Cosmologic red shift as consequence of theory of general relativity

Abstract

The cosmologic red shift is an effect of general relativity. It is possible to calculate cosmologic red shift of light, passing through a transparent continuum with mass. It is necessary to consider the mass variation caused from variation of potential energy demonstrated in [3] for this calculation. Result of this calculation is also an intensified increasing of cosmologic red shift at very long distances. Accelerated expansion of our universe or dark energy is not needed for this calculation, only pure theory general relativity. It is possible to calculate the average density of our universe, if you know the Hubble-constant. Calculated average density is $1,76 \cdot 10^{-26} \text{ kg/m}^3$ by using a Hubble-constant of 21km/s per million light years.
1. Introduction

Cosmologic red shift is yet a very unsatisfying explained physical phenomenon. This red shift has been explained at the beginning with a real expansion movement of all matter and the resulting Doppler-effect. Scientists need cosmologic expansion today also for explanation of cosmologic red shift. But they say today, there is no real expansion movement, there is new space, growing between the galaxy groups. It is very much more difficult to explain the intensified increasing of red shift at cosmologic distances. This intensified increasing of red shift is explained today with a very speculative dark energy with a pushing force. This very speculative dark energy cannot been detected with other interactions. Today dark energy is a speculation, only a speculation!

I will demonstrate in this script the general solution of the problem of cosmologic red shift and also of the problem of intensified increasing of red shift (accelerated expansion) as a result of the theory of general relativity. Necessary condition is only a static universe with constant density on large scales. This explanation of cosmologic red shift is in Einstein’s manner, without cosmologic expansion and also without speculative dark energy. Only pure theory of general relativity in a static universe with constant density is necessary.

I will demonstrate a general solution of this problem, as easy as possible. Therefore it is not necessary to use rotations. This does not change basics, but it makes it much easier, to describe this solution. I hope many people are interested in this new sight on cosmology and theory of relativity. Therefore I use only school mathematics and school physics (secondary school). It is not necessary to use the 4-dimensional space time (Minkowski space), I need only 3 geometric dimensions and time to explain the cosmologic red shift. But I need calculus.

I have to explain something to numbering of the equations. The number of an equation is in brackets in front of the equation. (3.4) signifies, this is the fourth equation in the third chapter. The equation was written in the text before; it is a repetition of this equation, if the number is behind the equation.
2. What causes the cosmologic red shift

Slipher, Hubble and other astronomers discovered at beginning of 20th century that all spectra of far lightning matter are shifted to red. What does this red shift cause? Scientists believed at the beginning, a real expansion movement of all matter with 21km/s per million light years distance causes this cosmologic red shift by the resulting Doppler-effect. Other scientists assert, there is no real expansion movement but only a kind of “floating in the space time“. I have to contradict this with two main arguments:

- Space time is not a kind of medium, not a kind of water of the universe, not a kind of aether, where floating is possible. Everybody, who speaks of “floating in the space time“, negates the energetic interaction (gravitation) between far parts of our universe. Sometime people assert, there is no real expansion movement but there “grows new space“ between the big cosmic objects in our universe. Also the model of “growing new space“ is a method to negate the energetic interaction of far objects in our universe. My opinion is, these models of “floating in space time“ and of “growing new space“ indicate a certain vagueness of the concepts “space“ and “time“. These concepts are explained detailed in [4]. Sorry, only in German language yet.

- Space and time are models of our environment to make it possible, to describe a juxtaposition (collocation, neighbourhood, different places) and a succession (sequence) of events and objects. Every variation needs succession, and every movement needs juxtaposition and succession (succession of places). Causal relationships and physics are impossible without juxtaposition and succession of events and objects. Space and time are co-ordinates. They define a place in space and a moment in time. But space and time have no more characteristics both. Differences are very clear between space and time. Self-defined movement is possible in space in every direction, if space is empty, and if necessary energy is existent. Self-defined movement is impossible in time. You will be moved constantly and permanently forward through time. Movement in
time itself needs no energy. Only the events need energy, that indicate progress in time. Movement in every direction is impossible by using space time co-ordinates in a 4-dimensional space time. Also standstill and rotations around a point are impossible in 4-dimensional space time. Space and time are explained detailed in [4]. Sorry, only in German language yet. I think, it is confusing, to speak about 4-dimensional space time and wipe out all differences between space and time. It is not wrong but very confusing, to speak of movement in space time only while time goes by. The free self-definable movement in the space is set equal to the continuous permanent forward moving in time. But in the fourth co-ordinate is not time direct, there is the action radius. And action radius rises if only time goes by.

