The dynamics of average temperature of Earth's surface and thermo-balance of heat fluxes suggest a quasi-periodicity

Sergey V. Ershkov

Institute for Time Nature Explorations, M.V. Lomonosov's Moscow State University,

Leninskie gory, 1-12, Moscow 119991, Russia, e-mail: sergej-ershkov@yandex.ru

Keywords: Earth's surface temperature; thermo-balance; Greenhouse effect.

An independent heat source inside the system "Earth + atmosphere", in addition to Earth's heating by solar radiation, is required to account for differences between the measured and calculated values of Earth's average surface temperature. A mathematical model of thermo-balance for Earth's heat fluxes is used for the calculations reported here. Using well-known data (albedo of the Earth's surface, constant of the heat radiation from the Sun, etc.), it is shown that the process of heating only by solar activity would yield an average value for Earth's surface temperature of circa eighteen degrees below zero (Celcius scale). Since this calculated value of Earth's surface temperature is cooler than the observed value, there must exist others, *inner* sources of heating for the Earth. The main source of such an internal heat generation within the system "Earth + atmosphere" is, of course, the Greenhouse effect.

Besides, the dynamics of the Earth's surface temperature is proved to be determined by the proper *elliptical* integral. Thus, by obtaining of the re-inverse dependence of a solution from the time-parameter *t* we could present the temperature $T(t)$ as a set of periodic cycles.

1. Introduction.

Let us assume the Earth to absorb and then partially re-radiates heat fluxes as a *black body* [1] with the shape of sphere with radius R, which has the average temperature of surface T.

To estimate thermo-balance of Earth's heat fluxes, we should account the amount of the heat flows as below:

- The Solar radiation energy flux [2-3]:

$$F_{ext} \; = \; f \cdot (\pi R^2) \cdot (1 - A) \, , \tag{1.1}$$

- here f – is a measure of flux density of the mean solar electromagnetic radiation (the solar irradiance) per unit area that would be incident on a plane perpendicular to the rays, at a distance of one astronomical unit (AU) from the Sun, f = 1.361 kilowatts per square meter (kW/m²) at solar minimum [2]; R – is the radius of the Earth; A – is the Earth's spherical albedo, A = 0.29 [3].

Let us also assume that the surface of Earth *re-radiates* all the fluxes of energy to the outer Space according to the Stefan-Boltzmann law [4]:

$$F_{out\,(1)} \; = \; 4 \pi R^2 \cdot (\sigma \cdot T^4) \cdot B \, , \tag{1.2}$$

- here σ – is the constant of Stefan-Boltzmann law [4], B – is the coefficient depending on the opaqueness of atmosphere for infrared wavelengths (coefficient B is known to be varying depending on density of water vapor in a clouds [2-3]); also, we should take into account the losses for the evaporation of water from the surface of Ocean [3]:

$$F_{out\ (2)} = 4\pi R^2 \cdot (h \cdot \rho_w \cdot Q)\,, \qquad (1.3)$$

- here $Q = Q_0 - \eta \cdot T = (25 - 0.024 \cdot T) \cdot 10^5$ [J/kg] – is the amount of heat of water evaporation from the unit of Ocean's square per unit of time ($25 \cdot 10^5$ J/kg – is the amount of heat of water evaporation at temperature 0°C); h – is the average height of water in mm, which is assumed to be evaporating from the surface of Ocean per unit of time, h ~ 1000 mm/year $=3 \cdot 10^{-8}$ m/s; $\rho_w \approx 1024$ kg/m^3 - is the density of sea water.

We should also take into account the losses for the convection according to the NASA Earth Observatory report [3] as given below

$$F_{out\ (3)} = \alpha \cdot F_{ext}\,, \qquad (1.4)$$

- where α - is the appropriate dimensionless coefficient of proportionality, $\alpha = 0.05$ (5%).

