Formula based on squares of primes which conducts to primes, c-primes and m-primes

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Abstract. In my previous paper "Conjecture that states that any Carmichael number is a cm-composite" I defined the notions of c-prime, m-prime, cm-prime, odd positive integers that can be either primes either semiprimes having certain properties, and also the notions of c-composite, m-composite, cm-composite, odd positive integers with two or more prime factors having certain properties. In this paper I present a formula based on squares of primes which seems to lead often (I conjecture that always) to primes, c-primes, m-primes, cm-primes or c-composites, m-composites, cm-composites.

Note:

For start I would like to revise the definitions of c-composites, m-composites and cm-composites given by me in the paper mentioned in Abstract, in order to give them a more general meaning.

Definition 1:

We name a c-composite the composite number n = p(1)*p(2)*...*p(m), where p(1), p(2), ..., p(m) are the prime factors of n, which has often the following property: there exist p(k) and p(h), where p(k) is the product of some distinct prime factors of n and p(h) the product of the other distinct prime factors such that the number p(k) - p(h) + 1 allows iterative the operation mentioned until eventually is reached a prime or the unit.

Example: 245761 = 53*4637 is a c-composite because 4637 – 53 + 1 = 4585 = 5*7*131 and 131 – 5*7 + 1 = 97, a prime.

Definition 2:

We name a m-composite the composite number n = p(1)*p(2)*...*p(m), where p(1), p(2), ..., p(m) are the prime factors of n, which has often the following property: there exist p(k) and p(h), where p(k) is the product of some distinct prime factors of n and p(h) the product of the other distinct prime factors such that the number p(k) + p(h) - 1 allows iterative the operation mentioned until eventually is reached a prime.

Example: 45761 = 53*4637 is a m-composite because 4637 + 53 – 1 = 4689 = 3^2*521 and 3^2 + 521 – 1 = 529 = 23^2 and 23 + 23 – 1 = 45 = 3^2*5 and 5 + 3*3 – 1 = 13, a prime.

Definition 3:

We name a cm-composite a number which is both c-composite and m-composite.

Conjecture:

Any term (beside the first) of the sequence obtained through the iterative formula a(n + 1) = 2*a(n) – 1, where a(1) is a square of prime minus nine, is either a prime, a c-prime, a m-prime, a cm-prime, a c-composite, a m-composite or a cm-composite.

Verifying the conjecture:
(for the first 15 terms of the sequence, beside a(1), when the prime is 5, 7 or 11)

For a(1) = 5^2 – 9 = 16 we obtain the following terms:
: a(2) = 31, a prime;
: a(3) = 61, a prime;
: a(4) = 121 = 11^2, a cm-prime (c-prime because is square of prime and p – p + 1 = 1, a c-prime by definition, and m-prime because 11 + 11 – 1 = 2 = 3*7 and 7 + 3 - 1 = 9 and 3 + 3 – 1 = 5, a prime);
: a(5) = 241, a prime;
: a(6) = 481 = 13*37, a cm-prime (c-prime because 37 – 13 + 1 = 25 = 5^2 and m-prime because 37 + 13 – 1 = 49 = 7*7 and 7 + 7 – 1 = 13, a prime;
: a(7) = 961 = 31^2, a cm-prime (c-prime because is a square of prime and m-prime because 31 + 31 – 1 = 61, a prime;
: a(8) = 1921 = 17*113, a c-prime because 113 – 7 + 1 = 97, a prime;
: a(9) = 3841 = 23*167, a c-prime because 167 – 23 = 145 = 5*29 and 29 – 5 + 1 = 25, a square;
: a(10) = 7681, a prime;
: a(11) = 15361, a prime;
: a(12) = 30721 = 31*991, a cm-prime (c-prime because 991 – 31 = 961 = 31^2, a square and m-prime because 31 + 991 – 1 = 1021, a prime;
: a(13) = 61441, a prime;
: a(14) = 122881 = 11*11171, a c-prime because 11171 – 11 + 1 = 11161, a prime;

: a(15) = 245761 = 53*4637, a cm-composite (c-composite because 4637 – 53 + 1 = 4585 = 5*7*131 and 131 – 5*7 + 1 = 97, a prime, and m-composite because 4637 + 53 – 1 = 4689 = 3^2*521 and 3^2 + 521 – 1 = 529 = 23^2 and 23 + 23 – 1 = 45 = 5*9 and 5 + 9 – 1 = 13, a prime.

