Estimation of mean-values of Earth's surface temperature:

Mathematical robust model of thermo-balance of Earth's heat fluxes.

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Keywords: Earth's surface temperature, *Riccati* type of ODE, thermo-balance, heat fluxes.

Estimation of mean-values of Earth's surface temperature in dependence on heating by the Solar radiation energy flow is investigated here.

Mathematical robust model of thermo-balance of Earth's heat fluxes is suggested for such an estimation. Using well-known data (albedo of the Earth's surface, constant of heat flux from the Sun per square meter of Earth's outer atmosphere surface, etc.), we could conclude that heating only by the Solar activity is not sufficient to obtain the real meaning of temperature of Earth's surface. Estimation yields the mean-valued temperature about -50 degrees below zero (Celcius scale) in this case. It means that there exist other sources of heating of Earth; we suppose such sources to be *the inner* sources of heating of Earth itself.

The inner sources could be associated with Kozyrev's theory of Time (with generating sources in celestial bodies; the additional effect is the heating of these celestial bodies). For example, N.A. Kozyrev suggested the inner heating of the Moon and such a fact was successfully proved in the year 1959 by providing the direct observations for confirmation that the Moon was volcanically active.

1. Introduction.

Let us assume the Earth to be the absolutely (or ideally) *black body* [1] with the shape of sphere with radius R, which has the mean-valued average temperature of the surface T.

To estimate thermo-balance of Earth's heat fluxes, we should account the amount of the heat flows as below:

- The Solar radiation energy flow [2-3]:

$$F_{ext} = f\cdot(\pi R^2)\cdot(1-A),$$

- here f – is a measure of flux density of the mean solar electromagnetic radiation (the solar irradiance) per unit area that would be incident on a plane perpendicular to the rays, at a distance of one astronomical unit (AU) from the Sun, f = 1.361 kilowatts per square meter (kW/m²) at solar minimum [2]; R – is the radius of the Earth; A – is the Earth's albedo, $A \cong 0{,}36$ [3].

Let us also assume that the surface of Earth *re-radiates* all the fluxes of energy flow to the outer Space in according to the Stefan-Boltzmann law [4]:

$$F_{out\,(1)} = 4\pi R^2\cdot(\sigma\cdot T^4),$$

- here σ – is the constant of Stefan-Boltzmann law [4]; also, we should take into account the losses for the evaporation of water from the surface of Ocean [3]:

$$F_{out\,(2)} = 4\pi R^2\cdot(h\cdot\rho_w\cdot Q),$$

- here $Q = Q_0 - \eta \cdot T = (25 - 0{,}024 \cdot T) * 10^5$ [J/kg] – is the amount of heat of water evaporation from the unit of Ocean square per unit of time ($25 \cdot 10^5$ J/kg – is the amount of heat of water evaporation at temperature $0°C$); h – is the mean-valued height of water in mm, which is assumed to be evaporating from the surface of Ocean per unit of time; $\rho_w \approx 1024\ kg/m^3$ - is the density of sea water.

Thus, we could estimate the thermo-balance of Earth's heat fluxes near the surface of Earth (without accounting of effects of diffusion as well as the effect of turbulence heat transfer):

$$\frac{dH}{dt} = F_{ext} + F_{\text{int}} - (F_{out\,(1)} + F_{out\,(2)}) \tag{1.1}$$

- here H – is the effective enthalpy of Earth's surface; F_{int}. – is the source of internal heat generation (of unknown nature). So, we should obtain:

$$\frac{d(C \cdot T)}{dt} = \frac{F_{ext} + F_{int}}{4\pi R^2} - \sigma \cdot T^4 - h \cdot \rho_w \cdot (Q_0 - \eta \cdot T)$$

- here t – is the time-parameter; C – is the isobaric heat capacity of the Earth's surface per the unit of square; $F_{ext} = f \cdot \pi R^2 \cdot (1 - A)$. If we designate:

$$\Delta f_{\text{int}} = \frac{F_{\text{int}}}{4\pi R^2},$$

- the last equation above could be transformed as

$$\frac{d(C \cdot T)}{dt} = \{ f \cdot \frac{(1-A)}{4} + \Delta f_{\text{int}} \} - \sigma \cdot T^4 - h \cdot \rho_w \cdot (Q_0 - \eta \cdot T) \tag{1.2}$$

We should especially note that in general case: $\Delta f_{int} = \Delta f_{int}(t)$.

So, we could conclude: - Eq. (1.2) is the generalization of the equations of *Riccati* and *Abel* types [5]. Due to a very special character of *Riccati's type* ordinary differential equation, it's general solution is proved to have a proper gap for the components of a solution at some definite meanings of time-parameter *t* or so-called *gradient catastrophe* [6].
It means the possibility of sudden global changing of temperature on the Earth's surface and the changes of the global climate on Earth and environment (let us remember about so called Little Ice Age at the Maunder minimum of Solar activity [7]).