Besides, let us note that only 48% of the initial Solar radiation energy flux F_{ext} reaches the Earth's surface (let us denote such a coefficient as $\beta = 0.48$).

It means that $B = \beta - \alpha - (F_{out\ (2)} / F_{ext})$, where ratio $(F_{out\ (2)} / F_{ext})$ is for meaning of temperature circa ~ 17 °C ≈ 290.2 K; but the losses for re-radiation according to the NASA Earth Observatory report [3] is circa 18% of the initial Solar radiation energy flux F_{ext}.

Thus, we could estimate the thermo-balance of Earth's heat fluxes near the surface of Earth (without accounting of effects of heat transfer by turbulence or diffusion in the Ocean):

$$\frac{d\,H}{d\,t} \;=\; \beta \cdot F_{ext} \;+\; F_{\text{int}} \;-\; (F_{out\,(1)} + F_{out\,(2)} + F_{out\,(3)}) \qquad (1.5)$$

- here H – is the effective enthalpy of Earth's surface; F_{int} – is the source of internal heat generation (of unknown nature which to be determined).

So, we should obtain from (1.1)-(1.5) as below:

$$\frac{d\,(C \cdot T)}{d\,t} \;=\; \frac{(\beta - \alpha) \cdot F_{ext} \;+\; F_{\text{int}}}{4\,\pi\, R^2} \;-\; \sigma \cdot T^{\,4} \cdot B - h \cdot \rho_{\,w} \cdot (Q_0 - \eta \cdot T)$$

- here t – is the time-parameter; C – is the isobaric heat capacity of the Earth's surface per the unit of square. If we designate:

$$\Delta\, f_{\text{int}} \;=\; \frac{F_{\text{int}}}{4\,\pi\, R^{\,2}}\,, \qquad (1.6)$$

- the equation (1.6) could be transformed as

$$\frac{d\,(C \cdot T)}{d\,t} \;=\; \{ f \cdot \frac{(\beta - \alpha) \cdot (1 - A)}{4} + \Delta\, f_{\text{int}} \} \;-\; \sigma \cdot T^{\,4} \cdot B - h \cdot \rho_{\,w} \cdot (Q_{\,0} - \eta \cdot T) \qquad (1.7)$$

We should especially note that in Eqs. (1.6)-(1.7): $\Delta\, f_{int} = \Delta\, f_{int}\,(t)$, in general case.

As we can see, Eq. (1.7) is the generalization of the equations of *Riccati* and *Abel* types [5]. Due to a very special character of *Riccati's type* ordinary differential equation, it's general solution is proven to have a proper jumping of the components of a solution at some definite meanings of time-parameter *t*, or there so-called *gradient catastrophe* could occur [6].

This means a possibility of sudden global changing of temperature on the Earth's surface and the change of the global climate on Earth and environment (let us remember about so called Little Ice Age at the Maunder's minimum of Solar activity [7]).

<u>2. Quasi-periodic solutions, Earth's surface temperature</u>.

According to the data of modern climatology, the average temperature near the surface of the World Ocean [8] is circa ~ 17 °C ≈ 290.2 K.

If we assume Δf_{int} = const, Eq. (1.7) could be presented as below (let us additionally note that for the solid surface of Earth, heat capacity is lower than at the Ocean more than 2 times, but density is higher more than 2 times):

$$\int \frac{dT}{\{f\cdot\frac{(\beta-\alpha)\cdot(1-A)}{4}+\Delta f_{\text{int}}\} - \sigma\cdot T^4\cdot B - h\cdot\rho_w\cdot(Q_0-\eta\cdot T)} = \int\left(\frac{1}{C}\right)dt \qquad (2.1)$$

- where the left side of Equation (2.1) could be transformed to the proper *elliptical* integral [9] in regard to the function $T(t)$; let us denote:

$$a = \left(\{0.25 f\cdot(\beta-\alpha)\cdot(1-A)+\Delta f_{\text{int}} - h\cdot\rho_w\cdot Q\}/\{\sigma\cdot B\}\right)^{\frac{1}{4}} \qquad (2.2)$$

