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Reflecting on Numbers: A Geometry of Time, Ether, and Potential Infinity

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"Nothing can exist if it doesn't contain continually and simultaneously the limited and the unlimited, the definite and the indefinite." Plato (428/427 – 348/347 B.C., Philebus, 16 [25])

Abstract: **This paper is considering geometrically the smallest possible spatial displacement of the elementary mass by equating the corresponding and thereby smallest possible historical time with the unit. This time is supposed to be the travel time under smooth rectilinear motion as the motion, with the least change in the physicochemical substance of this mass, change determining the age of the mass and owing exclusively to kinematics in void space. Yet, the passage of time as recorded by physicochemical change under other types of motion towards the same terminal point and at the same arrival time, call it "age-time", does not coincide with the arrival time. Therefore, although void space has been assumed, some other, age-shaping factor beyond kinematics is inferred to exist in this space, so that it can be held responsible for the variations in age time. It acts so as to be ensuring the uniqueness of events. The same factor should be cited as the source of motion too, given that motion is contemplated analytically with no reference to a cause of it. This factor appears to exhibit the properties of ether, with which it is identified. Ether provides momentum to mass, altering constantly the texture of the mass, that is, the age of the mass and subsequently, the velocity of it in historical time. The operation of ether is taken to be mathematically the reason for the real-world relevance of irrational numbers. The potential infinity, associated with these numbers and with the operation of ether that is filling space, becomes embedded in the actual infinite of space by itself, in this manner.**

Keywords: **Geometry Of Time, Historical Time, Age-Shaping Time, Ether, Uniqueness Of Events, Potential Infinity.**

I. Introduction

"Le grand ouvrier de la nature est le temps: The great workman of nature is time." *Comte Georges-Louis Leclerc de Buffon* (1707-1788, [20])

In a recent article, Soldatos [32] shows that if space is finitely infinite, any number in this actual infinity should be rational and hence, any number should be geometrically constructible given the any rational number is constructible and that constructability is a space problem. Consequently, the presence of irrational numbers should be related to motion, to time and spacetime and not to space by itself. This paper pursues this reasoning further by elaborating upon a Geometry of Time. It is considering geometrically the smallest possible spatial displacement of the elementary mass, which is taken to be the physical analogue of the mathematical point. This is done by equating the corresponding and thereby smallest possible historical time with the unit. And, this time is supposed to be the travel time under smooth rectilinear motion as the motion with the least change in the physicochemical substance of this mass.

Let $1 - 0$ designate birth and death, respectively, with the term "death" applying even to the slightest change of the physicochemical substance of this mass. And, consider the sequence $\ldots 1-0, 1-0, 1-0, 1-0, 1-0, \ldots$ Under instantaneous change of mass substance, i.e. over a single pair $(1-0)$, time cannot be defined. One has to consider more than one pair in order to have time going. That is, one has to consider the dead substance form as living and not as reborn altered and connected with the succeeding pair of $1-0$. Defining a concept like time violates *Gottfried Wilhelm von Leibniz*'s (1646-1716) law of indiscernibles as the solution to Theseus Problem [17, 21]: *X is the same as Y if, and only if, X and Y have all the same properties and relations; thus, whatever is true of X is also true of Y, and vice-versa*; unless it takes more than one instant for a newborn to change. But, it does not, *ex hypothesi*. Consequently, the Geometry of Time is actually the geometry of $(1-0)$ above.

By working it out, the presence of some unknown substance in space, is detected analytically without having made previously the slightest hint about it. It is detected indirectly through the emergence of large-scale irrationality accompanying phenomena describable exclusively by rational numbers. Therefore, irrationality is ascribed to the presence of some unknown substance distributed uniformly all over the assumed space and being as responsible for motion as force is, because it was not detected it as a variable as well. Logically, it should be this unknown substance which is responsible for the change in the physicochemical texture of mass.

Trying to characterize it physically with something more "tangible", beyond its mathematical description, this unknown substance is found to fit well with the notion of "ether" [6, 8, 35]. Indeed, as the founder of quantum field theory, *Paul Adrien Maurice Dirac* (1902-1984), stated in 1951 in an article in *Nature*, titled "Is there an Aether?", we are rather forced to have ether as a key ingredient of any serious effort toward the better understanding of the cosmos [6]. We shall see that ether affects matter-energy whenever there is a change in velocity and hence, direction of motion. Consequently, if *all is flux*, as *Heraclitus* (c.535-c.475 B.C.) says, any entity birth is taking place in motion, prompting an immediate spatial displacement of the new born as if from a static position, (like letting an apple fall from a stream bank into the streaming waters), and ether becomes operational right at birth.

The concept for ether found here, is in line with how *Albert Einstein* (1879-1955) [8] wanted it to be within the framework of general relativity: Ether is found to act and counteract with matter, as he wanted it, though it is mobile in such a way that one cannot speak of motion in relation to it. This is not to say that general relativity is endorsed or not by this work. The matter is irrelevant. What is underlined here is that there is no time as a magnitude so that frames of reference may be defined. Time might be seen as the counting of the continuum of ether operation on matter-energy, the counting of pairs $(1-0)$ in some modular and discreet for convenience fashion like that connected with the sun and the stars. Time, seen from this perspective, is a human convention for abstracting from the reality of the uniqueness of events when their change is imperceptible. One hour may not be like one meter, which does derive concrete empirical content from the reality of length-width-height, but again is a useful practical invention. Yet, one then should not expect that an observer might easily identify the $(1-0)$ state of say Athens that corresponds exactly to the $(1-0)$ state of say New Delhi at 12 noon high. The locational bijection of events ordered by time is taken away by the mere fact of abstraction from physicochemical change.

We proceed, with the overall in general approach being advanced under a Geminus-minded view to saving the phenomena studied herein; (*Geminus of Rhodes*; 1st century B.C.). That is, we are interested in explaining the cosmos rather than in the realism of the assumptions made toward this end. There are only few of them in line with *Occam*'s (1287-1347) razor, too. Some specific bibliographical references on the key concepts pervading the discussion below and which need not be mentioned repeatedly are: Barrow [1], Dainton [3], Davies [4], Dieter [5], Friedman [10], Greene [13], Heller [16], Horwich [18], North [23], Price [26], Rucker [27], Sean [30], and Sider [31]. At each step of exposition, the lessons derived for Arithmetic are discussed too, focusing explicitly on it in the concluding section.

II. Time, Rational and Irrational Numbers

"At ubi materia, ibi Geometria: Where there is matter, there is Geometry." *Johannes Kepler* (1571-1630, [19, p. 188])

Assume that point A is the origin of the three-dimensional space, $tx_1 - tx_2 - tx_3$, as in Fig. 1. Point A gives the initial position of elementary mass, M, corresponding to the physical as opposed to the mathematical point. M, at time $t=0$, is motionless. Its position in the three-dimensional space at time $t=t' \geq 0$ comprises an event in space-time. Two events may differ only in the value of t, and not necessarily in position. Because the world, the universe is not empty, the number 0 can be defined in space-time, only conventionally, for reasons of measurability regarding time and space. Moreover, we might contemplate the existence of an inter-event line linking an initial to a terminal event. Point A will be here the initial event. The reference terminal event, B, consists of: (a) the smallest possible displacement of M under smooth rectilinear planar motion, which will bring M to position B_i, in the direction of dimension i, $i = 1,2,3$, under constant velocity, u, (b) the time $T=1$ recorded by M and the time $t=1$ registered by a hypothetical observer at B_i for this minimal linear movement to B_i, and (c) the alteration of the physicochemical texture of M owing exclusively to such a motion in space and time by itself. The change in texture cannot be attributed to any other factor beyond the kinematics, because simply no other factor is assumed to exist at present.