I will demonstrate here, that cosmologic red shift is caused only by theory of general relativity. There is no real expansion movement, and of course, there is no dark energy. There are only two conditions necessary for this demonstration:

- The universe is filled with matter of constant density at very long distances. We calculate with about $1.7 \cdot 10^{-26}$ kg/m$^3$ density (about 10 protons/m$^3$).

- The universe is filled with matter of constant density in the same manner also beyond the visible space very far away.

We imagine, there is a light source in $1.3 \cdot 10^{26}$ m distance (about $14 \cdot 10^9$ light years), and we can observe it. We imagine also a spheric border in $1.3 \cdot 10^{26}$ m distance around the light source. This border passes our observing place. This configuration is shown in picture 1. The sphere around the light source is continuous filled with transparent matter with a density of $1.7 \cdot 10^{-26}$ kg/m$^3$. A sphere with a radius of $1.3 \cdot 10^{26}$ m has a volume of $9.2 \cdot 10^{78}$ m$^3$. This sphere has a mass of $1.6 \cdot 10^{53}$ kg at specified density. A mass of $1.6 \cdot 10^{53}$ kg has a Schwarzschild radius of $2.3 \cdot 10^{26}$ m. Our observing position is more then half of Schwarzschild radius (insight) distant from light source. The spectrum of this light source is clear shifted to red. This red shift is a result of the mass of the sphere around the light source. The red shift results on slowing down of the clock, we can see at the light
source in the center of the transparent mass. This is named gravitational red shift or gravitational time dilatation. We can see an identical red shift on all places of the light beam, if there is some dust or smoke, and we can see the light beam from the light source to observer. This red shift depends only on observing place, and on the quantity of the transparent mass (the density of the mass) around the light source. The matter outside of the border does not care; it has no influence to red shift of the light source in the center of the sphere.

Some attentive readers will make the objection, it is also possible to put the observer in the mass center and the light source to the border. This is impossible because the condition was a continuum around. There is also a mass around the observer if you will put a second observer to the light source in the center. Therefore the second observer in the center will see the clock at the observer slowed down also. Every observer will see a clock in a distance slowed down, more or less, depending on distance and density of the

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**Picture 1:** An observer at the border of the sphere with transparent mass sees the light source in the center of the sphere red shifted by the mass of the sphere.
continuum. This slowing down of every clock in a distance causes the red shift. Therefore every light from a light source is red shifted in a continuum in a distance.

This configuration like picture 1 is possible around observer in every direction from observer. This red shift exists in all directions from the observer. And this red shift is caused only by the mass around the light source, without any real moving between light source and observer. A calculation will be demonstrated in the chapters 3 and 4. This description shows, our universe does not expand, it is essential static. The observed red shift is an effect of gravitational time dilatation of general relativity and not an effect of a real moving.

This red shift works with all light sources, if our universe is filled with matter of constant density at very long cosmologic distances, much more far, then we can see.

The demonstrated possibility to explain red shift is tempting because the calculated progress of red shift is according to measured values of red shift; including the intensified increasing of red shift at cosmologic distances. No accelerated expansion, no dark energy is necessary to explain this red shift. The calculation is shown in next chapter.

This description demonstrates, it is possible to explain cosmologic red shift by gravitational time dilatation of theory of general relativity. Clocks at light sources very far away move more slowly, because of the giant masses around the light sources on the way of the light to the observer.