For a(1) = 7^2 – 9 = 40 we obtain the following terms:
: a(2) = 79, a prime;
: a(3) = 157, a prime;
: a(4) = 313, a prime;
: a(5) = 625 = 5^4, a mc-composite (c-composite because 5*5 - 5*5 + 1 = 1, a c-prime by definition, and m-composite because 5*5 + 5*5 – 1 = 49 = 7*7, a m-prime because 7 – 7 + 1 = 1);
: a(6) = 1249, a prime;
: a(7) = 2497 = 11*227, a c-prime because 227 - 11 + 1 = 217 = 7*31 and 31 – 7 + 1 = 25 = 5*5 and 5 – 5 + 1 = 1;
: a(8) = 4993, a prime;
: a(9) = 9985 = 5*1997, a c-prime because 1997 – 5 + 1 = 1993, a prime;
: a(10) = 19969 = 19*1051, a cm-prime (c-prime because 1051 – 19 + 1 = 1033, a prime, and m-prime because 19 + 1051 – 1 = 1069, a prime;
: a(11) = 39937, a prime;
: a(12) = 79873, a prime;
: a(13) = 159745 = 5*43*743, a c-composite because 5*743 – 43 + 1 = 3673, a prime;
: a(14) = 319489, a prime;
: a(15) = 638977, a prime;
: a(16) = 1277953 = 101*12653, a c-prime because 12653 – 101 + 1 = 12553, a prime.

For a(1) = 11^2 – 9 = 112 we obtain the following terms:
: a(2) = 223, a prime;
: a(3) = 445 = 5*89, a cm-prime (a c-prime because 89 – 5 + 1 = 85 = 5*17 and 17 – 5 + 1 = 13, a prime and m-prime because 89 + 5 – 1 = 93 = 3*31 and 3 + 31 – 1 = 33 = 3*11 and 3 + 11 – 1 = 13, a prime);
: a(4) = 889 = 7*127, a cm-prime (c-prime because 127 – 7 + 1 = 11^2, a square and m-prime because 7 + 127 = 133, a prime);
: a(5) = 1777, a prime;
: a(6) = 3553 = 11*17*19, a c-composite because 11*17 – 19 + 1 = 169 = 13^2, a square;
: a(7) = 7105 = 5*7^2*29, a cm-composite (c-composite because 5*29 – 7*7 + 1 = 97, a prime and m-composite because 5*29 + 7*7 – 1 = 193, a prime);
: a(8) = 14209 = 13*1093, a c-prime because 1093 – 13 + 1 = 1081 = 23*47 and 47 – 23 + 1 = 25 = 5^2, a square;

: a(9) = 28417 = 157*181, a cm-prime (c-prime because 181 – 157 + 1 = 25 = 5^2, a square and m=prime because 157 + 181 – 1 = 337, a prime);

: a(10) = 56833 = 7*23*353, a c-prime because 353 – 7*23 = 193, a prime;

: a(11) = 113665 = 5*127*179, a cm-prime (c-prime because 5*179 – 127 + 1 = 769, a prime and m-prime because 5*179 + 127 – 1 = 1021, a prime;

: a(12) = 227329 = 281*809, a c-prime because 809 – 281 + 1 = 529 = 23^2, a square;

: a(13) = 454657 = 7*64951, a cm-composite (c-composite because 64951 – 7 + 1 = 64945 = 5*31*419 and 419 – 5*31 + 1 = 265 = 5*53 and 53 – 5 + 1 = 47, a prime and m-composite because 64951 + 7 – 1 = 454663 = 11*41333 and 41333 + 11 – 1 = 41343 = 3*13781 and 13781 + 3 – 1 = 13783 = 7*11*179 and 179 + 7*11 – 1 = 255 = 3*5*17 and 3*5 + 17 – 1 = 31, a prime);

: a(14) = 909313 = 17*89*601, a cm-composite (c-composite because 17*89 – 601 + 1 = 913 = 11*83 and 83 – 11 + 1 = 73, a prime and m-composite because 17*89 + 601 – 1 = 2113, a prime;

: a(15) = 1818625 = 5^3*14549 is a c-composite because 5^2*14549 – 5 + 1 = 557*653 and 653 – 557 + 1 = 97, a prime.