Any other event is connected with curvilinear motion and hence, with a velocity $v > u$, and with a recorded time $T < 1$, and is given by the triplet $[M, T, t]$. For the initial event we have $[M(0), T=0, t=0]$, for the reference terminal event we have $[M(1), T=1, t=1]$ while $[M(n=T<1), T<1, t=1]$ for curvilinear motion. The parentheses indexing M, capture the alteration of the physicochemical texture of M, taking place under any movement of M. $M(1)$ reflects the fact that the movement itself alters M while $M(n=T<1)$ shows that alteration differs not only between smooth rectilinear and curvilinear motion under u and v, respectively, but also among the various curvilinear motions under the different v_n, as captured by the value of natural number n. t appears in any triplet, because $t=1$ is arrival time registered by the hypothetical observer at B regardless type of motion and regardless time, T, recorded by M. t dates the occurrence of an event independently of its kinematics and of the physicochemical hypostasis of M, and is defined as such to be "historical time".

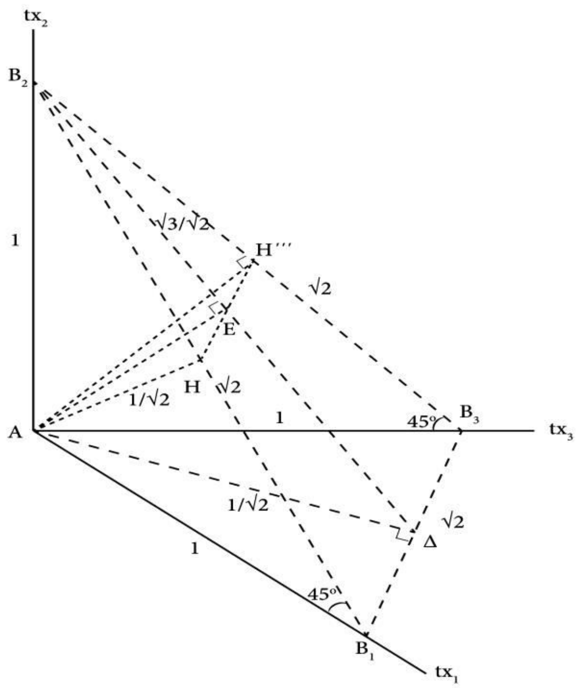

Fig. 1

Elementary kinematics can help relate t with T by considering the example of space $tx_1 - tx_2$ in Fig. 2 as follows: (a) Recall that v is a velocity, which, were to have been applied to smooth rectilinear motion, would had given a travel time, $T < 1$, the moment the velocity under the latter motion, u, gives a travel time, $T = 1$. The distance that would have been traveled linearly under u at $T < 1$, is less than AB_i, $i = 1,2,$, say $AH = AC_2$. Hence, points in general on B_1B_2, are, (except B_1 and B_2 themselves), if seen from the origin of the axes, A, endpoints of vectors, capturing distances at smooth rectilinear velocity, u, but under the T's of curvilinear motion, with $T < 1$. Each of these vectors is the product of some $T < 1$ with the constant u, and B_1B_2 is the locus of the endpoints of these vectors. (b) But, the actual distances run are those under the v's, which bring M onto the circular arc $B_1H'B_2$ centered at A. Hence, points like H' on this arc reflect distances according to T's and v's of curvilinear motion, consistent with $t = 1$. Arc $B_1H'B_2$ is the locus of such points. (c) The distances that would have been run under the various v's at $T = t = 1$ are given by points like H'' on arc $B_1H''B_2$. Points on this arc are endpoints of vectors, capturing the distances that would have been run linearly by v's at $T = t = 1$ rather than at $T < t = 1$. Arc $B_1H''B_2$ is the locus of the endpoints of vectors which are the product of some v with the unit of $T = t = 1$.

In sum, line segment B_1B_2 and arc $B_1H''B_2$, are spatial representations of time lengths in the form of linear distances; the former by keeping the velocity constant at u and letting T vary, and the latter by keeping T constant at $t = 1$ and letting the velocity v vary. More insight into the nature of these two loci may be obtained by noting that: (a) If distance vectors are really time-length vectors, which is the notion of time underlying them the moment curvilinear motion should be ending on a circular arc like $B_1H'B_2$, which is consistent with $t = 1$? Simply, by conception, there can be no motion taking time less than $t = 1$ or traveling distance longer than AB_i. (b) Moreover, note that the vectors might be seen also as complex numbers, as position vectors. Hence, AC_2 may be seen as the circular projection of AH on the horizontal axis. And, although the real part, AC_1, of the complex number describing AH might be given by a real-line rational number, the number corresponding to AC_2 cannot be seen as such a number, because simply AC_2 represents time, not a spatial length. Consequently, T stands for some notion of time different than that of historical time, and is associated with real-line irrational numbers provided that the trigonometric number accompanying the argument of a composite vector is such a number when detached from the actual infinite of spatial considerations. (c) Finally, note that curvilinear motion does have to end on $B_1H'B_2$; if not, it would be like we were saying that there can be no such motion given that the only mass displacement that there can be is on this arc *ex hypothesi*.

So, on the one hand, T has to be less than one for any B-point other than B_i, on the other hand, t has to equal one, and the two have to go together. It follows that T cannot be just travel time. T appears to be something innate in curvilinear motion, related with some factor making a mass M reach its destination on $B_1H'B_2$ by having to undergo first some kind of a phenomenon captured by irrationality. The phenomenon is induced by the change not just of faster but of curvilinearly faster motion relative to the motion under the reference terminal event. And, given an event by the triplet $[M, T, t]$, the phenomenon must be pertaining to an alteration of the physicochemical hypostasis of M beyond the alteration assumed by the reference terminal event; an alteration reflected on T. Therefore, T must be related with some kind of physical entity, having also the property of timing travel indirectly *via* the extent of mass alteration induced by it; different alteration, different T. And, subsequently, T must be traveling time from the viewpoint of the age at which a curvilinearly traveling mass arrives at the same destination as that under the reference event, because such motion induces this entity to incite a phenomenon, affecting age and described by irrationality.

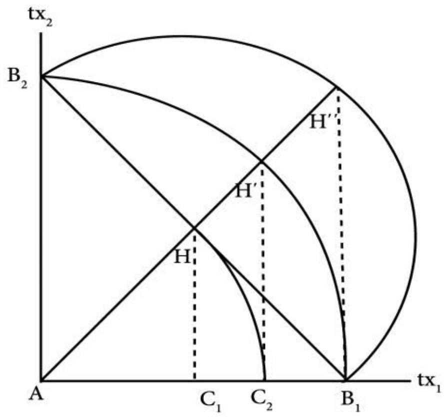

Fig. 2

Moreover, independently of the extent of mass alteration, the phenomenon always takes place regardless direction of motion, which in turn implies the omnipresence of this age-shaping physical entity in space. It reminds one of the concept of "ether", but again the age-shaping phenomenon suggests that it does have consequences for matter, and at the other end, matter does affect it, because this substance cannot be detected unless there is motion and hence, release of energy. The age-shaping phenomenon must be connected with energy release; but, with what kind of energy? To answer this question, note that we assumed motion but not force causing it so that to have the effects just mentioned. It follows that this substance, the contact of matter with it, must also be responsible for the motion of matter and hence, the kind of energy which is intrinsic in ether is kinetic energy. Ether is kinetic energy, one of the building blocks of the cosmos beyond space.