If we exclude the influence of any source of Internal heat generation Δf_{int}, the maximal temperature of the Ocean surface could be calculated for the case of thermodynamic equilibrium {$d\,T = 0$ in (1.7)} from the expression (2.2) as shown below:

$$a_0 = \left(\{0.25 f \cdot (\beta - \alpha) \cdot (1 - A) - h \cdot \rho_w \cdot Q\} / \{\sigma \cdot B\} \right)^{\frac{1}{4}} = 255.4\ K \cong -17.8\,^{\circ}C.$$

Parameters above are taken as follows: $f = 1.361$ (kW/m²), $\beta = 0.48$, $\alpha = 0.05$, $A = 0.29$; $Q = 18.9 \cdot 10^5$ [J/kg], $h \sim 3 \cdot 10^{-8}$ m/s, $\rho_w \approx 1024$ kg/m³; $\sigma = 5.67 \cdot 10^{-8}$ [W/(m²·K⁴)], $B = 0.19$.

In this case we obtain from (1.1)-(1.4):

$$F_{out\,(1)} = 0.19 F_{ext},\ F_{out\,(2)} = 0.24 F_{ext},\ F_{out\,(3)} = 0.05 F_{ext}$$

$$\rightarrow\ F_{out\,(1)} + F_{out\,(2)} + F_{out\,(3)} = 0.48\, F_{ext} \qquad (2.3)$$

Thus, we should conclude that the process of heating only by solar activity would yield an average value for Earth's surface temperature of circa eighteen degrees below zero (Celcius scale). So, $\Delta f_{int} \neq 0$; moreover, the essential part of the sum of sources for Internal heat generation Δf_{int} is, of course, a *Greenhouse effect* [3]. NASA Earth Observatory' reported in [3] that $0.48\, F_{ext}$ is the total sum of heat fluxes to be absorbed by the Earth's surface (from the incoming flux of Solar radiation F_{ext}) and then such a total flux should be outgoing from the Earth's surface to the Earth's atmosphere.

According to the 'NASA Earth Observatory' report [3], meanings of heat fluxes from Earth's surface to Earth's atmosphere should be given as below:

$$F_{out\,(1)} = 0.18F_{ext},\ F_{out\,(2)} = 0.25F_{ext},\ F_{out\,(3)} = 0.05F_{ext}$$

$$\rightarrow\ F_{out\,(1)} + F_{out\,(2)} + F_{out\,(3)} = 0.48\ F_{ext}\quad, \qquad (2.4)$$

- which closely coincide with the meanings of heat fluxes (2.3) above.

To achieve the observed temperature on Earth's surface, which is circa 290.2 K [8], we should take appropriate parameters in (2.2) as follows: f = 1.361 (kW/m²), β = 0.48, α = 0.05, A = 0.29; $\Delta f_{int} = 0.134 \cdot F_{ext}$, $Q = 18 \cdot 10^5$ [J/kg], $h \sim 3 \cdot 10^{-8}$ m/s, $\rho_w \approx 1024$ kg/m³; σ = $5.67 \cdot 10^{-8}$ [W/(m²·K⁴)], B = 0.201. For such a case, we obtain from (1.1)-(1.4):

$$(F_{out\,(1)} + \Delta f_{int}) = (0.335 - 0.134)F_{ext} \cong 0.201F_{ext},\ \ F_{out\,(2)} = 0.23F_{ext},\ F_{out\,(3)} = 0.05F_{ext}$$

$$\rightarrow\ F_{out\,(1)} + F_{out\,(2)} + F_{out\,(3)} \cong 0.48\ F_{ext} \qquad (2.5)$$

- which also closely coincide with the ideal meanings of heat fluxes (2.4) above: $F_{out\,(1)}$ increases from the proper meaning in (2.4) on 11% (equals to $0.02F_{ext}$), $F_{out\,(2)}$ decreases from the proper meaning in (2.4) on 9% (equals to the same $0.02F_{ext}$). In total sum of heat fluxes in (2.5), we obtain the previous meaning of 0.48 F_{ext}. as in Eqs. (2.3)-(2.4).