3. The mathematic model of red shift

An event horizon, a singularity, is impossible as shown in [3]. There are no singularities around any masses. This makes consequences for seeing to infinity. Once the radius of the sphere is equal to the Schwarzschild radius if you enlarge this radius of the continuous filled sphere around a light source. This Schwarzschild radius is bigger, if the density is lower. Our universe has a Schwarzschild radius of about \(9,7 \cdot 10^{25} \text{ m}\) if you use a density of \(1,7 \cdot 10^{-26} \text{ kg/m}^3\).
A sphere with a radius of $9.7 \cdot 10^{25} \text{ m}$ has a volume of $3.82 \cdot 10^{78} \text{ m}^3$. This sphere has a mass of $6.5 \cdot 10^{52} \text{ kg}$ at density of $1.7 \cdot 10^{-26} \text{ kg/m}^3$. A mass of $6.5 \cdot 10^{52} \text{ kg}$ has a Schwarzschild radius of $9.7 \cdot 10^{25} \text{ m}$.

You cannot see more far away, if you use the model of event horizon at Schwarzschild radius. There is a sphere with a mass of $6.5 \cdot 10^{52} \text{ kg}$ and an event horizon in every direction around the observer. There is an event horizon in every direction around an observer in a continuum and you cannot look more far than this event horizon. The impossibility of the event horizon is shown in [3] and therefore you can look (theoretical, without red shift) infinite far.

But there are very big technical problems. You need a telescope with 100 m diameter for looking in a distance of $100 \cdot 10^9 \text{ light years}$ on the same galaxy, you can see in a distance of $10 \cdot 10^9 \text{ light years}$ with a telescope of 10 m diameter. But red shift is increasing giantly above Schwarzschild radius; and so you cannot recognize any optical object in a distance of $100 \cdot 10^9 \text{ light years}$. May be, objects so far away will be recognized by radio astronomy. You can see, a big, expensive technical apparatus is necessary to recognize objects so far away. You can recognize only the brightest objects in this giant distance, and there are technical limits for observation of such far objects. The biggest optical telescopes today have a (mirror-) diameter of about 10 meters. There is a very big advance at radio telescopes at time (ALMA).

It is very difficult to determinate the distance of such far objects. This determination will be done by two ways: brightness of supernovae type 1A and the red shift of a far object. Therefore it is important to know, how red shift depends on distance. Perlmutter and other astronomers compared distance by supernovae 1A with red shift. They found an intensified increasing of red shift at cosmologic distances. They called it accelerated expansion of our universe by dark energy. The discovery of this intensified increasing of red shift was important, but their explanation is not correct. There is no expansion of the universe, there is no dark energy.

You need to investigate the mass increasing of a small test mass for the determination the cosmologic red shift. This mass increasing has to be transformed to frequency shift of a photon. Therefore we
use equation (2.9) from [3]. This equation describes the mass variation by a variation of the distance of two masses. Start point is now a sphere in the center with a mass, determined by the density of the mass and the radius of the sphere. It will be calculated the variation of a small test mass by lifting it up from \( r \) (surface of the inner sphere) to \( r + \Delta r \). You have to increase \( r \) to \( r + \Delta r \) after this calculation and you have to repeat the calculation. This configuration is shown in picture 2.

The mass of the inner sphere will enlarge by increasing of radius \( r \) of inner sphere (and inner spheric shells) at constant density. This process starts in the center and goes to outside this time. Test mass increases by enlarging distance to the center. This mass increasing will be converted to a red shift.

![Diagram showing mass variation](image)

**Picture 2**: test mass will be moved away from light source. Potential energy against mass of inner sphere and inner sphere shells has to be added at every step \( \Delta r \) of motion of test mass.
It is necessary to use equation (2.9) from [3] (J. Altenbrunn, Die Relativitätstheorie ohne Singularitäten / Theory of relativity without singularities) to calculate the cosmologic red shift. This equation describes the dependence of the mass variation of two masses on variation of their distance together and is:

\[ \Delta m = \frac{G \cdot m_1 \cdot m_2}{2 \cdot c^2 \cdot r^2} \cdot \Delta r \]  \hspace{1cm} (2.9) from [3]