We will focus on its "timing side", referring to it as "age-shaping time", that is, to the irrational number capturing the phenomenon which is shaping age. It is age-shaping and not age by itself, which is connected with irrationality. Ether, either in the absence of motion or in the presence of smooth motion, would be uniformly distributed in space, or could be taken to be motionless, measurable according to space measures. One meter of distance in space would be equivalent to one meter of distance inside ether. Consequently, the integer part of an age-shaping irrational time corresponds to the distance that

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a moving mass has traveled in this fashion before entering the age-shaping phenomenon, with the decimal part of the irrational time capturing the culminating actually end of age-shaping time.

Such a perception in general of the cosmos might be challenged on the grounds that as long as loci B_1B_2 and $B_1H''B_2$ describe the same quantity, the area of the isosceles right triangle B_1AB_2 should be equal to the area of the lune $B_1H'B_2H''$, which as such is the Lune of *Hippocrates of Chios* (c.470-c.410 BC). That is, we have a lune which is the result of the products of v's with $T = t = 1$ and hence, of endpoints of seemingly purely spatial and subsequently, geometrically constructible vectors, reflecting decimal expansions consistent with actual infinite at most. One is thus tempted to conclude that the irrationality connected with B_1B_2 is only fictitious, because it is "trapped" by the rationality of the Lune through the "Quadrature" given that this Lune was advanced as an approximate solution to the problem of the Quadrature. Put differently, the finite infinite of the spatial nature of the Quadrature appears to have trapped the infinite infinity of age-shaping time! Nevertheless, note that the arc $B_1H''B_2$ and by extension the Lune, might be thought of as answering alternatively to the question: How longer than $T = t = 1$ time T would have taken motion under u to cover the same distances as under the v's. T varies as much as under the locus B_1B_2, but now for $T > 1$. The vectors ending in the Lune are spatial representations of time lengths in the form of linear distances, with circular projections on real-line irrational numbers.

When there is motion, another building block of the cosmos, ether with its age-shaping time, becomes visible in addition to space. And, it is the physicality of this kind of time which is this block and not the mere artifact of dating events through registration numbers like 25 March 1821 or 28 October 1940; an artifact even if as elaborate as within the context of reference frames. The potential infinite is as built-in the cosmos through age-shaping time as actual infinite is through space. But, the potential infinite cannot be exhibited unless there is motion. But, let us take things one at a time and see how these claims about the cosmos tie in physically.

III. Ether and Age-Shaping Time

"The meaning of time has become terribly problematic in contemporary physics. The situation is so uncomfortable that by far the best thing to do is declare oneself an agnostic." *Simon Saunders* [28]

The terminal event is certainly one and only, depending on whichever is the direction of the movement. Note, however, that even if the difference in the slopes of the vectors originating in A was infinitesimally small, the points at which these lines would be meeting hypotenuse B_1B_2 would not be exhibiting continuity. To ensure continuity, we should allow for such curvilinear movements that given the slope of a line like $A\epsilon$ in Fig. 3, $A\epsilon$ could act on its way to the hypotenuse as a tree trunk of as many branch-vectors as needed from the viewpoint of slope to fill the gaps left by two consecutive trunk lines, say ψ branches with ψ slopes as Fig. 3 tries to illustrate. In addition to the continuity of the locus B_1B_2, the existence of such a vastness of tree-like temporal lines ensures through them the better representation of the diversity of curvilinear motions, from the smooth and accelerating/decelerating motion, up to the most complex one. The term "temporal line" connotes either straight- or angled-line spatial representation of time lengths. The shortest one is given by the bisector, median, and altitude, $AH = 1/\sqrt{2}$ since, $B_1B_2 = \sqrt{AB_1^2 + AB_2^2} = \sqrt{1+1} = \sqrt{2}$ and hence, $B_1H = B_2H = AH = \sqrt{2}/2 = 1/\sqrt{2}$. In general, $T = 1/\sqrt{2}$ is the minimal age-shaping time, and corresponds to the maximal velocity $v = AA' = \sqrt{2}$ in the planes $tx_i - tx_j$, $i, j = 1,2,3$, $i \neq j$, (and to maximum mass alteration).

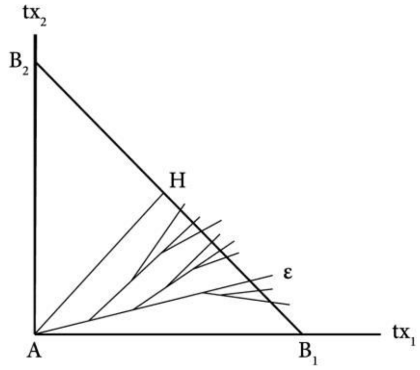

Fig. 3

Next, note that a locus B_iB_j may be seen not only from the perspective of point A but also from the perspective of a point B, i.e. as an inter-event line from B_i to B_j, representing the reproduction, the transformation, of B_i to B_j from dimension i to dimension j, under smooth rectilinear motion with the same speed, u, as that from A to B_i or B_j. Such a movement of $M(1)$ in space from position B_i to position B_j, might well be contemplated spatially simply because, the required age-shaping time $T = \sqrt{2}$ is greater than the time $T = t = 1$, which is required by the minimal spatial movement. This, however, under a smooth rectilinear velocity equaling to u. No other movement may be assumed, because in terms of the temporal side of the movement, first, we have $dT_i/dT_j = ct < 0$, i.e. conversion of age-shaping time T_i of dimension i to such time T_j of dimension j, and time conversion can only be given by a negative constant, ct. And secondly, given the coincidence of age-shaping and historical time, $T = t = 1$, regarding events B_i and B_j, it must be the case that $ct = 1$: The counting of time by the clock of an observer either from A to B_i or from A to B_j is the same, and therefore it should not change if of course this observer wants to go from B_i to B_j and not somewhere else.

But, can actually an observer go from B_i to B_j, can actually an event be transformed into another identical, be repeated in another dimension of space, since each event is unique and non-reproducible by assumption? That $T = \sqrt{2}$, that reproduction involves the lapse of a physical time period given by an irrational number, that the clock of the observer which records the time, (which time, physical and historical, cannot but to coincide in the end due to smooth rectilinear motion), that this clock will stop at the last rational number before $\sqrt{2}$, because immediately then will start the endless parade of the decimal digits of $\sqrt{2}$, leads one to conclude that the reproduction, repetition of an event is impossible. It is prevented by some kind of "age-shaping-time whirl", because which other phenomenon the boundlessness of the decimal digits of $\sqrt{2}$ should may be taken to reflect?