Let as especially note that amount of the *Greenhouse effect* $\Delta f_{int} = \chi \cdot F_{ext}$, where χ = 0.134 or 13.4% of the initial Solar radiation energy flux F_{ext}.

As for estimating of Solar radiation energy fluxes, it is an easy matter (see (1.1), radius of the Earth, R = 6,371,000 m): having calculated, we obtain the meaning $F_{ext.}$ = 0.48·111,072 Terawatts = 53,315 (TW). So, for the 13.4% of the *Greenhouse effect* we obtain 7,144 (TW).

Discussions.

The above calculations demonstrate that the process of heating only by solar activity is not sufficient to obtain the observed value for the temperature of Earth's surface. This means there must exist other sources of heating for planet Earth, i.e., the *inner* sources of global heating for Earth itself.

The inner sources could be those associated with Kuroda's theory [10-15] of natural nuclear reactors. Similar heat sources have been identified in Jupiter [16] and in the Sun [17].

Also, the inner sources could be associated with Kozyrev's [18] theory for generating heat in celestial bodies. For example, N.A. Kozyrev year 1959 by direct observations that confirmed the Moon is volcanically active.

An additional mechanism of internal heat generation within the system "Earth + atmosphere" may be produced as follows: high-intensity changing outbursts of solar wind (which brings the magnetic field of the Sun, frozen into the plasma of solar wind), through the intermediary of the geomagnetic field, will induce electric currents [19] in the outer atmosphere and Earth, causing ohmic heating, which should form an additional source of Earth's heating. We should especially note that the existence of stable Earth's magnetic field should decrease the ohmic heating above; in the case of absence of a magnetic field of the planet (see the case of Venus [19]), such a process should have a chaotic character, so the temperature of the outer atmosphere would be circa ~ 400-500 degrees C.

But the main source of internal heat generation within the system "Earth + atmosphere" is, of course, the Greenhouse effect. The main reason is much of the radiation emitted from the surface of the Earth is absorbed by the atmosphere, and re-radiated back to the Earth. The

Earth is consequently inefficient at re-radiating the incoming solar radiation, and the surface temperature is much higher than the black body temperature corresponding to the energy flux of the incoming solar radiation.

But the main result, which should be outlined, is that the dynamics of the Earth's surface temperature is proved to be determined by the proper *elliptical* integral.

Thus, by obtaining of re-inverse dependence of a solution from time-parameter we could present the expression of $T(t)$ as a set of periodic cycles (Fig.1-2).

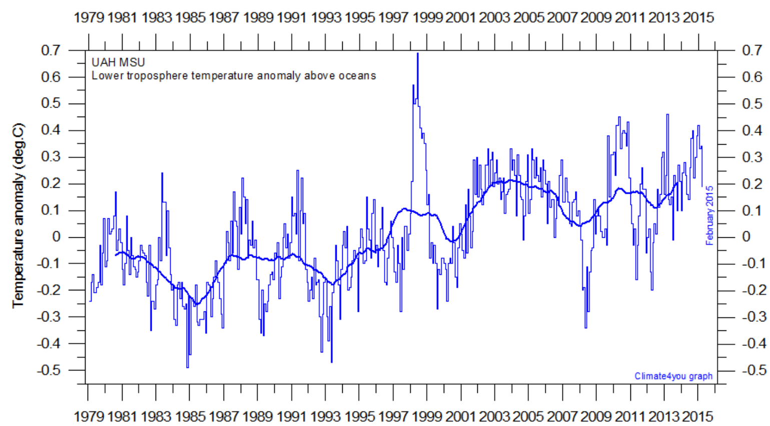

Fig.1. Global monthly average lower troposphere temperature above oceans since 1979, according to University of Alabama at Huntsville, USA