\( m_1 \) is the mass of the central sphere and of the inner spheric shells. \( m_1 \) is very big compared to the test mass \( m_2 \). Mass \( m_1 \) does not really vary by increasing potential energy. But \( m_1 \) increases by increasing of the volume of the central sphere and inner spheric shells by moving test mass to outside. Therefore \( m_1 \) is mostly depending on distance \( r \). It is possible to calculate the mass by using constant density \( D \). The equation is:

\[ m_1 = \frac{4}{3} \pi r^3 \cdot D \]  \hspace{1cm} (3.1)  \hspace{1cm} (mass = sphere volume \cdot density)

You have to put equation (3.1) into equation (2.9) from [3] and rearrange, so that \( m_2 \) is on the left side of the equation and you get:

\[ \frac{1}{m_2} \cdot \Delta m = \frac{G}{2 \cdot c^2 \cdot r^2} \cdot \frac{4}{3} \pi r^3 \cdot D \cdot \Delta r \]  \hspace{1cm} (3.2)

Equation (3.2) can be simplified to:

\[ \frac{1}{m_2} \cdot \Delta m = \frac{2 \cdot \pi \cdot G \cdot D \cdot r}{3 \cdot c^2} \cdot \Delta r \]  \hspace{1cm} (3.3)

This equation (3.3) has to be written as integrals:

\[ \int \frac{1}{m_2} \, dm = \int \frac{2 \cdot \pi \cdot G \cdot D \cdot r}{3 \cdot c^2} \, dr \]  \hspace{1cm} (3.4)

The solutions of the integrals (3.4) are:

\[ \ln(m_2) = \frac{\pi \cdot G \cdot D \cdot r^2}{3 \cdot c^2} + k \]  \hspace{1cm} (3.5)

\( k \) is constant of integration. You have to put both sides of the equation (3.5) into the exponent to the base \( e \), the base of the natural logarithm, to get \( m_2 \) separate on the left side of equation (3.5). You will get:
(3.6) \[ m_2 = e^{\frac{\pi \cdot G \cdot D \cdot r^2}{3 \cdot c^2} + k} \]

Constant of integration \( e^k \) has to be substituted by constant \( k_2 \) and it will be extracted from the exponent by this way. Then you will get:

(3.7) \[ m_2 = k_2 \cdot e^{\frac{\pi \cdot G \cdot D \cdot r^2}{3 \cdot c^2}} \]

Constant of integration \( k_2 \) has to be substituted. A responsible value for \( k_2 \) is \( m_{20} \). \( m_{20} \) is the original mass \( m_2 \) at start without any added potential energy. You will get by substitution:

(3.8) \[ m_2 = m_{20} \cdot e^{\frac{\pi \cdot G \cdot D \cdot r^2}{3 \cdot c^2}} \]

This equation (3.8) describes the increasing of a test mass in a configuration shown in picture 2 by moving this test mass away from the center with the light source.

The equation for red shift of light at a mass was in chapter 3 of [3]:

(3.5) \[ z = 2 \cdot \left( \frac{m_{K0}}{m_{KA}} - 1 \right) \] from [3]

Two masses had been approached in [3] and potential energy had been released by this approaching and masses did decrease. Distance increases in this calculation, and therefore potential energy and mass increase also in this calculation. You have to use the ratio \( m_{K0}/m_{KA} \) inverse (reciprocal) because of that. You will get if you substitute \( m_k \) by \( m_2 \):

(3.9) \[ z = 2 \cdot \left( \frac{m_2}{m_{20}} - 1 \right) \]

You have to insert equation (3.8), cancel \( m_{20} \) and you will get:

(3.10) \[ z = 2 \cdot \left( e^{\frac{\pi \cdot G \cdot D \cdot r^2}{3 \cdot c^2}} - 1 \right) \]

