MICRO BLACK HOLES

Hypothetical Terrestrial Flux and a Re-Visitation of Astrophysical Safety Assurances

Thomas B Kerwick.

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Abstract

Although studies into the relation between the effects of hypothetical stable TeV-scale black holes produced in particle colliders relative to those which may be produced in nature due to cosmic ray (CR) collisions have already been conducted in great detail [3] this short paper re-examines the relationship between the two.

Herein the same figures of CR flux and the planned number of collisions over the lifetime of the Large Hadron Collider (LHC) are used to determine a comparison between the two, though different results are concluded due to account taken of the relative flux of sub-Keplerian phenomena produced at colliders (<11.186km/s) to faster naturally occurring phenomena which would evade gravity capture.

A re-visititation of alternative astrophysical safety assurances follows, where some minor concerns are raised.
1. Terrestrial Flux of Hypothetical MBH

a. Factors for Consideration

The similarity between cosmic ray collisions of equivalent centre-of-mass energies which occur all the time in nature and p-p collisions which occur under experiment in particle colliders is often cited in safety assurances of such experiments. It has been argued [3] that $3 \times 10^{22}$ cosmic rays with similar energies or more have struck the Earth since its formation, which greatly dwarfs the planned $2 \times 10^{17}$ p-p collisions over the duration of the planned experiments [12] at the Large Hadron Collider at CERN (which in itself is unprecedented in industry). However, it is argued herein that a direct comparison cannot be made between these two figures, as products of the latter can be influenced by gravity capture whereas the products of the former are not. In this context I consider a relative flux of hypothetical stable phenomena from nature and from the experiments.

It shall be defined for the purpose of this analysis that $\partial_{MBH}$ is the ratio of collisions which produce such hypothetical stable MBH, the same figure applied to cosmic ray collisions of equivalent center-of-mass energies to those collisions which occur within particle colliders. In this case, nature has produced $\partial_{MBH} \times 3 \times 10^{22}$ stable MBH over the course of the lifetime of the Earth, with $\partial_{MBH} \times 2 \times 10^{17}$ to be produced by the experiments. The ratio of MBH produced in experiments which are below 11.186 km/s and subject to gravity capture, will be represented as $\phi_R$ here, while one can assume that none created in nature are sub-Keplerian – on the basis that for the centre-of-mass energy of CR-atmosphere collisions to be larger than the MBH mass, the CR momentum must be very large - and the produced BH would always be ultra-relativistic [11] prior to accretion.

With an average velocity of sub-Keplerian MBH at $K_v$ km/s, such MBH would traverse a distance equivalent to one Earth diameter in approximately 12,756km/K_v seconds, in orbital motion within the Earth. MBH with velocities greater than 11.186 km/s can traverse only one Earth diameter or less – depending on the trajectory - with an average chord length traverse through the circular plane at $c = R \pi/2$, where R is the Earth radius. If one considers a hypothetical flux of sub-Keplerian MBH through the Earth, introduced over the course of LHC experiments, this approximates to $\phi_R \times \partial_{MBH} \times 2 \times 10^{17} \times K_v/12,756km$ – measured in MBH traversals per second. This can be compared directly to $\partial_{MBH} \times 3 \times 10^{22}$ natural traversals over 4.54 billion years since the formation of our Earth.

b. Approximating the Relative Flux

One can compare the flux of captured MBH traversals per second into the overall number of traversals due to natural cosmic ray exposure since the formation of Earth, and from this determine the time duration $T$, one would require for this flux to equal that of the natural flux.

$$T = \frac{\partial_{MBH} \times 3 \times 10^{22} \times \frac{\pi}{4}}{\phi_R \times \partial_{MBH} \times 2 \times 10^{17} \times \frac{K_v}{12,756km}}$$

$$= \frac{1.502 \times 10^9}{\phi_R \times K_v}$$

Consider a sub-Keplerian MBH with an initial velocity $v$ km/s - this would fall freely under acceleration due to gravity $g_0$ - initially at approx. 35,30394 km/h/s. This would accelerate/decelerate as it orbits towards and away from the Earth’s center of gravity, and for the
purposes of this analysis it will be assumed that such will remain within the sub-Keplerian limit - and so here we can apply 11.185 km/s as the extreme upper-bound for such MBH velocities:

\[ T = \frac{1.502 \times 10^9}{\phi_K \times 11.185} \text{ seconds} \]
\[ = \frac{4.278}{\phi_K} \text{ years} \]