To be precise, this boundlessness is open to two interpretations. Either is the cause of unrealized reproduction, just as claimed, or is the result of the unrealized in the sense that if there was a clock recording rational and irrational time, the accumulation of decimal digits would reflect how close the reproduction of the event is nearing to its completion. Increasingly being completed and increasingly some detail remaining to be settled. The endless infinite of the decimal digits manifests that in any case, the completion never comes. And, we can only accept the first interpretation of infinity, that completion is inhibited by an age-shaping-time vortex, simply because the progress temporally of the completion could well be measured by rational and not necessarily by irrational numbers. It is good at this point to start calling these vortices through the term "ether ruptures/ whirls/spin-outs", too, because as we shall see, it is the phenomenon responsible for the irrationality of age-shaping time and for age-shaping by itself.

The general conclusion so far is that irrationality describes ether ruptures, which guarantee the uniqueness of events by preventing their reproduction, and that because of this precisely reason, infinity has to be the potential one. Ether must have some physical hypostasis of its own, it must be some unknown self-existent entity, serving and as a sort of physical time, and certainly not as any dimension additional to the three known through our senses. Indeed, this is precisely the conclusion which obtains from the following sequence of arguments, prompted by noting that an ether rupture inhibits the reproduction of events by altering their material side. The material hypostasis of M, which at B_i is $M_i(1)$, moving M to position B_j, will have to become $M_j(1)$. But, how exactly given that such an alteration may also be brought about by a spatial movement *per se*?

Toward this direction, note that any spatial distance in space, be a line segment or portion of a curve, has a beginning and an end and can thus be described only by a rational number. Hence, $B_iB_j = \sqrt{2}$ is time, physical time, and the corresponding spatial distance is the first rational either before the $\sqrt{2}$; This implies in turn that space is never rectilinear, because $\sqrt{2}$ and the rational will have to coincide, and this can happen only if the spatial representation of one of them is curved. And, because time, either age or historical, cannot but to be rectilinear, it must be space which is curved. Given now that any division and subdivision of time cannot but to be maintained with its passage, this division and subdivision breaks down at the end of a curved movement when the linear of time has to co-occur with the curved of space the way Fig. 4 tries to illustrate with regard to the one-dimensional space. Curve AA′H is supposed to be equal to AH′, and AH, which is time, equals AC_2. The dots in this Fig. show how the age-shaping time is "forced" to end with the end of the path of motion in space, and *vice versa*; the cause for this forced marriage between the rectilinear and the curvilinear being the end of the trip of the moving mass. The dots represent the ether spin-out; where it starts and where it ends. This is how the linearity of time is reconciled with the curvity of space; through ether spin-outs.

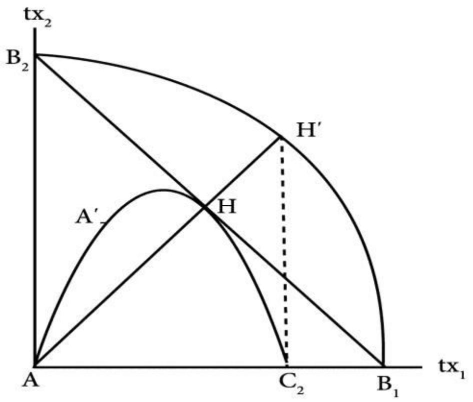

Fig. 4

The "irrationality" of age-shaping time reflects the fact that the moving mass attracts so to speak, it and space to co-occur "at least" at the end of the motion. It reflects that this happens suddenly and this is the reason for the emergence of ether ruptures. And, this in turn, corroborates further the view that time is not some fourth dimension, but a manifestation of some intangible physical entity as real as light, heat, smell, etc. The order or disorder of an ether whirl is captured by the way the eternal recurrence of decimal digits takes place under irrationality, by the structure of the seeming only chaos of non-repeating decimals. We shall see below the two ways through which ether and ether vortices become as physically sensible as light, heat, smell, etc.

The greater the curvature of space, the greater a vortex, a whirl is. Since the size of a spatial distance can be from subtle to infinite, vortices are everywhere, ranging they too, from subtle, up to gigantic. And, since a given distance length could be considered as consisting of a continuum of smaller lengths, a great vortex could perhaps be seen as an accumulation of the smaller vortices associated with the smaller spatial lengths. The phrase "at least at the end of the motion," we used in the previous paragraph concerning the "forced" co-occurrence of age-shaping time with space, could be seen from this exactly point of view. And, it is only in connection with this kind of time that time might be seen as being interwoven with space as is the tendency to do nowadays. How could not it be so the moment historical time is just registration numbers devoid of any empirical content?

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The way specifically irrationality expresses the structure of a vortex, is a matter as already remarked of the structure of decimal digits. Their infinity captures the fact that time stops and yet goes on. But, why the endless unwinding of decimal digits does not stop somewhere given that a vortex does lead somewhere, ending there? Why the infinite of the decimal digits should not be the actual rather than the potential infinite? The answer is that irrationality captures not only the vortex itself but also the vortex' consequences on matter-cum-energy, and notably doubly as follows: Note, first, that the span of a curved line into a straight line, would give a line longer than that representing time like for example, AH' vs. $AA'H$ in Fig. 4. Therefore, given the length of time as it is captured by the length of the segment representing time, the drive speed on the curved space would have to be greater than the speed on straight course. Otherwise the movement on the linear spread of the curve would stop before the end of this spread, producing segment equal to that representing time.

However, as already mentioned, when a mass on a curved path reaches the same destination at the same date as an identical at the starting point mass but on a straight path and therefore with a slower velocity *vis a vis* that along the curve, the former mass reaches at a younger age relative to the age of the latter mass; ($T < 1$). The two masses will be found at the terminal together again as in the beginning, but now with an age difference. We are now in a position to say that this is caused by the vortex triggered by the stoppage of motion, making age-shaping time co-occur with space. A mass may well be aging by the movement in space and by the passage of historical time independently of movement, but the vortices appear to revive it so to speak. And, they revive it in such a way that the "without this revitalization" mass that would exist if it were traveling together with the "older upon arrival at the same destination and historical time" mass, to have disappeared as such since it will have been transformed into a younger mass. $v > u$ is the cause of $T < 1$, but the phenomenon of the vortex is the cause of arrival at $t = 1$ at age $T < 1$.

It is these two interrelated facets of the impact of vortex on matter-energy that irrationality captures beyond the vortex itself. One might say metaphorically that irrationality captures one the one hand the death of the initial mass and the stoppage of the clock there, and on the other hand the mass' revival in another form and the clock continues to tick. How can then the clock counting rational and irrational times not to stop at the last rational number before irrationality when death occurs, and to have at the same time not to stop in view of a quasi resurrection? But, only by allowing the markers to move on to the irrational number. Died and rose again the mass, or did not die and did not rise? This very hazy, undecided state is what irrationality is all about.