2. Estimation of the source of Internal heat generation.

According to the data of modern climatology, the average mean-valued temperature near the surface of the World Ocean [8] is about ~ *17 °C ≈ 290,2 K.*

If we assume Δf_{int} = const, Eq. (1.2) for modeling of mean-valued temperature near the surface of the World Ocean could be presented as below (for the solid surface of Earth, heat capacity is low than at the Ocean more than 2 times, density is higher more than 2 times):

$$\frac{dT}{\{0{,}25 f \cdot (1-A) + \Delta f_{\text{int}} - h \cdot \rho_w \cdot Q_0\}/\sigma - T^4} = \frac{\sigma}{C}\,dt \qquad (2.1)$$

- here for $F_{out\,(2)}$ in (1.1) we assume $Q = Q_o - \eta \cdot T = (25 - 0{,}024 \cdot T)*10^5$ [J/kg], where $\eta = 0$;
- $Q_o = 18 \cdot 10^5$ [J/kg] for temperature $T = 290{,}2\ K$ near the surface of the Ocean.
- $h \sim 1325$ *mm/year* $= 4{,}2 \cdot 10^{-8}$ *m/s;*
- $C = C_{Ocean} + C_{Air}$, $C_{Ocean} = c_w \cdot \rho_w \cdot H_w$ ($c_w \approx 4186{,}8$ [J/(kg·K)], $\rho_w \approx 1024\ kg/m^3$, $H_w \approx 150\ m$);

the mean-valued Heat capacity of the Air near the surface of the World Ocean:

- $C_{Air} = c_{Air} \cdot \rho_{Air} \cdot H_{Air}$ ($c_{Air} \approx 1005$ [J/(kg·K)], $\rho_{Air} \approx 1{,}2\ kg/m^3$, $H_{Air} = 50\ m$)

Let us denote:

$$a = \left(\{0{,}25 f \cdot (1 - A) + \Delta f_{\text{int}} - h \cdot \rho_w \cdot Q_0\} / \sigma \right)^{\frac{1}{4}} \qquad (2.2)$$

If we exclude the influence of any source of Internal heat generation Δf_{int}, the maximal temperature of the Ocean surface could be calculated from expression (2.2) above as below:

$$a_0 = \left(\{0{,}25 f \cdot (1 - A) - h \cdot \rho_w \cdot Q_0\} / \sigma \right)^{\frac{1}{4}} = 223{,}16\ K \cong -50\ ^{\circ}C.$$

But in reality such temperature is about ~ $17\ °C \approx 290{,}2\ K$, see [8].

It means that $\Delta f_{int} \neq 0$; moreover, we could estimate it as below:

$$\Delta f_{int} \approx 1{,}2 \cdot \{0{,}25 f \cdot (1 - A)\}$$

- so, the influence of the source of Internal heat generation Δf_{int} is more than even the influence of solar irradiance per unit area of Earth's surface.

Besides, by analyzing Eq. (2.1) we could conclude:

$$\begin{cases} dT/dt \geq 0 & \Leftrightarrow \quad T \leq a \\ dT/dt < 0 & \Leftrightarrow \quad T > a \end{cases}$$

It means that the main feature of the dynamics of temperature from (2.1) should be formulated as follows: - temperature should be increasing up to the proper critical level *a*, then temperature should be decreasing.

Let us obtain the solution of (2.1) under condition Δf_{int} = const:

$$\int_{T}^{T+\Delta T} \frac{dT}{a^4 - T^4} = \frac{\sigma}{C}\Delta t \ ,$$

or

$$\left. \frac{1}{4a^3}\cdot \ln\left(\frac{a+T}{a-T}\right) + \frac{1}{2a^3}\cdot \arctan\left(\frac{T}{a}\right)\right|_{T}^{T+\Delta T} = \frac{\sigma}{C}\Delta t \ ,$$

- which could be reduced at $\Delta T \to 0$ as below:

$$\frac{\Delta T}{a^4}\cdot\left(1 + \frac{T^2}{2\cdot(a^2 - T^2)}\right) \cong \frac{\sigma}{C}\cdot \Delta t \ .$$

So, we could conclude: - the critical level of temperature *a* could be calculated currently (up to the present moment), if we know the time-period Δt (for which the additional temperature changing ΔT is occured), initial level of temperature T and the amount of the additional temperature changing ΔT

$$u = a^2 \quad \Rightarrow \quad u^3 - T^2 u^2 - \left(\frac{\Delta T}{\Delta t}\cdot\frac{C}{\sigma}\right) u + \frac{\Delta T}{\Delta t}\cdot\frac{C}{\sigma}\frac{T^2}{2} = 0 \ .$$

Let us solve the polynomial algebraic equation of 3-rd extent in regard to the function *u*, using the conditions below:

$$T = 290{,}2\ K,\ \ \Delta T = 0{,}5\ K,\ \ \Delta t = 10\ years \approx 315'576'000\ sec.,$$

- then we obtain the meaning of the critical level of temperature *a*:

$$u^3 - T^2u^2 - \left(\frac{\Delta T}{\Delta t}\cdot\frac{C}{\sigma}\right)u + \frac{\Delta T}{\Delta t}\cdot\frac{C}{\sigma}\frac{T^2}{2} = 0\ ,$$

$$\frac{C}{\sigma} = \frac{4186{,}8\cdot 1024\cdot 150 + 1005\cdot 1{,}2\cdot 50}{5{,}6697\cdot 10^{-8}}\ [s\cdot K^4] = 11'343'682'734'536'200\,[s\cdot K^4]\ ,$$

$$T^2 = 84216{,}01\ [K^2],\quad \frac{\Delta T}{\Delta t}\cdot\frac{C}{\sigma} = 17'972'980{,}73\ [K^4]\,,$$

$$\frac{\Delta T}{\Delta t}\cdot\frac{C}{\sigma}\frac{T^2}{2} = 756'806'632'094{,}180\ [K^6]\,,\quad \Rightarrow$$

$$\Rightarrow\quad u^3 - 84216{,}01u^2 - 17'972'980{,}73u + 756'806'632'094{,}180 = 0\ ,$$

$$u_2 = 84322{,}707\quad\Rightarrow\quad \mathrm{a} = 290{,}38\ [K]\ .$$

As we can see, the critical level of temperature *a* is more than the current temperature on ~ *0,2 K* (*the date of calculation above was ~ the summer of year 2010*); so, it means that during next 5 years the temperature of World Ocean should be decreasing (immediately after the reaching of the critical level ~ *290,4 K*).

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