The ratio increase in flux, FR, due to hypothetical stable MBH through Earth is calculated as:

\[ FR = \frac{\phi_K \times 4.54 \text{ billion years}}{4.278 \text{ years}} \]
\[ = \phi_K \times 1.061 \times 10^9 \]

Applying a sub-Keplerian ratio as derived in previous analysis [3] in the region of 5.7 \times 10^{-4}, where M = 4 TeV, applying worst case scenarios to the analysis, this will set an upper bound:

\[ FR \leq 6.049 \times 10^5 \]

The resultant relative flux of sub-Keplerian MBH through the Earth due to the complete run of LHC collisions over its lifetime could therefore be as great as 6.049 \times 10^5 times higher compared to the flux of such MBH through the Earth as caused by natural CR collisions in the atmosphere.

c. Implications of an Elevated Flux

The consequences of p-p collisions could have a lasting imprint on the flux of stable MBH through the Earth if such can be produced – hypothetically, an increase of 6.049 \times 10^5 : 1 km/km based on the planned number of LHC collisions and their sub-Keplerian ratio. This equates to a lower bound of approx. 7,500 years over which a similar traversal distance could accumulate due to sub-Keplerian MBH produced at the LHC as has already occurred naturally over the lifetime of Earth.

With public material [2][27][22][21] still disputing the effectiveness of Hawking Radiation (HR), an elevated flux could signify a greater risk of MBH accretion than that which can occur in nature.

An elevated flux could also have a heating effect on the Earth if any such produced MBH do emit Hawking Radiation - a concept explored in earlier work by Plaga [4]. However, to sustain the flux derived herein one would require the accretion rates to balance or exceed the rate of evaporation or such MBH would not sustain their mass - a scenario which could only materialize if either rate was vastly different to those explored in detail in the LSAG report, a point returned to in conclusions.

In either case, a re-visitation to alternative astrophysical safety assurances, based on white dwarf and neutron star measurements and longevity should be reasoned in the context of an elevated flux.
2. White Dwarf Assurances

a. The Case Study of Sirius B

Whereas a detailed analysis of CR exposure on WD was included in the LSAG safety report [3], here I focus on the simple case study of Sirius B, which although newer than those used in the LSAG safety report, could be considered a more tangible case study, due to its proximity.

Sirius B, the white dwarf partner to Sirius A, provides an ideal case study, in that it relatively well understood, as proximate to our local surrounding in space - just a few light years distant.

One can reason that the region of Sirius B, just a few light years from Earth, is subjected to similar levels of background CR to that which Earth is subjected to, though in the case of Sirius B, with a far more powerful magnetic field, significant magnetic deflection also occurs.

Here some basic data on the two bodies is used in consideration, first an uncorrected exposure on Sirius B, and secondly, an exposure on Sirius B with consideration for magnetic screening:

1. $3 \times 10^{22}$ CR of similar CM or more have struck the Earth since formation. [3]
2. The estimated age of the Earth: $4.54 \pm 0.05$ billion years ($4.54 \times 10^9$ years $\pm 1\%$).
3. The estimated age of Sirius B: $1.6 \times 10^8$ years. [7]
4. The mean radius of Earth: $6,371$ km.
5. The radius of Sirius B: $R= 0.0084 \pm 0.00025$ solar radius $= 5,842$ km. [7]
6. Magnetic Field of Sirius B: Estimated at 200,000 to 400,000 Tesla. [10]
7. Magnetic Field of Earth: In range $25,000 - 65,000$ nano-Tesla $(0.25 – 0.65$ G). [9]

