Abstract

The problem I shall address in this paper is concerned with the mean lifetimes of leptons. After proposing a formula for the lifetime of the tau particle and following two simple rules, I build a formula for the lifetime of the muon. Based on these two formulas and based on the same rules, I derive the formula for the mean lifetime of the electron. The formula I found through this extrapolation process indicates that the electron is unstable and that its mean lifetime is, approximately, \((\pi/2) \times 10^{90}\) years, which is about \(10^{80}\) times the age of the universe. Thus according to this formulation the lifetime of the electron is extraordinarily long but not infinite as previously thought.

Keywords: mean lifetime, fine-structure constant, tau particle, muon, electron, Planck's constant.

1. Introduction

The electron has always been considered to be a stable particle. Or should I say that it was the case until now. If this analysis is correct, we can put the electron in the “bag” of unstable particles. Therefore, should the electron proved to indeed be unstable, it is likely that the whole universe have a finite lifetime as well unless there is a lighter version of the electron.

Let us consider the decays caused by weak and strong forces. The Hyperphysics web page entitled Decays Caused by Weak and Strong Forces [1] quotes:

“The lifetime of a decay is proportional to the inverse square of the coupling constant between the initial and final products”

\[
\text{lifetime} \propto \frac{1}{\alpha^2} \tag{1}
\]

I shall generalize this law by postulating that the lifetimes of the leptons are proportional to the inverse of the \(n\) power of the electromagnetic coupling constant; where \(n\) is an integer. Mathematically I postulate that

\[
\text{mean lepton lifetime} \propto \frac{1}{\alpha^n} \tag{2}
\]

Based on this postulate I shall build the mean lifetime formulas starting from the heaviest generation or third generation of matter, then the second generation and, lastly, the first generation. Thus, if this assumption is correct, we should get a very long mean lifetime for the electron.
2. Nomenclature

I shall use the following nomenclature for the constants and variables used in this paper:

\( \alpha = \) fine-structure constant (atomic structure constant)
\( c = \) speed of light in vacuum
\( h = \) Planck’s constant
\( m_e = \) electron rest mass
\( \tau_{\tau} = \) tau particle mean lifetime
\( \tau_{\mu} = \) muon mean lifetime
\( \tau_e = \) electron mean lifetime
\( a_{\tau} = \) numerator of the first factor for the tau particle
\( a_{\mu} = \) numerator of the first factor for the muon
\( a_e = \) numerator of the first factor for the electron
\( n_{\tau} = \) main part of the exponent \( N_{\tau} \) of the formula for the tau particle mean lifetime
\( n_{\mu} = \) main part of the exponent \( N_{\mu} \) of the formula for the muon mean lifetime
\( n_e = \) main part of the exponent \( N_e \) of the formula for the electron mean lifetime
\( F = \) total number of generations of matter or families (3)
\( f = \) family quantum number or generation quantum number
\( a_0 = \) Bohr radius (hydrogen atomic radius corresponding to the quantum number \( n = 1 \))
\( t_H = \) hydrogen unit of time

3. The Leptons Mean Lifetime Formulas

Leptons (and all particles) are divided into three generations: the lepton generation 1 comprising the electron and the electron neutrino, the lepton generation 2 comprising the muon and the muon neutrino and the lepton generation 3 comprising the tau particle and the tau neutrino. In this article I shall only deal with the electron, the muon and the tau particle. Neutrinos lifetimes will not be addressed. Starting from the tau particle and taking into consideration the generalized lifetime formula (2), I shall, firstly, build the formula for the mean lifetime of the tau particle, secondly, the formula for the mean lifetime of the muon and, lastly, the formula for the mean lifetime of the electron.

3.1 The Tau Particle Mean Lifetime Formula

The formula for the lifetime of the tau particle is

\[
\tau_{\tau} = \frac{a_{\tau}}{\pi^2} \frac{h}{m_e c^2} \left( \frac{1}{\alpha} \right)^{n_{\tau}+1}
\]

\( n_{\tau} = F \)  \hspace{1cm} (3.1-2)
\( a_{\tau} = 1 \)  \hspace{1cm} (3.1-3)
Where $F$ is the total number of known families or generations of matter, which is 3, and $\alpha_\tau$ is the numerator of the first factor, which is 1. Therefore we can build up the formula as follows

$$n_\tau = 3$$

The exponent for the tau particle lifetime formula is simply $n_\tau$ plus 1

$$n_\tau + 1 = 3 + 1 = 4$$

Thus, the final formula for the tau particle is

$$\tau_\tau = \frac{1}{\pi^2} \frac{h}{m_e c^2} \left( \frac{1}{\alpha} \right)^4$$

It is worthy to observe that we have used the electron rest mass and not the tau particle mass. This has been done to be able to apply the same rules twice. These rules will be explained in subsection 3.3.