It remains one more point to clarify: If space is never rectilinear, what exactly does the term "smooth rectilinear motion" mean? What exactly the three dimensions not only in Fig. 1 but in all Figs., represent? Let us consider again triangle $\vartriangle B_1AB_2$ in Fig. 1. The segment $AH = 1/\sqrt{2}$ is clearly a purely temporal line, and specifically an age-shaping time continuum. The catheti AB_1 and B_2 , what are they? To answer, note that as soon as the average speed v' behind $AH = 1/\sqrt{2}$ is the maximum within the context of the two-dimensional space of triangle $\vartriangle B_1AB_2$, any formula involving v', would give $AB_1 = AB_2 = v'/\sqrt{2}$ It would give, in other words, an irrational number describing spatial distance, and spatial quantities can never be described by such a number. Consequently, temporal (either physical or historical) continua are lines AB_1 and AB_2, and by extension AB_3 as well. Space is curved, and the three dimensions in our Figures are clearly purely temporal lines, the dimensions of time corresponding to the three curved spatial dimensions. Space in reality is what it is; but, as far as motion in it is concerned, it is curvilinear, being linear only as a description of the omnipresence of ether, of ether's manifestation as age-shaping time before a vortex and hence, of the hypothetical movement of this time in space, and finally, of dating event occurrence.

Consequently, when we say "smooth rectilinear motion of mass", we mean a smooth, non-accelerating or decelerating motion on smooth curved line or surface; motion "of mass", because only age-shaping time, can be thought of as having genuine smooth rectilinear motion, motion along a straight line. And, when we say "curvilinear motion (of mass)", we mean either smooth or accelerated or decelerated motion over and above a curved line or surface. And, to be even more precise, line segments AB_i, $i = 1,2,3$, represent the same age-shaping time for the least curved smooth curvilinear motion compared to all other curvilinear motions, with the tangents at A forming a solid right angle. This is the case where the age-shaping is equated with historical time and the case for the shortest possible vortex: The case of the minimum after the straight line curvature. In reality, however, if only age-shaping time is traveling straight, there is not something tangible upon which we can base a definition of minimum curvature empirically. This concept has merit only on analytical grounds.

IV. Time and the Uniqueness of Events

"Die Zeit ist selbst ein Element: Time is itself an element." *Johann Wolfgang von Goethe* (1749-1832, [12])

Observe next that a locus is formed by the purely temporal lines of historical time too, by the unit-length vectors drawn from point A depending on the type of curvilinear motion. Vector $AH' = 1$, for example, in Fig. 1, provides the historical time corresponding to curvilinear motion with physical time $AH = 1/\sqrt{2}$. Point H belongs to an arc joining the points B_1 and B_2 and equaling to the fourth of a circle circumference with center A and radius $R = 1 = AB_i$, $i = 1,2$. Such in general circular arc represents the locus of historical times depending on the curvature of movement, just as a hypotenuse is the locus of age-shaping time. Each point on this arc indicates arrival at the same historical time and at equidistant from

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A terminal depending on the type of motion and mass that arrives at a particular terminal at that time.

The vectors prompting the emergence of the arc are purely temporal lines, because simply they are straight lines; and the locus they form, the arc, is thereby a purely temporal line as well. But, historical time cannot take on irrational values and hence, there cannot be continuity with the strict sense of the term, which in turn implies that the vectors and the locus they form are both discontinuous. Let us elaborate on this matter by asking: Which exactly is the correspondence between the points on the arc and the points of the hypotenuse.

The observation made earlier that even an infinitesimally small difference in the slope of vectors drawn from A, would rupture the continuity of the hypotenuse, is applicable to the case of the arc, too. The problem with the continuity of the hypotenuse, were treated through the concept of tree-like temporal continua as in Fig. 3: Suffices the branches from the trunks to meet the hypotenuse in a manner such that two consecutive intersection points leave no empty space between them. But the length of an arc such as the arc $B_1H'B_2$ exceeds the length of a chord-hypotenuse such as line B_1B_2, which implies in turn that the method of ensuring the continuity of the hypotenuse cannot solve the continuity problem with the arc. It follows that although the locus of age-shaping time, the hypotenuse, may be claimed to be continuous, the locus of historical times, the arc corresponding to the hypotenuse, may not.

It may be odd that the latter locus is discontinuous, but the discontinuity simply reflects the impossibility of the reproduction of an event, like for example the reproduction of B_i as event B_j, from the viewpoint of historical time as well. The discontinuity tells us that an attempt to reproduce B_i as B_j, will fail long before point B_j, with the first vortex that will be encountered by the moving mass, cut short subsequently the historical time too, at its first discontinuity. The presence of a given mass at two places at a given moment in history is impossible. And, any attempt towards such a goal will be cut short right at its roots. This is what the discontinuity of historical time clarifies by stopping a reproduction attempt of B_i to B_j not at the last moment just before the completion of the reproduction, but much earlier, with the first discontinuity.

Next note that event reproduction has been linked up to the one-to-one conversion of recorded, age-shaping, time from one dimension to another, because simply it is about smooth rectilinear motion with identical recorded and historical time recorded at B_i and B_j in the two-dimensional temporal space $tx_i - tx_j$. That is, we have $dT_i/dT_j = 1$, but this stops being valid in the three-dimensional space where as we shall see immediately, we have $dT_i/dT_j = ct$ in general, ct=constant.

Toward this end, consider at first the three identical isosceles right triangles $\vartriangle B_iAB_j$, which form in Fig. 1, the three faces of a triangular pyramid, of a unit-legs trirectangular at vertex A tetrahedron, with base the equilateral triangle $\vartriangle B_1B_2B_3$, having sides equal to $\sqrt{2}$ as in Figs. 5 and 6. In Figs. 1 and 6, E is the point at which the altitude of the tetrahedron meets the base. As such, E gives the minimum recorded, age-shaping, time in the three-dimensional temporal space $tx_1 - tx_2 - tx_3$. Time, certainly, which is less than the minimum age-shaping time, $T = 1/\sqrt{2}$, in the two-dimensional temporal space $tx_i - tx_j$. AE is actually equal to $(3\sqrt{3}-4)/2$, which may be found by solving for AE (and $E\Delta$) the system of the two equations: $(AE)^2 + (E\Delta)^2 = (1/\sqrt{2})^2 = (\sqrt{2}/2)^2$ and $(AE)^2 = (A\Delta)^2 + (E\Delta)^2 - 2(A\Delta)(E\Delta)cosE\hat{\Delta}A$, through the auxiliary variable $z = \sqrt{1-2(AE)^2}$ and with $cosE\hat{\Delta}A = 1/\sqrt{3}$. The impression one would have about E from the two-dimensional spacetime, is that given by point E' in Fig. 7. Impression, thereby, of an age-shaping time locus being convex towards A, in the two-dimensional always spacetime. In the three-dimensional space, this locus is given by the equilateral triangle $\vartriangle B_1B_2B_3$ while the historical-time locus is given by the 1/8th of the surface of a unit-radius sphere with center A.