In the context of the corrected and uncorrected rates of CR exposure on Sirius B, the validity of white dwarf stars of similar and weaker fields as a reasoned safety assurance is considered.
First, a calculation of the relative age of Sirius B to Earth, and the relative surface area of Sirius B to Earth, is used to calculate an uncorrected figure, $CR_U$, for CR exposure on Sirius B, attaining a value therefore consistent with estimates typically used for CR exposure on Earth.

$$\text{Relative age of Sirius B to Earth: } \frac{1.6 \times 10^8}{4.54 \times 10^9} = 3.5 \times 10^{-2}$$

$$\text{Relative surface area of Sirius B to Earth: } \frac{4\pi \times 5,842 \text{km}^2}{4\pi \times 6,371 \text{km}^2} = 8.4 \times 10^{-1}$$

$$CR_U = 3 \times 10^{22} \times 3.5 \times 10^{-2} \times 8.4 \times 10^{-1} = 8.82 \times 10^{20}$$

In the detailed analysis of black hole production on white dwarfs in the official LSAG safety report [3], it is suggested that to avoid significant magnetic screening we must consider white dwarfs with magnetic fields of: $B_p \leq N \times 10^5 G$. In the LSAG derivation [3] for an effective maximum energy for CR that penetrates to the surface of such a star, for protons, normalising to white dwarf parameters similar to Sirius B (i.e. with a radius circa 5,000km):

$$E_{\text{max}}(\theta = \pi/2) = 3.6 \times 10^{18} eV \frac{5000 \text{km}}{R_0} \left( \frac{10^5 G}{B_p} \right)^2.$$

This equates to a requirement for a field far weaker than that of Sirius B, by perhaps a factor of 10,000:1 for an assurance that CR exposure is not significantly deflected. In this context, one could postulate that magnetic field effects on Sirius B ensure that no such CR collisions occur despite an estimated $8.82 \times 10^{20}$ CR exposure. This sets a dilemma in such LHC safety assurances [12], as the most well understood of white dwarfs, Sirius B, therefore has characteristics which run against safety arguments presented in the LSAG report [3], where examples of far more distant white dwarf stars were chosen - for which CR flux is less certain.

If one considers exposure on Sirius A, which has a relatively weak magnetic field of no greater than 2 G [17], a small portion of MBH produced on Sirius A, as assumed uncharged, could subsequently become captured in Sirius B. However, at an average distance of 19.8 AUs of an orbital semi-major axis [6], that portion is quite low. The ratio of MBH produced on Sirius A to those which can be expected to reach Sirius B can be approximated based on the size of the disc of Sirius B relative to the area of a sphere 19.8 AU from Sirius A derived as:

$$\Rightarrow \text{ Sirius B Disc Ratio: } \frac{(2\pi \times 5,842 \text{km})}{(4\pi \times (19.8 \times 149,597,870 \text{km})^2)} = 3.33 \times 10^{-16}.$$

It should also be noted, that the production of MBH due to equivalent centre-of-mass CR exposure may be less efficient than in LHC conditions, with CR estimated to include heavy elements at a ratio in the region 1:2 [19]. This does not compare favourably to the expected number of MBH produced on Sirius A over its lifetime, and so consideration of hypothetical production of MBH due to CR exposure directly on WD shall remain the focus here instead.
b. Production of MBH on White Dwarf Stars

The potential exposure of \(8.82 \times 10^{20}\) CR over the lifetime of Sirius B (equating to approx. 10,000 such CR per second) was deemed to be a non-reliable figure due to the strong magnetic screening effects on Sirius B. In fact, it could be considered implausible that any CR collisions of equivalent centre-of-mass energy to LHC collisions can occur on Sirius B.

The consequence of MBH production on lower field white dwarf stars shall be considered here instead, the purpose of this exercise to explore the durability of the white dwarf safety assurance when applied to the more distant white dwarf stars discussed in the LSAG report [3] which are believed to have far weaker magnetic fields, and similar levels of background CR.

Sirius B is considered a ‘young’ white dwarf – in that more distant cousins are estimated to be considerably older – the oldest often cited in the region of 1.27 +/- 0.7 billion years [13], and as such the exposure of CR over the lifetime of these alternative candidates is also far greater.