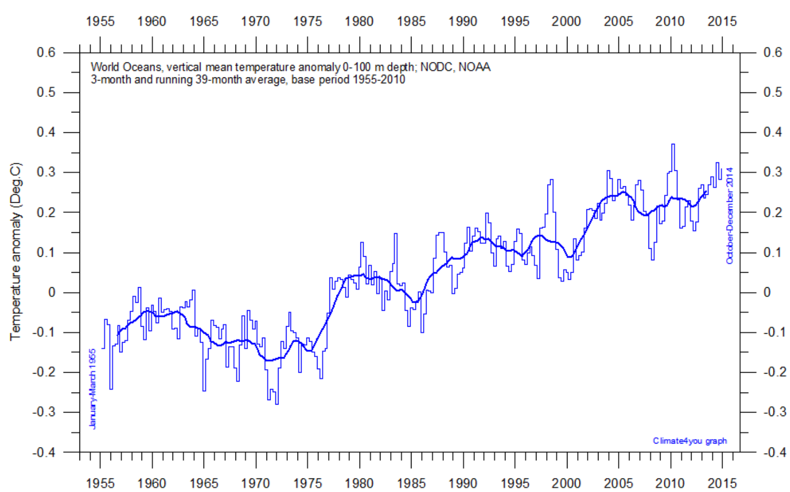

Fig.2. World Oceans vertical average temperature 0-100 m depth since 1955.

The thin line indicates 3-month values, and the thick line represents the 39-month average.

<u>Conclusion</u>.

Estimation of average temperature of Earth's surface in dependence on heating by the Solar radiation energy flow is investigated here.

Mathematical model of thermo-balance of Earth's heat fluxes is suggested for such an estimation. Using well-known data (albedo of the Earth's surface, constant of the heat radiation from the Sun per square meter of Earth's outer atmosphere surface, etc.), we could conclude that the process of heating only by the Solar activity is not sufficient to obtain the real meaning of the temperature of Earth's surface. Estimation yields the average temperature circa -18 degrees below zero (Celcius scale) in this case. It means that there exist other sources of heating of Earth; we suppose such sources should be *the inner* sources of heating of Earth itself.

The inner source, associated with Kuroda's type natural nuclear reactor [10-15], which is assumed to be located in the core of Earth [16], gives the meaning of 'Global heat loss to space' in amount of 44.2 Terawatts (TW) [20], including 15 Terawatts (TW) of natural radioactivity without nuclear fission (we should note that there might be other existing inner sources, associated with Kuroda's theory (natural nuclear reactor), for example being located not at the surface of Earth as Okla paleo-reactor [10-15], but somewhere deep in the Ocean, moreover it might be a variety of natural nuclear reactors).

Finally, the invocation of Ohmic heating through currents driven by the solar-wind is not consistent with our knowledge of the solar wind interaction with the Earth's magnetosphere. It is known that currents flow in the Earth's ionosphere during geomagnetically disturbed periods of time. These currents are most intense in the polar ionosphere. But even then the ionospheric potential is at most a few 100 kV across the polar cap [3], and the current is several 10's of MA. But this only gives a few 1000 TW of Joule dissipation, and that is in the ionosphere - not below the surface of the Earth.

Thus, the main source of internal heat generation within the system “Earth + atmosphere” is, of course, the Greenhouse effect. The main reason is much of the radiation emitted from the surface of the Earth is absorbed by the atmosphere, and re-radiated back to the Earth.

Acknowledgements

I am thankful to unknown Editor of the “Journal of Atmospheric and Solar-Terrestrial Physics” for valuable advices in preparing of this manuscript.