### 3.2 The Muon Mean Lifetime Formula

The formula for the lifetime of the muon is

$$\tau_\mu = \frac{a_\mu}{\pi^2} \frac{h}{m_e c^2} \left( \frac{1}{\alpha} \right)^{n_\mu + 1}$$

$$n_\mu = n_\tau (F + 1 - f)$$

Because the constant $a_\mu$ is equal to $n_\tau$ (the main part of the exponent of the tau particle – this is the rule!) we can write

$$\tau_\mu = \frac{n_\tau}{\pi^2} \frac{h}{m_e c^2} \left( \frac{1}{\alpha} \right)^{n_\mu + 1}$$

Once again $F$ is the total number of families or generations of matter, $F = 3$ and $n_\mu$ is main part of the exponent for the muon lifetime formula. Because the muon belongs to the second generation of matter, the family quantum number, $f$, is 2. Thus the value of $n_\mu$ turns out to be

$$n_\mu = 3 (3 + 1 - 2) = 6$$

$$n_\mu + 1 = 6 + 1 = 7$$

Thus the final formula for the muon is

$$\tau_\mu = \frac{3}{\pi^2} \frac{h}{m_e c^2} \left( \frac{1}{\alpha} \right)^7$$

It is worthy to observe that we have used the electron rest mass and not the muon mass.
3.3 The Electron Mean Lifetime Formula

The formula for the lifetime of the electron is

\[
\tau_e = \frac{a_e}{\pi^2} \frac{h}{m_e c^2} \left( \frac{1}{\alpha} \right)^{n_e+1}
\]  

(3.3-1)

\[
n_e = n_\tau n_\mu (F + 1 - f)
\]  

(3.3-2)

Because the constant \( a_e \) is equal to \( n_\tau n_\mu \) (the product of the main part of the exponent for the tau particle and the muon – this is the rule again!) we can write

\[
\tau_e = \frac{n_\tau n_\mu}{\pi^2} \frac{h}{m_e c^2} \left( \frac{1}{\alpha} \right)^{n_e+1}
\]  

(3.3-3)

Thus we can announce the two rules:

<table>
<thead>
<tr>
<th>Factor Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>The factor ( a_f ) for generation ( f ) is the product of the main part of the exponents of the heavier generations. For the heaviest generation ( a_f = 1 ).</td>
</tr>
</tbody>
</table>

Thus, for the muon there is only one heavier generation: the generation that corresponds to the tau particle, as a consequence \( a_\mu = 1 \times n_\tau = n_\tau \). For the electron there are two heavier generations: one that corresponds to the tau particle and another one that corresponds to the muon, consequently \( a_e = 1 \times n_\tau n_\mu = n_\tau n_\mu \).

<table>
<thead>
<tr>
<th>Exponent Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>It is easier to express this rule mathematically than in words. Thus we shall express it as follows</td>
</tr>
</tbody>
</table>

\[
N_\tau = n_\tau + 1 \quad (F + 1 - f) \quad \text{is a new factor for both the muon and the electron only}
\]

\[
N_\mu = n_\tau (F + 1 - f) + 1 \quad \text{where} \quad f = 2
\]

\[
N_e = n_\tau n_\mu (F + 1 - f) + 1 \quad \text{where} \quad f = 1; \quad \text{and} \quad n_\mu = n_\tau (F + 1 - f) \quad \text{with} \quad f = 2
\]

Here, as in the previous two formulas, \( F \) is the total number of generations of matter \( F = 3 \) and \( n_e \) is part of the exponent for the electron lifetime formula. Because the electron belongs to the first generation of matter, the family quantum number, \( f \), is 1. Thus the value of \( n_e \) can be computed as follows

\[
n_e = 3 \times 6 \left( 3 + 1 - 1 \right) = 54
\]

To get the complete exponent for the formula of the electron we simply add one.

\[
n_e + 1 = 55
\]  

(3.3-4)

The numerator, \( a_e \), of the factor that contains the number \( \pi^2 \) turns out to be
Thus, the final formula for the electron lifetime is

\[ \tau_e = \frac{18}{\pi^2} \frac{h}{m_e c^2} \left( \frac{1}{\alpha} \right)^{55} \]  

(3.3-6)

4. Summary

The following table shows the mean lifetime formulas for the three heaviest leptons.