MBH production on WD would initially travel at near-luminal speed, and follow a near-linear path, continuing the trajectory of the colliding CR due to conservation of momentum. In order to consider such to have a lasting effect on the WD, comparable to the production of sub-keplerian MBH in the LHC on Earth, one would require MBH to reduce to a sub-keplerian speed within the maximum one full traversal through the WD, in order to be captured. For WD of similar mass to Sirius B, MBH must reduce to less than an escape velocity of 5,200 km/sec within the traversal distance, i.e. at most 10,000 km or so, and as MBH are reasoned to be uncharged [2], this deceleration must be achievable through gravitational effects alone.

MBH deceleration due to gravitation effects can be caused by both a Coulomb effect where collisions result in a particle scattering and by accretion slow-down where the MBH absorbs particles [3]. The length required to slow-down to the non-relativistic regime was calculated in the LSAG report at 1.5km [3] on a detailed analysis of such processes, with subsequent stopping bounds shorter than WD diameters (for D<=8 MBH at 14 TeV or less).

It has been suggested [1] that a smaller initial MBH radius would result in a longer stopping distance in WD, though to date there has been no credible challenge to the derivation of these.
As an MBH increases in size, such that it is no longer smaller than the particles that it would accrete, the capture radius becomes much more significant to the rates of accretion [3]. Therefore although the capture radius is not considered to be significant in the initial accretion slowdown phase, it must be considered for subsequent stages of MBH accretion models.

One should therefore consider the thermal velocity of an MBH growing at the core of Earth (5700K) relative to an MBH growing in a typical white dwarf core (10 million K) [20], as such has a direct impact on the capture radius (\(R_c = \frac{R_s}{v_T}\)) [3][20]. As such the capture radius of an MBH in a white dwarf core would be significantly smaller to an equivalent mass MBH at the core of Earth, by a factor of: \(\sqrt{\frac{1 \times 10^7 K}{\sqrt{5.7 \times 10^3 K}}} = 41.9\).

Thermal velocity effects on the capture radius of MBH in WD relative to Earth decreases any safety assurance based on the longevity of white dwarfs – where the capture radius is considerably smaller. Returning to the case study, the accretion rate of an MBH in rotation through Sirius B for the past 150 million years through matter of density of \(1 \times 10^8 \text{ kg/m}^3\) [14], compared to Earth’s \(5,520 \text{ kg/m}^3\) (5.52 g/cm³), and taking into account that MBH growth is proportional to the square of the capture radius here [3], gives a safety assurance of:

\[
T = \frac{(150 \times 10^6) \times (1 \times 10^9)}{41.9^2 \times (5.52 \times 10^3)} = 1.5 \times 10^{10}\text{years}
\]

To put a safety assurance of \(T = 1.5 \times 10^{10}\text{years}\) in context, this figure is comparable to the current estimated age of the Universe at \(13.7 \times 10^9\) years. Therefore the observation of WD older than Sirius B with magnetic fields of \(B_p \leq N \times 10^5 G\) can be taken as evidence that MBH of dimensions D<8 cannot be produced under 14 TeV, or if they could be created, the accretion process would not affect Earth, even on Universal time scales - without requiring one to consider the absolute accretion rates specific to either body, as this is a relative metric.

c. Safety Assurance based on WD Measurement

WD related safety assurances are applicable to MBH with dimensions D<8, and at no greater than 14 TeV energy levels [3]. To suffice as assurance, one requires the accuracy of the magnetic field strengths to within the range applied to such stars used for safety assurances in the LSAG report to within the standard industrial safety tolerances, and although the examples sampled [18] in that LSAG report conclude a mere 99% confidence - L745-46A (WD0738-172) at 7kG with 99% confidence interval of ±6 kG, and GD 40 (WD0300-013) with an upper limit of 12kG with 99% confidence, some more recent measurements do provide for this [23].
3. Neutron Star Assurances

a. Basis of Neutron Star Assurances

Safety assurance based on neutron star (NS) observation lies in the assertion that cosmic-ray (CR) to surface collisions on NS are analogous to p-p collisions at particle colliders such as the LHC, the products of which would successfully captured in such neutron stars. As with WD, these are sufficiently dense to capture any naturally occurring MBH produced due to CR exposure – though the greater density of NS ensures capture even at energy levels far greater than 14 TeV, and/or MBH of greater number of dimensions D>=8. The continued existence of NS is seen as evidence that such hypothetical stable MBH are not produced, or are not dangerous if such are produced.