Now, point E belongs to the bisector, median, and height, $B_2\Delta$, of the equilateral triangle $\vartriangle B_1B_2B_3$, which is forming the base of the above pyramid. $B_2\Delta$ is also the hypotenuse of the right triangle $\vartriangle B_2A\Delta$ in Fig. 1, with legs $AB_2 = 1$ and $A\Delta = 1/\sqrt{2}$ $(= \sqrt{2}/2$, because triangle

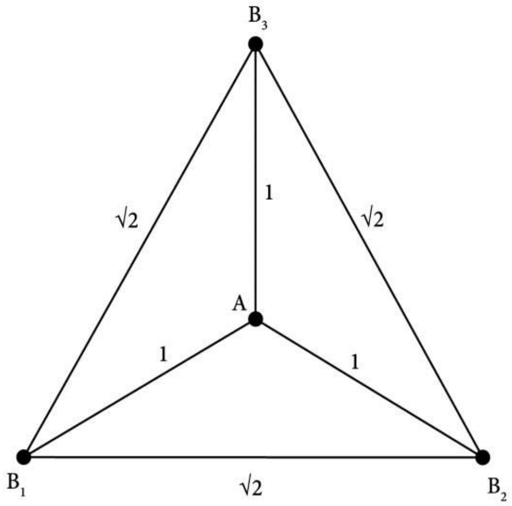

Fig. 5

$\vartriangle A\Delta B_1$ is an isosceles right triangle), and thus, $B_2\Delta = \sqrt{3}/\sqrt{2}$. These values of the sides imply moreover that the tangent of the acute angle $\angle B_2\Delta A$ is equal to $\sqrt{2}$ and that the tangent of the other acute angle, $\angle\Delta B_2A$, of triangle $\vartriangle B_2A\Delta$, is equal to $1/\sqrt{2}$. It follows that a would-be conversion of the recorded time $T = 1/\sqrt{2}$ at Δ to the recorded and historical terminal time $T = t = 1$ at B_2, would be possible at a rate not 1 but $\sqrt{2}$. Similarly, a potential conversion of age-shaping time at B_2 to such time at Δ, would be possible at the rate of $1/\sqrt{2}$. In any case, we have $dT_i/dT_j = ct$, since each conversion presumes smooth and steady rectilinear movement from one event to the other, but no longer applies that $ct = 1$.

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This, i.e, $ct = 1$, applies only to the sides of the equilateral triangle of the base.

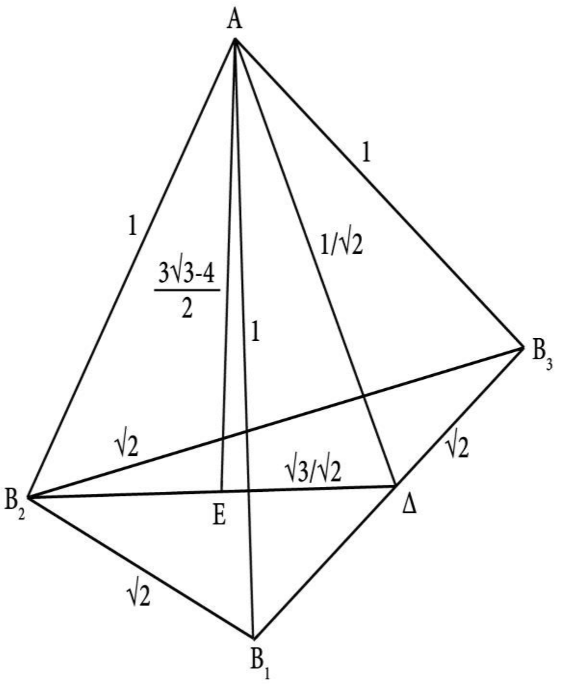

Fig. 6

Both, the hypotenuse and the one leg of the right triangles like $\triangle B_2A\Delta$ that are formed with fixed the leg AB_2, decrease in length as we move either from B_1 to Δ, or from B_3 to Δ, with minima the lengths $A\Delta = 1/\sqrt{2}$ and $B_2\Delta = \sqrt{3}/\sqrt{2}$. But, the decreasing leg, decreases more than the decreasing hypotenuse, with the ratio e.g. $(B_2B_1/AB_1) = (\sqrt{2}/1) = \sqrt{2}$ becoming eventually, $(B_2\Delta/A\Delta) = [(\sqrt{3}/\sqrt{2})/(1/\sqrt{2})] = \sqrt{3} > \sqrt{2}$. Consequently, the rate of conversion of the time recorded at points between $B_1\Delta$ and between $B_3\Delta$ to the time of point B_2, increases steadily as we move from B_1 or B_3 towards Δ; increases steadily from $ct = 1$ at B_1 or B_3 to $ct = \sqrt{2}$ at Δ, (or decreases from 1 to $1/\sqrt{2}$ if the conversion is being done from B_2 towards these points).

Intuitively, one temporal moment should be convertible to one precisely other such moment and not e.g. to $\sqrt{2}$ or $1/\sqrt{2}$. The subdivision of time on an observer's watch should not change. But, this is exactly what should happen if one of the events was the outcome of curvilinear motion like those along B_1B_3 (excluding endpoints), and the other event was brought about by smooth rectilinear motion as is the case with event B_2. If an observer was possible to travel from the former to the latter (but linearly), the counting of time by his/her watch should accelerate to coordinate with the flow of time under the smooth rectilinear motion, because simply the counting of time under the curvilinear motion is slower *vis a vis* the counting under the smooth rectilinear motion. And, if an observer was possible to travel from an event induced by smooth rectilinear motion like event B_2, to another event caused by curvilinear motion, like event Δ, the counting of time by his/her watch should decelerate to coordinate with the slower flow of time that accompanies the curvilinear motion.

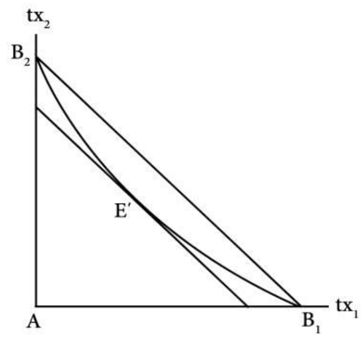

Fig. 7

Of course, the adjustment of the counting should be done only once, because we presume that the course from the one event to the other, is smooth rectilinear steady course. If the course were curvilinear, the derivative dT_i/dT_j would no longer be equal to some constant, ct. But, the mode of inter-event travel, be it curvilinear or rectilinear, does not really matter, because note that the watch should be adjusted according to an irrational time. Indeed, any attempt towards such travel, towards the reproduction of an event, is doomed to fail right at its roots in the three-dimensional world as well. This is to what the last paragraphs come down.

V. The Delian Problem

"That the knowledge at which geometry aims is knowledge of the eternal and not of aught perishing and transient." *Plato* (Republic, 527, [24])

Having clarified this point, let us resume the discussion about the two loci. The locus of historical time is given by the $1/8$th of the surface of a unit-radius sphere with center A, but with this surface being discontinuous, with the points comprising it having gaps among them, for the same reason this locus is discontinuous in the two-dimensional space, too. The total area of the surface of this sphere is $4\pi(1)^2$ and hence, the area of the $1/8$th of this surface is $4\pi/8 = \pi/2$. From this $\pi/2$, the part $(\pi/2) - (\sqrt{3}/2)$ does not belong to the historical time locus, because simply the area of this locus has to be equal with the area of the physical time locus, which is given by the area of triangle $\triangle B_1B_2B_3$ and which by the formula of *Heron of Alexandria* (c.10-70 A.D.), is $\sqrt{3}/2$. The scenaria regarding the alternative routes of the moving M, which produce points on $\triangle B_1B_2B_3$ leaving no gaps among them, filling out fully the area of $\triangle B_1B_2B_3$, are recorded historically as well, by the part of the surface of the sphere corresponding to $\triangle B_1B_2B_3$, but scattered on this surface section given that the area of this section is larger *vis a vis* the area of triangle $\triangle B_1B_2B_3$. Consequently, pooling the scattered points to form a homogeneous space, should yield a spherical cap area equal to $\pi h^2 = \sqrt{3}/2$, where h is the hypotenuse depicted in Fig. 8.