This equation (3.10) describes the dependence of cosmologic red shift \( z \) on very far distances \( r \) at the mean density \( D \) of the universe. This equation (3.10) is the fundamental equation of cosmologic red shift in our universe.
This dependence of cosmologic red shift $z$ on distance $r$ is shown in pictures 3 and 4 for a mean density of $1.762 \cdot 10^{-26}$ kg/m$^3$. This density has been chosen because observed Hubble constant is equal to calculated Hubble constant (equation (4.7)) at this value. Schwarzschild radius of sphere $(9.55 \cdot 10^{25}$ m) with mean density is marked in pictures 3 and 4. In pictures 3 and 4 is shown the dependence of red shift $z$ on distance $r$ to the light source. You can see in picture 3 the big intensifying of the increasing of red shift $z$ above Schwarzschild radius of our universe. This red shift $z$ has increased to a value above $10^6$ at a distance of $10^{27}$ m $(106 \cdot 10^9$ light-years). Red shift $z$ is $4.24 \cdot 10^5$ at a distance of $100 \cdot 10^9$ light years $(9.461 \cdot 10^{26}$ m).

Picture 4 shows a part of picture 3. Picture 4 is shows only the range of $z$ from 0.1 to 10. The square increasing of $z$ is shown with the pink line. You can see in this picture 4 also the intensified increasing of $z$, comparable to the publications on accelerated expansion of universe. But the reason for this intensified increasing of red shift is not a dark energy with a pushing effect! This reason for intensified increasing of red shift $z$ is different, it is general relativity only. The sphere around the light source has a mass of about $1.6 \cdot 10^{55}$ kg, if the observer has a distance of $6 \cdot 10^{26}$ m to the light source. This sphere has a Schwarzschild radius of $2.4 \cdot 10^{28}$ m! The observer is located wide inside the Schwarzschild radius of the sphere of mass between the light source and the observer. And this is the reason for intensified increasing of red shift. There is no dark energy (accelerated expansion) necessary for explanation of intensified increasing of red shift $z$! Only general relativity is necessary.

The calculation of red shift shows a square proportionality between red shift $z$ and distance $r$, if the distance is clearly less than Schwarzschild radius $R_S$ of the continuum at matching to radius $r$ of the sphere. This distance is $R_S = 9.55 \cdot 10^{25}$ m in our universe with a density of $1.762 \cdot 10^{-26}$ kg/m$^3$.

You can calculate the mean density of our universe (see chapter 4) by using this proportionality of red shift $z$ and distance $r$. 
Picture 3: dependence of red shift $z$ on distance $r$ in meters

Picture 4: dependence of red shift $z$ on distance $r$ in meters
Red shift $z$ increases very much intensified (seen from us) outside of Schwarzschild radius ($9.55 \cdot 10^{25}$ m). Red shift $z$ reaches values of about $10^6$ at a distance of $100 \cdot 10^9$ light years. Therefore the accelerated expansion is an effect of the theory of general relativity and not of a real expansion. Red shift gives no cause to postulate a dark energy that causes an imaginary accelerated expansion. You can calculate red shift complete — inclusive intensified increasing of red shift — with theory of general relativity only. The effects of theory of general relativity in a continuum are according to observed effects at cosmologic distances without “dark energy“, without “flowing in space time“ and also without “growing of new space“; simple with theory of general relativity pure in an essential static universe with a constant density.

4. Consequences of red shift by general relativity

You cannot say anything about the age of our universe, or time after big bang, if theory of general relativity causes cosmologic red shift. Time of big bang has to be investigated by another way. Red shift is only a sign of the distance between the light source and the observer and of the mean density of the matter in our universe.

Cosmologic red shift cannot substantiate a big bang ! The assumption of a big bang may be useful, but it is impossible to substantiate big bang by backward calculation of an effect of cosmologic red shift. This description demonstrates, our universe is essential static (in cosmologic distances), universe does not expand. Big bang can be substantiated by cosmic microwave background radiation only.

Another consequence of this consideration is : a constant continuous movement over very big (cosmologic) distances needs energy. A mass will be slowed down; the kinetic energy will be converted to potential energy, if the mass moves over very high distances through a cosmic continuum. The standstill mass of moving mass will grow up by this movement over cosmologic distances.
In the range of very small values of the e-function of cosmologic red shift (3.10) results a square curve. It is a straight line in the double logarithmic diagram.