In considering such as a safety assurance, CR deflection was assessed in detail by Giddings and Mangano [3] due to the ultra-high magnetic field strengths associated with NS, and in doing so also considered both neutrino flux, and more elaborate scenarios of secondary MBH capture from production in binary pairs as alternatives. Excluding the more elaborate circumstances of secondary MBH capture in binary pairs as over-elaborate, and assurances based on neutrino flux as based on unreliable data, only the hypothetical direct production of MBH on NS, from non-deflected CR impacts on the surface of NS, is reasoned here as a qualifiable safety assurance.

b. Magnetic Fields & Magnetic Deflection

Whereas a lower limit of a 100kG magnetic field strength for CR deflection was set as a criteria in determining safety assurance based on WD observation and measurement [23][3], higher upper limits are applicable [3] for determination of safety assurance based on NS observation and measurement, as magnetic field strength decreases proportional to radius R0 at distance r from a star (R0/r)^3, with NS requiring magnetic fields at 10^3 stronger for equivalent magnetic deflection, equating to a 10^9 G limit. Instead, as all known NS have far stronger magnetic fields, upwards from 10^9 G, an approximation of maximum energy penetrable to a typical NS was determined (without error tolerance values provided), for perpendicular impingement of NS with lowest known field strengths, giving collisions just above the LHC CM energy, with other impingements less effective. Some consideration in the safety report was also given to CR penetration of theorised weaker magnetic fields at the polar regions of NS, though such assurances cannot be supported with verifiable astrophysical measurement. Indeed, the basic structure can be considered multipolar which would dampen weakness at any specific pole [25].

It is also noted that a number of alleged discrepancies can be found in the safety report, regarding the use of uncorrected production rate figures in conclusions on NS safety assurance, and were highlighted in an ‘outsider’ independent study [24], to the effect that magnetic fields of all known NS are too strong for significant MBH production from direct CR. It is reasonable to surmise that NS safety assurance based on direct impact of CR is less reliable than assurance based on WD observation and measurement, where magnetic field strengths are far weaker, of the order 1,000:1.

c. Critical Consideration for Sub-Millisecond Pulsars

Pulsars – rapidly rotating highly magnetised neutron stars – have provided accurate references in our Universe due to their regularity and predictability, of which there are now over 2,000 pulsars observed and measured [8]. Each of these pulsars have been measured to have a very distinct and
consistent Barycentric period, with periods in a range from several seconds to just over 1 millisecond [8], with older pulsars typically of shorter periods. When one generates a plot of pulsar periods relative to the estimated age of such pulsars (Figure 1), there is a noticeable cut-off at approximately 1.5 milliseconds. In Figure 1, pulsars are plotted in ascending order of Barycentric period, in which it can be seen that there are in fact very few pulsars below 2 milliseconds, and none below 1 millisecond - with the lowest known pulsar measured at just under 1.4 milliseconds (0.00139595482s). It has been noted that most neutron star equations of state allow shorter periods, and it has been speculated [26] that the lack of such pulsars with P < 1.5 ms is caused by gravitational wave emission from R-mode instabilities – though such speculation is inconclusive.

![PSRCAT plot](http://www.atnf.csiro.au/research/pulsar/psrcat)

**Figure 1:** Generated plot of all known pulsars, illustrating Barycentric period (p0) vs estimated Age. Note the distinct lack of pulsars with sub-millisecond p0 periods.