This precisely matter, the geometry of age-shaping time and ether, is the one which motivates empirically the broader problem of squaring the circle, as we had already the chance to realize again but less emphatically through the Lune of Hippocrates. This precisely matter is also the one behind the Delian problem of doubling the cube, which reads: "Given line segment a, construct another line segment x such that $x^3 = 2a^3 \Rightarrow x = a\sqrt[3]{2}$, or if $a = 1$, $x = \sqrt[3]{2}$" [14, 15, 33, 34]. Now, note that the volume of our pyramid is $1/3$rd of the product of the area $\sqrt{3}/2$ with the apex $AE = (3\sqrt{3} - 4)/2$: that is, $(3\sqrt{3} - 4)/4\sqrt{3}$. The volume of the regular octahedron (Fig. 3.10) formed by bases like those of $\triangle B_1B_2B_3$, is $\sqrt{2}/3$ times the cube of edge length $\sqrt{2}$: $(\sqrt{2}/3)(\sqrt{2})^3 = 4/3$. This implies two things: The first is that this volume, which is equal to eight times $1/6$, should be eight times the volume of the pyramid and hence, that pyramid volume should be equal to $1/6$: $(3\sqrt{3} - 4)/4\sqrt{3} = 1/6 \Rightarrow \sqrt{3} = 12/7$. The number $\sqrt{3}$ emerges as the rational $12/7 = 1.7142857142857\ldots$ rather than the irrational $1.7320508075689\ldots$! Either there is something wrong with the formulas, which I do not think so, because they are Geometry, or there is something wrong with the Arithmetic!

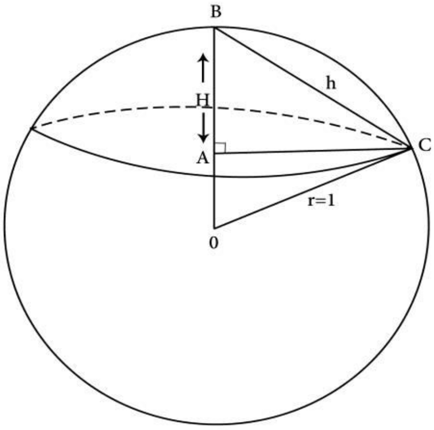

Fig. 8

The second is that the volume of the octahedron must also be equal with the volume of the eight spherical caps, one for each $1/8$th of the octahedron: $[(8\pi H^2/3)(3(1) - H)] = 4/3 \Rightarrow \pi H^3 = [3\pi H^2 - (1/2)](1)^3$, where H is as in Fig. 8. From the last expression, one obtains the cubic equation in H: $2\pi H^3 - 6\pi H^2 + 1 = 0$, having real all roots, because its discriminant $\pi^2(8\pi - 1) > 0$ since $\pi > 1/8$. Letting $w = H^2 \Rightarrow wH = H^3$, this equation becomes the quadratic one: $2\pi w^2 - 6\pi w + 1 = 0$, with roots, $w_{1,2} = (3\pi \pm \sqrt{9\pi^2 - 2\pi})/2\pi$. Of course, $9\pi^2 - 2\pi > 0 \Rightarrow \pi > 2/9$, which is true, and $(3\pi)^2 > (\sqrt{9\pi^2 - 2\pi})^2 \Rightarrow 0 > -2\pi$, which is also true. Hence, two of the H's are $H_{1,2} = \sqrt{(3\pi \pm \sqrt{9\pi^2 - 2\pi})/2\pi}$, and one of the H^3, based for convenience on w_1 as being the w with the positive sign of the square root, is $H^3 = \left[\sqrt{(3\pi + \sqrt{9\pi^2 - 2\pi})/2\pi\sqrt{2}}\right]^3$ Next, note that the quadratic equation may be rewritten as follows: $w^2 - 3w + (1/2\pi) = 0$, which implies that the w's and subsequently, the H's are geometrically constructible. The w's emerge from the problem: "Given square of side length equal to $\sqrt{1/2\pi}$, construct parallelogram of sides w_1 and w_2 such that $w_1 + w_2 = 3$". And, once they have been constructed, their square roots may be constructed through various methods as, for instance, the geometric mean method. Finally, the multiplication w_1H_1 may be constructed to give H^3, or by doubling it, $2H^3$. Now, let H be the a of the Delian problem: The line segment $2H^3$ is the cube of the line segment x sought by this problem.

It remains to determine x by returning back to the beginning of the construction procedure. It is not proper to assume that a line segment such as the side length $\sqrt{1/2\pi}$ is given, because it contains the number π. This length has to be constructed through the Quadrature. There are two ways to do so given than in any case an acute angle of a right triangle will have to be formed, having a trigonometric tangent equal to $\sqrt{\pi}$: One is to take as given a unit-length line segment, and the other is to consider as given a segment $y \neq 1$, provided that in either case we have to have $y^2 = \pi(\sqrt{2})^2$. To choose, note that in order to construct a square root, a unit-length line segment has to be given; and we did have to construct such a root earlier. Consequently, having line segments of length 1 and H, and noting that $x = H\sqrt[3]{2}$, segments x and $\sqrt[3]{2}$ might be constructed too, through the intercept theorem of Thales of Miletus (c.624-c.546 B.C.) from the ratio: $x/\sqrt[3]{2} = H/2$. Alternatively, inserting $y = \sqrt{2\pi}$ in H_1, yields, $H_1 = y\left(\sqrt{(3\pi \pm \sqrt{9\pi^2 - 2\pi})}\right)$, which when inserted in the expression for $2H^3$, gives, $2H^3 = (H_1^3/y^3)(1/y\pi)$ or the same, $x = (H_1/y)(1/\sqrt[3]{y\pi})$. Hence, the product $x\sqrt[3]{y\pi}$ may be constructed through Thales' intercept theorem from the ratio: $(x\sqrt[3]{y\pi})/1 = H_1/y$. And, having this construction, which also refers to $H\sqrt[3]{2}(\sqrt[3]{y\pi})$, length $\sqrt[3]{2}(\sqrt[3]{y\pi})$ may be constructed via the same theorem, from the ratio, $x\sqrt[3]{y\pi}/H = \sqrt[3]{2}(\sqrt[3]{y\pi})/1$, or from the ratio, $\sqrt[3]{2}(\sqrt[3]{y\pi})/1 = H_1/Hy$...