<table>
<thead>
<tr>
<th>Particle</th>
<th>(f)</th>
<th>Mean lifetime formula (algebraic factors and exponents)</th>
<th>Mean lifetime formula (numeric factors and exponents)</th>
<th>Predicted value of the mean lifetime</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\tau) (tauon)</td>
<td>3</td>
<td>[ \tau_{\tau} = \frac{a_{\tau}}{\pi^2} \frac{h}{m_e c^2} \left( \frac{1}{\alpha} \right)^{n_{\tau}+1} ]</td>
<td>[ \tau_{\tau} = \frac{1}{\pi^2} \frac{h}{m_e c^2} \left( \frac{1}{\alpha} \right)^{4} ]</td>
<td>(2.89 \times 10^{-13} \mu S)</td>
</tr>
<tr>
<td>(\mu) (muon)</td>
<td>2</td>
<td>[ \tau_{\mu} = \frac{a_{\mu}}{\pi^2} \frac{h}{m_e c^2} \left( \frac{1}{\alpha} \right)^{n_{\mu}+1} ]</td>
<td>[ \tau_{\mu} = \frac{3}{\pi^2} \frac{h}{m_e c^2} \left( \frac{1}{\alpha} \right)^{3} ]</td>
<td>(2.23 \mu S)</td>
</tr>
<tr>
<td>(e) (electron)</td>
<td>1</td>
<td>[ \tau_{e} = \frac{a_{e}}{\pi^2} \frac{h}{m_e c^2} \left( \frac{1}{\alpha} \right)^{n_{e}+1} ]</td>
<td>[ \tau_{e} = \frac{18}{\pi^2} \frac{h}{m_e c^2} \left( \frac{1}{\alpha} \right)^{55} ]</td>
<td>(4.96 \times 10^{97} S = \frac{\pi}{2} \times 10^{90} \text{ years})</td>
</tr>
</tbody>
</table>
Table 1: Mean lifetime formulas for the tau particle, the muon and the electron. It is worthy to emphasize that the formulas were built from the tau particle (generation 3); this is from the “bottom up”. Note that the quantum number \( f \) is different for each generation while \( F \) remains constant and that the mass of the electron is common to all formulas.

5. Original Formulas

The formulas shown on Table 1 are based on the *hydrogen unit of time*, \( t_H \), which is defined as

\[
\text{Hydrogen unit of time}
\]

*Time taken by a photon to travel a distance equal to the diameter, \( 2a_0 \), of the hydrogen atom. Where \( a_0 \) is the Bohr radius: \( a_0 = 0.529 \times 10^{-10} \text{ m} \)

Mathematically

\[
t_H 
= \frac{2a_0}{c}
= \frac{1}{\pi \alpha} \frac{h}{m_e c^2}
\] (5.1)

\[ t_H \approx 3.530 \times 10^{-19} \text{ S} \]

The original formulas are shown below

The formulas shown on Table 1 were derived from the corresponding formulas shown in the above picture by substituting \( t_H \) with the second side of equation (5.1). The rules
are shown in green, red and blue. The additional number 1 in the exponent of the final formulas shown on Table 1 is due to the fact that the hydrogen unit of time contains an extra $1/\alpha$ factor. The exponent rule for the original formulas is

$$\text{Exponent for the tau particle } = n_\tau$$
$$\text{Exponent for the muon } = n_\tau(F + 1 - f) = n_\mu \quad (\text{where } f = 2)$$
$$\text{Exponent for the electron } = n_\tau n_\mu(F + 1 - f) \quad (\text{where } f = 1)$$

It is worthy to note that both rules are equivalent.

6. Experimental Values

The following table shows the experimental values for the tau particle and the muon.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Experimental value of the mean lifetime</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau$ (tauon)</td>
<td>$(2.97 \pm 0.09,\text{(stat.)} \pm 0.05,\text{(syst.)}) \times 10^{-13},\mu S$</td>
<td>[2]</td>
</tr>
<tr>
<td>$\mu$ (muon)</td>
<td>$(2.22 \pm 0.03),\mu S$ \text{ (2.19703 \pm 0.00004) } \mu S$</td>
<td>[3] [4] [5]</td>
</tr>
<tr>
<td>$e$ (electron)</td>
<td>Is it really infinite?</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2: Measured values for the lifetimes of the tau particle and the muon.

7. Conclusions

From these three formulas we can draw the following conclusions

1) The predicted lifetimes for both the tau particle and the muon are in excellent agreement with the observed values.

2) The predicted lifetime for the electron: $\frac{\pi}{2} \times 10^{90}$ years, is extraordinarily long but not infinite.
3) The Universe (as a Meta-transformation) could have a finite lifetime.

4) The decay products of the electron could comprise a negatively charged particle that I shall call: electrino. This hypothetical particle must be lighter than the electron. If the electrino exists, then there is either another generation of matter or this particle is sterile.

In summary, the tau particle turned out to be the lepton with the simplest lifetime formula and the electron the one with the most complex one. The formula for the electron is based on the other two formulas and this seems to indicate that there are only three generations of matter (unless conclusion 4 to be true). It also suggests that a) the structure of the electron is, somehow, connected to the structures of the tauon and the muon and that b) the electron incorporates the parameters that make up the lifetime formulas of the other two heavy leptons. If this were the case, then we are before a new and mysterious quantum mechanical phenomenon for which we have no explanation.

REFERENCES