If one considers the pulsar as a rotating magnetic dipole, the surface magnetic field strength can be derived relative to the Barycentric period $B \propto (PP)^{1/2}$ [26] giving typical values of $10^{12} - 10^9$ G.
As the existence of millisecond pulsars with $10^8$ G fields were used as a safety assurance to LHC collisions, the non-existence of sub-millisecond pulsars which would have fields lower than $10^8$ could also be considered relevant. In the context of such safety assurances, one should consider non-existence of sub-millisecond pulsars may be due to their lower magnetic fields, allowing for more significant MBH production from direct CR collisions. In other words – a non-existence of sub-millisecond pulsars could be considered as observational evidence of not only the production of stable MBH on NS with lower order magnetic field strengths, but NS transformation into BH.

In consideration of the NS safety assurance of p-p collisions in the case of MBH of dimensions $D\geq8$, and/or for MBH created at energy levels $> 14$ TeV, one can consider that this is not only a discrepancy in safety assurance, but observational evidence suggesting stable MBH might be created from comparable astrophysical processes. However, other explanations for the absence of observable sub millisecond pulsars are latent, and orthogonal safety assurances also remain in both Hawking Radiation theory and in the LSAG accretion estimates of such hypothetical stable MBH.

Indeed, it has been noted [11] that as pulsars approach 1.0ms rotation, the speed of rotation at the equator would be approximately 60,000km/s (given pulsar radii of the order of 10km), and before reducing to 0.1ms, a pulsar would require an equatorial velocity close to the speed of light. As such it is surmised that it would not be realistic to expect to observe pulsars below 0.5ms, with an equatorial velocity 1/3 of the speed of light, nor perhaps any sub-millisecond pulsars. However, this does not consider relativistic effects, and effects such as Frame Dragging, or Lense-Thirring effects, similar to the processes theorised within the ergosphere of rotating BH, may allow for this.
10. Conclusions

The net flux of hypothetical stable MBH from LHC collisions has been shown to be far greater than that which could occur due to natural CR exposure - hypothetically, an increase of $6.049 \times 10^5 : 1$ km/km, which could be considered a concern to MBH accretion processes, or a heating effect due to radiation from the elevated flux.

Alternative astrophysical safety assurances, based on white dwarf (for hypothetical stable MBH of dimensions $D<8$) and neutron stars (for hypothetical stable MBH of dimensions $D>=8$) were re-visited. These provide some assurance against MBH accretion processes, but neither provides an assurance against a proposed heating effect.

It was also noted that although assurances based on white dwarf were reliant on select magnetic field data of just 99% confidence, and the case study of our closest and most understood white dwarf, Sirius B, provides no safety assurance whatsoever, recent magnetic field data does provide a safety assurance with full confidence for $D<8$.

The absence of sub-millisecond pulsars was also questioned as potential evidence of NS transformation into BH due to heightened MBH flux, although other explanations to a lack of such observable phenomena are reasoned.

The obscure notion of ‘missing pulsars’, and over-liberal use of unreliable measurements in magnetic field data in astrophysical safety assurances, raises some concern to the case of non-radiating MBH for dimensions $D>=8$. This is compounded by realisation of a permanently elevated MBH flux due to experiments ($1 : 6.049 \times 10^5$), which even in the case of radiating MBH may present a heating effect if radiation rates are lower than expected and such MBH stabilise at an accretion/radiating equilibrium. However, both of these scenarios require both Hawking Radiation and accretion estimates to be far less effective in practice than in their mathematical model.

11. Acknowledgements

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<td>ATNF</td>
<td>Australia Telescope National Facility</td>
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<tr>
<td>BH</td>
<td>Black Hole</td>
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<tr>
<td>CERN</td>
<td>Organisation européenne pour la recherche nucléaire (European Organisation for Nuclear Research)</td>
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<tr>
<td>CM</td>
<td>Centre of Mass / Centre of Mass/Energy</td>
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<td>CR</td>
<td>Cosmic Ray(s)</td>
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<td>D</td>
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<td>G</td>
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<td>MBH</td>
<td>Micro Black Hole</td>
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<td>Neutron Star</td>
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