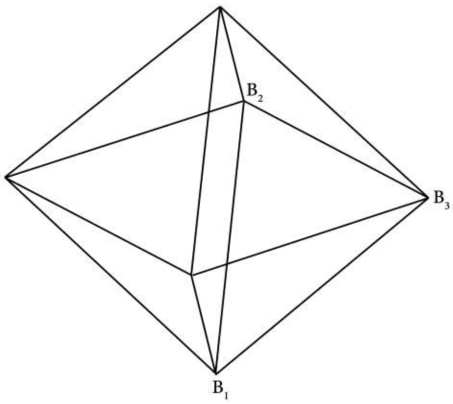

Fig. 9

And, what exactly is the "trisection of an acute angle"? Consider again the equilateral triangle $\Delta\, B_1B_2B_3$, as in Fig. 9. We have contemplated the theoretical possibility of traveling from B_i to B_j through smooth rectilinear motion. Let us continue theorizing by contemplating curvilinear motions along the plane defined by $\Delta\, B_1B_2B_3$ the same way such motions were considered earlier within the context of right triangles $\Delta\, AB_iB_j$. One might conclude then that the sides of $\Delta\, B_1B_2B_3$ are also loci of age-shaping time for motion originating in the opposite vertex. The corresponding loci of historical time would be obtained in the form of sharp *Franz Reuleaux* (1829-1905, [9, 11]) triangles by drawing circular arcs from each vertex of our equilateral triangle between the other two vertices, arcs like B_2UB_3 in Fig. 10.

Now, let B_1Q' be one of the trisectors of angle $\angle B_2B_1B_3$, point U be the intersection point of the extension of this trisector with arc B_2UB_3, and $UH = U'Q$ be parallel to B_1B_3, cutting B_2B_3 at point Q. The subsequent angle metrics are as in [32, pp.23-24]. Are H and Q the midpoints of B_1B_2 and B_2B_3, respectively? They are, because from the law of cosines, one obtains that:
$(B_1Q)^2 =$
$(B_1B_2)^2 + (B_2Q)^2 - 2(B_1B_2)(B_2Q)cos(\pi/3) \Rightarrow$
$(B_2Q)^2 - (B_2Q)\sqrt{2} + [2 - (B_1Q)^2] = 0$, and $(B_1Q)^2 =$
$(B_1B_3)^2 + (B_3Q)^2 - 2(B_1B_3)(B_3Q)cos(\pi/3) \Rightarrow$
$(B_3Q)^2 - (B_3Q)\sqrt{2} + [2 - (B_1Q)^2] = 0$ which two quadratic equations give for (B_2Q) and (B_3Q) the same root, $\left[1 \pm \sqrt{2(B_1Q)^2 - 3}\right]/\sqrt{2}$, and hence, $(B_2Q) = (B_3Q)$. (B_1Q) is the median, height, and bisector originating from B_1: $\sqrt{2(B_1Q)^2 - 3} = 0 \Rightarrow (B_1Q) = \sqrt{3/2}$ and $(B_2Q) = (B_3Q) = 1/\sqrt{2}$.

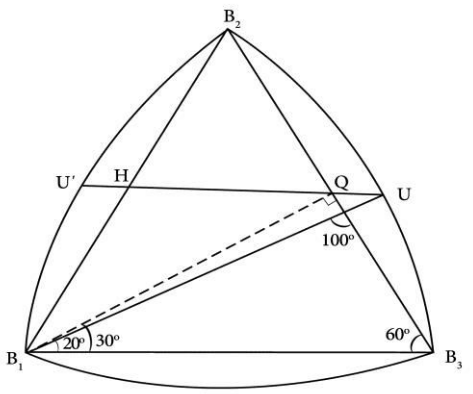

Fig. 10

The connection of trisection with the Reuleaux circular arcs [9, 11] is I think clear. The discussion about trisection might be easily extended to Reuleaux-like circular arcs in connection with the two equal sides of an isosceles triangle, guided by the vertices of the equilateral triangle formed by these two sides. All our two quadratic equations above need to hold is the equality of two angles: All the line segment analogous to B_1Q needs to be is a median. Having trisected the two equal angles, one can easily form then a *Frank Morley* (1860-1937, [22]) triangle to find the trisectors of the third angle… And, of course, given that we are dealing with an equilateral triangle in any case, one should not be surprised if *George Phillips Odom*-like golden sections [2] were found as well…

VI. Concluding Remarks

"There is no space without aether, and no aether which does not occupy space." *Sir Arthur Stanley Eddington* (1882-1944, [7])

In sum, ether is motion, and motion is life. Put differently, life presupposes motion, and ether is what ensures it. As far as Arithmetic and Mathematics are concerned, although the continuity of the locus B_iB_j allows the treatment of age time as a continuous variable, when one does so, one disregards that age time is defined over hypostasis differing events. Any function containing time as an argument should be continuous but non-differentiable. Much more so with respect to historic time, which is subject to "continuous discontinuity", and should be treated at best as being coincident with age time? These are the empirical considerations I think that explain why differentiable functions are only a meager subset of all continuous functions, and why a piecewise non-differentiable function defined between successive $(0-1)$'s could be used to capture the dynamics of hypostasis change through a self-excited repellor connected with an unstable equilibrium at 1 (birth) and through a self-excited attractor at the 0 of the next $(0-1)$. The "self-excitement" has been explained in Section VI above, but the aspect of instability is I think clear: If the equilibrium was stable, there would not be hypostasis change;... eternal youth... And, if we had attraction instead of repelling for a given $(0-1)$, the same entity would be born and reborn *ad infinitum*... Either case is inconsistent with our findings here. For example, if hypostasis change was a matter of age time alone according to a Riemann-Weierstrass function, we would have a sequence of fractals, each differing from and serving as an attractor of the previous one.

The uniqueness of events dictates in addition the non-modular character of the piecewise function, which implies that the splines should be differing in at least one rational or irrational number. It is remarkable that as soon as the vast majority of numbers is irrational ones, age time, inorganic and organic life inside space, exhibits much less discontinuity than space itself. Does life serves as patches to space? Or, it is the discontinuity of space that serves as a source of life? Soldatos [32] concludes that irrationals are geometrically constructible too, which in turn implies that life is patches to space. Indeed, empty space might be a good analytical abstraction, but the cosmos is a single unified volume in the sense that it could not be thought of as a container plus whatever is inside it; all is one. Consequently, the numbers which are deemed to be inconstructible irrationals, are deemed so on the basis of the unreal one-dimensional world of the real line. An irrational number does contain an infinite number of non-repeating decimal digits, but it is constructible simply because it is a construction stone of the cosmos, much as rationals are, but more important given that the bulk of numbers is irrationals.

The real issue is the geometry of the connection between the potential and the actual infinite as it comes up arithmetically by noting that age-shaping time is a special entity and as such it needs its own unit of measurement. Space, liquids, gases, temperature, etc., have each its own unit of measurement. But, all units are based on rationals…1.2 yards, 3.5 acres, 10 bushels, 6 gallons, 20 centigrade,...The unit of measurement of age-shaping time should be one reflecting its irrationality, exclusively irrationality, because the end of even the slightest motion is accompanied by a vortex. Rational numbers pertain to all phenomena except physical time, and irrational numbers are pertinent to physical time exclusively. And, all rational numbers are irrationalizable by starting adding to any one of them randomly decimal digits endlessly. A potentially infinite number of potentially irrationals may emanate from a given rational by considering potentially infinite combinations of never ending decimal digits attached to it. Doesn't the number 1 above, the radius of the sphere of historical time, correspond to all irrationals, potential irrationals, on the locus of physical time?...

Genuine continuity can exist only with regard to equilibrium ether. The real line makes sense empirically only in connection with this entity, with the irrationality referring to ether whirls. These, in general, are the arithmetic considerations: the coexistence of two number systems, which are distinct in one domain but complementary in another one, and with this difference being in context indiscernible.

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