Another infinite sequence based on mar function that abounds in primes and semiprimes

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Abstract. In one of my previous paper, namely “The mar reduced form of a natural number”, I introduced the notion of mar function, which is, essentially, nothing else than the digital root of a number, and I also presented, in another paper, a sequence based on mar function that abounds in primes. In this paper I present another sequence, based on a relation between a number and the value of its mar reduced form (of course not the intrinsic one), sequence that seem also to abound in primes and semiprimes.

Let’s consider the sequence of numbers \( m(n) \), where \( n \) is odd and \( m \) is equal to \( x*y + n \), where \( x = 2*\text{mar} \ n + 2 \) and \( y = 2*\text{mar} n - 2 \), if \( y \neq 0 \), respectively \( m \) is equal to \( x + n \), where \( x = 2*\text{mar} n + 2 \), if \( y = 0 \):

: for \( n = 1 \), we have \( \text{mar} n = 1 \), \( x = 4 \) and \( y = 0 \) so \( m = 5 \);
: for \( n = 3 \), we have \( \text{mar} n = 3 \), \( x = 8 \) and \( y = 4 \) so \( m = 35 \);
: for \( n = 5 \), we have \( \text{mar} n = 5 \), \( x = 12 \) and \( y = 8 \) so \( m = 101 \);
: for \( n = 7 \), we have \( \text{mar} n = 7 \), \( x = 16 \) and \( y = 12 \) so \( m = 199 \);
: for \( n = 9 \), we have \( \text{mar} n = 9 \), \( x = 20 \) and \( y = 16 \) so \( m = 329 \);
: for \( n = 11 \), we have \( \text{mar} n = 2 \), \( x = 6 \) and \( y = 2 \) so \( m = 23 \);
: for \( n = 13 \), we have \( \text{mar} n = 4 \), \( x = 10 \) and \( y = 6 \) so \( m = 73 \);
: for \( n = 15 \), we have \( \text{mar} n = 6 \), \( x = 14 \) and \( y = 10 \) so \( m = 155 \);
: for \( n = 17 \), we have \( \text{mar} n = 8 \), \( x = 18 \) and \( y = 14 \) so \( m = 269 \);
: for \( n = 19 \), we have \( \text{mar} n = 1 \), \( x = 4 \) and \( y = 0 \) so \( m = 23 \);
: for \( n = 21 \), we have \( \text{mar} n = 3 \), \( x = 8 \) and \( y = 4 \) so \( m = 53 \);
: for \( n = 23 \), we have \( \text{mar} n = 5 \), \( x = 12 \) and \( y = 8 \) so \( m = 119 \);
: for \( n = 25 \), we have \( \text{mar} n = 7 \), \( x = 16 \) and \( y = 12 \) so \( m = 217 \);
: for \( n = 27 \), we have \( \text{mar} n = 9 \), \( x = 20 \) and \( y = 16 \) so \( m = 347 \);
: for \( n = 29 \), we have \( \text{mar} n = 2 \), \( x = 6 \) and \( y = 2 \) so \( m = 41 \);
: for \( n = 31 \), we have \( \text{mar} n = 4 \), \( x = 10 \) and \( y = 6 \) so \( m = 91 \);
for $n = 33$, we have $m = 6$, $x = 14$ and $y = 10$ so $m = 173$;
for $n = 35$, we have $m = 8$, $x = 18$ and $y = 14$ so $m = 287$;
for $n = 37$, we have $m = 1$, $x = 4$ and $y = 0$ so $m = 41$;
for $n = 39$, we have $m = 3$, $x = 8$ and $y = 4$ so $m = 71$;
for $n = 41$, we have $m = 5$, $x = 12$ and $y = 8$ so $m = 137$;
for $n = 43$, we have $m = 7$, $x = 16$ and $y = 12$ so $m = 235$;
for $n = 45$, we have $m = 9$, $x = 20$ and $y = 16$ so $m = 365$;
for $n = 47$, we have $m = 2$, $x = 6$ and $y = 2$ so $m = 59$;
for $n = 49$, we have $m = 4$, $x = 10$ and $y = 6$ so $m = 109$;
for $n = 51$, we have $m = 6$, $x = 14$ and $y = 10$ so $m = 191$;
for $n = 53$, we have $m = 8$, $x = 18$ and $y = 14$ so $m = 305$;
for $n = 55$, we have $m = 1$, $x = 4$ and $y = 0$ so $m = 59$;
for $n = 57$, we have $m = 3$, $x = 8$ and $y = 4$ so $m = 89$;
for $n = 59$, we have $m = 5$, $x = 12$ and $y = 8$ so $m = 155$;
for $n = 61$, we have $m = 7$, $x = 16$ and $y = 12$ so $m = 253$;
for $n = 63$, we have $m = 9$, $x = 20$ and $y = 16$ so $m = 383$;
for $n = 65$, we have $m = 2$, $x = 6$ and $y = 2$ so $m = 77$;
for $n = 67$, we have $m = 4$, $x = 10$ and $y = 6$ so $m = 127$;
for $n = 69$, we have $m = 6$, $x = 14$ and $y = 10$ so $m = 209$;
for $n = 71$, we have $m = 8$, $x = 18$ and $y = 14$ so $m = 323$;
for $n = 73$, we have $m = 1$, $x = 4$ and $y = 0$ so $m = 77$;
for $n = 75$, we have $m = 3$, $x = 8$ and $y = 4$ so $m = 107$;
for $n = 77$, we have $m = 5$, $x = 12$ and $y = 8$ so $m = 137$;
for $n = 79$, we have $m = 7$, $x = 16$ and $y = 12$ so $m = 271$;
for $n = 81$, we have $m = 9$, $x = 20$ and $y = 16$ so $m = 401$;
for $n = 83$, we have $m = 2$, $x = 6$ and $y = 2$ so $m = 95$;
for $n = 85$, we have $m = 4$, $x = 10$ and $y = 6$ so $m = 145$;
for $n = 87$, we have $m = 6$, $x = 14$ and $y = 10$ so $m = 227$;
for $n = 89$, we have $m = 8$, $x = 18$ and $y = 14$ so $m = 341$;
for $n = 91$, we have $m = 1$, $x = 4$ and $y = 0$ so $m = 95$;
for $n = 93$, we have $m = 3$, $x = 8$ and $y = 4$ so $m = 125$;
for $n = 95$, we have $m = 5$, $x = 12$ and $y = 8$ so $m = 191$;
for $n = 97$, we have $m = 7$, $x = 16$ and $y = 12$ so $m = 289$;
for $n = 99$, we have $m = 9$, $x = 20$ and $y = 16$ so $m = 419$;
for $n = 101$, we have $m = 2$, $x = 6$ and $y = 2$ so $m = 113$;
for $n = 103$, we have $m = 4$, $x = 10$ and $y = 6$ so $m = 163$;
for $n = 105$, we have $m = 6$, $x = 14$ and $y = 10$ so $m = 325$;
for $n = 107$, we have $m = 8$, $x = 18$ and $y = 14$ so $m = 359$;
for $n = 109$, we have $m = 1$, $x = 4$ and $y = 0$ so $m = 149$;
for $n = 111$, we have $m = 3$, $x = 8$ and $y = 4$ so $m = 143$;
for $n = 113$, we have $m = 5$, $x = 12$ and $y = 8$ so $m = 209$;
for $n = 115$, we have $m = 7$, $x = 16$ and $y = 12$ so $m = 307$;
for $n = 117$, we have $m = 9$, $x = 20$ and $y = 16$ so $m = 437$;
for $n = 119$, we have $m = 2$, $x = 6$ and $y = 2$ so $m = 131$;
for $n = 121$, we have $m = 4$, $x = 10$ and $y = 6$ so $m = 181$;
for $n = 123$, we have $m = 6$, $x = 14$ and $y = 10$ so $m = 263$;
for $n = 125$, we have $m = 8$, $x = 18$ and $y = 14$ so $m = 377$;
for $n = 127$, we have $m = 1$, $x = 4$ and $y = 0$ so $m = 131$;
for $n = 129$, we have $m = 3$, $x = 8$ and $y = 4$ so $m = 161$;
for $n = 131$, we have $m = 5$, $x = 12$ and $y = 8$ so $m = 227$;
for $n = 133$, we have $m = 7$, $x = 16$ and $y = 12$ so $m = 325$;
for $n = 135$, we have $m = 9$, $x = 20$ and $y = 16$ so $m = 455$;
for $n = 137$, we have $m = 2$, $x = 6$ and $y = 2$ so $m = 149$;
for $n = 139$, we have $m = 4$, $x = 10$ and $y = 6$ so $m = 199$;
for $n = 141$, we have $m = 6$, $x = 14$ and $y = 10$ so $m = 281$. 
So the sequence \( m(n) \) is:

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Comment:

It is notable that, from the first 71 terms of this sequence, 44 are primes, 24 are semiprimes and 2 are products of two distinct prime factors \((245 = 5\times7^2 \text{ and } 325 = 13\times5^2)\)!

Conjectures:

(1) The sequence above has an infinity of terms which are distinct primes.

(2) The sequence above has an infinity of terms which are squares of distinct primes \((289 = 17^2, \ldots)\).

(3) The sequence above has an infinity of terms which are cubes of distinct primes \((125 = 5^3, \ldots)\).

(4) The sequence above has an infinity of terms which are products of twin primes \((35 = 5\times7, 323 = 17\times19, 143 = 11\times13)\).

(5) The sequence above has an infinity of terms which are products of a Sophie Germain prime and a safe prime \((253 = 11\times23, \ldots)\).

(6) The sequence above has an infinity of terms which are products of a prime \( p \) and a prime \( q = k\times p - (k - 1) \), such, for instance: \( 91 = 7\times13 \), \( 217 = 7\times31 \), \( 365 = 5\times73 \), \( 305 = 5\times61 \), \( 145 = 5\times29 \).

(7) The sequence above has an infinity of terms which are products of a prime \( p \) and a prime \( q = k\times p - (k + 1) \), such, for instance: \( 329 = 7\times47 \), \( 119 = 7\times17 \), \( 287 = 7\times41 \), \( 209 = 11\times19 \), \( 95 = 5\times19 \), \( 161 = 7\times23 \).