References:

[1]. Mahmoud Massoud (2005). *§2.1 Blackbody radiation*. Engineering thermofluids: thermodynamics, fluid mechanics, and heat transfer. Springer. p. 568.

[2]. Kopp G., Lean J. L. (2011). *A new, lower value of total solar irradiance: Evidence and climate significance*. Geophysical Research Letters **38**: n/a.

[3]. Environmental Encyclopedia, 3rd ed., Thompson Gale, 2003. See also:

http://earthobservatory.nasa.gov/Features/EnergyBalance

[4]. Cole, George H. A.; Woolfson, Michael M. (2002). *Planetary Science: The Science of Planets Around Stars (1st ed.)*. Institute of Physics Publishing. pp. 36–37, 380–382.

[5]. Kamke E. (1971), *Hand-book for Ordinary Differential Eq*. Moscow: Science.

[6]. Arnold V.I . (1992). *Catastrophe Theory, 3-rd ed*. Berlin: Springer-Verlag.

[7]. Legrand, JP; Le Goff, M; Mazaudier, C; Schröder, W (1992). *Solar and auroral activities during the seventeenth century*. Acta Geodaetica et Geophysica Hungarica **27** (2–4): 251–282

See also: http://en.wikipedia.org/wiki/Maunder_Minimum

[8]. NASA's Physical Oceanography Distributed Active Archive Center (PO.DAAC)

Provider of historic and near real time SST data from 14 satellites, from 1981 through yesterday

See also: http://en.wikipedia.org/wiki/Sea_surface_temperature

[9]. Lawden D. (1989) Elliptic Functions and Applications. *Springer-Verlag*. See also:

http://mathworld.wolfram.com/EllipticIntegral.html

[10]. Kuroda P.K. (1982). *The Origin of the Chemical Elements and the Oklo Phenomenon*. Springer-Verlag, Berlin, 165 pages.

http://www.amazon.com/Origin-Chemical-Elements-Oklo-Phenomenon/dp/3540116796

[11]. Kuroda P.K. (1956). *On the nuclear physical stability of the uranium minerals*. Journal of Chemical Physics 25 (4): 781-782 (1956). See also ibid., 1295–1296.

[12]. Kuroda P.K. (1960). *Nuclear fission in the early history of the Earth*. Nature 187, 36-38. http://www.nature.com/nature/journal/v187/n4731/abs/187036a0.html

[13]. Bodu R., et al. (1972). C. R. Acad Sci. Paris, Ser. D 237, pages 1731, 1847 and 2291.

[14]. Boulos M.S. and Manuel O.K. (1971). *The xenon record of extinct radioactivities in the Earth*. Science 174, 1334-1336.

[15]. Braun Tibor (2013). *The Nuclear Chemist Who Foresaw the Past: Paul Kazuo Kuroda and the Oklo Paleoreactors*. (4 pages)

https://dl.dropboxusercontent.com/u/10640850/A_nuclear_chemist_who_foresaw_the_past_Paul_Kazuo_Kuroda.pdf

[16]. Herndon J.M. (1992). *Nuclear fission reactors as energy sources for the giant outer planets*. Naturwissenschaften 79, 7-14.

[17]. Manuel Oliver K (2014). *Solar energy*. Advances in Astronomy (submitted 1 Sept 2014). https://dl.dropboxusercontent.com/u/10640850/Solar_Energy.pdf

[18]. Kozyrev N.A. (2005), *Sources of Stellar Energy and the Theory of the Internal Constitution of Stars*, Progress in Physics, v.3, 61-99. See also:

http://en.wikipedia.org/wiki/Nikolai_Aleksandrovich_Kozyrev

[19]. Herndon J.M. (2013). *New indivisible planetary science paradigm*. General article, Current Science, 105 (4): 450-460.

[20]. Herndon J.M. (2014). *Terracentric nuclear fission georeactor: background, basis, feasibility, structure, evidence and geophysical* implications. Review article, Current Science, 106 (4): 528-541.