The approximation of equation (3.10) is at very small values:

(4.1) \[ z = \frac{2 \cdot \pi \cdot G \cdot D \cdot r^2}{3 \cdot c^2} \]  
(approximation up to Rs / 10)

This approximation works at a density of \(10^{-26}\) kg/m³ up to more than \(10^{25}\) m. Look at pictures 3 and 4.

The red shift by Doppler-effect of a radiant away moving object is at very small values of \(z\) (or \(v\)):

(4.2) \[ z = \frac{v^2}{2 \cdot c^2} \]  
(approximation for slow velocities)

You have to equate the equations (4.1) and (4.2) if you will calculate the imaginary radiant velocity of a star. You will get:

(4.3) \[ \frac{v^2}{2 \cdot c^2} = \frac{2 \cdot \pi \cdot G \cdot D \cdot r^2}{3 \cdot c^2} \]

This equation (4.3) can be simplified to:

(4.4) \[ v^2 = \frac{4 \cdot \pi \cdot G \cdot D}{3} \cdot r^2 \]

You can use the square root and you get:

(4.5) \[ v = \sqrt{\frac{4 \cdot \pi \cdot G \cdot D}{3}} \cdot r \]

You can see, radiant velocity \(v\) of a star in cosmologic distances is linear proportional to the distance \(r\) of the star to the observer. This equation (4.5) has an error of about 10\% at \(c/10\) (30 000 km/s) or 1\% at \(c/100\) (3 000 km/s) because of the usage of the approximations (4.1) and (4.2). It is possible to calculate Hubble-constant (near us) by rearranging equation (4.5) and you get:

(4.6) \[ \frac{v}{r} = \sqrt{\frac{4 \cdot \pi \cdot G \cdot D}{3}} \]
You can also calculate the mean density of our universe if you know Hubble-constant by rearranging equation (4.4) and you get:

\[ D = \frac{3 \cdot v^2}{4 \cdot \pi \cdot G \cdot r^2} \]  

You will get a mean density of our universe of $1,762 \cdot 10^{-26}$ kg/m$^3$ if you use a Hubble-constant of 21 km/s per million light years ($9,461 \cdot 10^{21}$ m) in equation (4.7). This is a mean density of about 10 protons per cubic meter.

Equation (4.3) would be very complicated without the two approximations (4.1) and (4.2). The resulting equation is rearrangeable to density D but it is not rearrangeable to Hubble constant or to proportionality of distance r to the seeming radiant velocity v. Therefore this approximation is useful for small values.

It is possible to reorganize equation (3.10) to density D. You can calculate directly the mean density D of our universe if you know the true distance r of the light source.
5. Literature

I recommend some literature for presupposing facts and for extended studies:

[1] Albert Einstein, Über die spezielle und die allgemeine Relativitätstheorie, Verlag Vieweg & Sohn
Neuausgabe: Springer Verlag, Print on demand

(beschreibt nur die Spezielle Relativitätstheorie)

[3] Jürgen Altenbrunn, Die Relativitätstheorie ohne Singularitäten Selbstverlag, also available in English
(PDF from internet, www.altenbrunn.de/wissen.htm)

(PDF from internet, www.altenbrunn.de/wissen.htm)

[5] Andreas Müller, Lexikon der Astrophysik, aus dem Wissensportal Astrophysik
(PDF from internet)

German language is used in literature. May be, some documents are also available in English language ([1],[3]). The discovered facts are of course not yet in literature. There are black holes, event horizons, cosmic censors, accelerated expansion, dark energy etc. in most of the documents in literature([1],[2],[5]).

The original document is in German language “Die kosmologische Rotverschiebung als Folge der Allgemeinen Relativitätstheorie“. This is the translation. I made it myself. Please excuse me, my native language is not English. May be, it sounds not really perfect. And may be, there are some mistakes in writing. Please send me a mail with the correction, if you will find a mistake.