do with modern art ? See [16].

App.7 Feynman's integrals and other formulations of QM

In my opinion GLPEI is used most visually and intensively in the Feynman integrals also called the path integral formulation. In this approach to QM is shown immence amount of possibilities for particle displacements caused by the lack of information. Another approach, the Heisenberg operator approach, expresses an influence of measurements on information about a system. In the Heisenberg approach to QM (the Heisenberg picture), the dynamical variables are substituted by operators (e.g. matrices)

$$q(t) \to \hat{q}(t) \tag{51}$$

which satisfies the operator Newton equations. Considering all possible n-'point' information (n-pi):

$$< \Phi|\hat{q}(t_1)\cdots\hat{q}(t_n)|\Psi > \tag{52}$$

for n=0,1,..., where $|\Phi, \Psi >$are given arbitrary vectors from, e.g., Hilbert space, we get for them exactly the same equations as for n-pi (n-point correlation functions) in the case of **classical Newton equations with random initial conditions**. Basic difference is such that classical correlation functions are permutationally invariant but quantum n-pi are not. However, in the **free Fock space,** for the both type of n-pi satisfying identical equations, we can use the same methods of solutions considered e.g. in previous author papers, see [7, 3]. And that, in my opinion, is a reminiscence of mythical hidden local variables excluded by the Bell's theorem. We think that the Heisenberg representation of QM described in the free Fock space, in which n-pi 52 satisfy linear equations considered e,g, in [7, 3] are properly handled, may prove to be more effective than the Schrodinger formulation.

App.8 Insight into the nature

is possible due to connection of macro and micro phenomena through the GLPEI. It is amazing that a simple function:

$$R \to S^1 : e^{ix} \tag{53}$$

which combines various Sectors of mathematics also combines various sectors of physics and the fact that this function is mapping a straight line in an infinite times winding circle is also worthy of reflection.

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