An Accumulative Model for Quantum Theories

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Abstract. For a general quantum theory that is describable by a path integral formalism, we construct a mathematical model of an accumulation-to-threshold process whose outcomes give predictions that are nearly identical to the given quantum theory. The model is neither local nor causal in spacetime, but is both local and causal is in a non-observable path space. The probabilistic nature of the squared wavefunction is a natural consequence of the model. We verify the model with simulations, and we discuss possible discrepancies from conventional quantum theory that might be detectable via experiment. Finally, we discuss the physical implications of the model.

Keywords: quantum theory, quantum mechanics, Born rule, signal processing, threshold process, path integral

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1. Introduction

In a previous paper [1], a model of quantum particle transmission was proposed that explains some puzzling aspects of the quantum theory. According to the model, particle detection is the outcome of a signal accumulation process which occurs in an extra, non-spacetime dimension (which we refer to as the a-dimension). The complex wavefunction corresponds to in-phase and quadrature-phase components of an amplitude and phase-modulated carrier signal field that is present throughout spacetime augmented by the a-dimension. We postulated that the location of particle detection is determined when an accumulated signal reaches a threshold, and proved that the Born probability rule is a mathematical consequence. The apparent collapse of the wavefunction corresponds to the fact that the observed detection corresponds to a single "a-slice", while the wavefunction represents the accumulation process that occurs throughout the a-dimension. Entanglement has a similar explanation: conservation laws (such as conservation of spin) apply in each individual a-slice and produce correlations within the a-slice corresponding to detection; while on the other hand, the different a-slices that cumulatively determine the probability of the final outcome include contributions from all different possible final outcomes. However, the paper gives no explanation of the origin or formation of the carrier signal field.

The current paper provides a more comprehensive interpretation of quantum probabilities by taking a related, but somewhat different approach. The approach is based on the observation that both quantum mechanics and quantum field theory may be derived from a path integral formalism. We conjecture that path integrals correspond to a universal physical process which essentially performs a numerical integration. As in the previous paper, this process unfold in a non-spacetime dimension, and the observable universe is the outcome of the process upon attaining a threshold.

The paper is organized as follows. Section 2 presents a simplified preliminary mathematical model which illustrates the basic model structure. We demonstrates the model's ability to generate quantum probabilities both theoretically and with simulations. Section 3 gives a more detailed model which is designed to conform more closely with the hypothesized physical processes involved. Section 4 discusses the possibility of experimental verification of the model; and Section 5 gives a summary discussion. The simulation code is given in Section 6.

2. Preliminary model

Let $\mathcal{U}$ represent the space of all possible configurations of the observable universe. We emphasize that any $u \in \mathcal{U}$ expresses the entire configuration of the universe over all times, not just its configuration at a single time. We do not need to specify whether we are employing a quantum-mechanical or field-theoretic representation of the universe's configuration space – our argument does not depend on the specific nature of $\mathcal{U}$.

In both quantum-mechanical or the field-theoretic representations of $\mathcal{U}$, the

wavefunction can be expressed in terms of a path integral $\Psi : \mathcal{U} \to \mathbb{C}$ of the form::

$$\Psi(u) \equiv \frac{1}{|\Gamma_u|} \sum_{\gamma \in \Gamma_u} e^{iS(\gamma)}, \tag{1}$$

where Γ_u is a space of paths corresponding to the configuration u, and $S(\gamma)$ is the action associated with the path γ. Here we have used summation notation to facilitate the connection with simulations that we will describe later.. We shall suppose that $|\Gamma_u|$ is independent of u, so that $|\Gamma_u| = |\Gamma|/|\mathcal{U}|$ where $\Gamma \equiv \cup_u \Gamma_u$. We also suppose that the $\{\Gamma_u\}_{u \in \mathcal{U}}$ are disjoint, which implies that for every $\gamma \in \Gamma$ there exists a unique $u_\gamma \in \mathcal{U}$ such that $\gamma \in \Gamma_{u_\gamma}$.

The path integral is associated with a probability distribution:

$$p_S(u) \equiv \frac{|\Psi(u)|^2}{\sum_{v \in \mathcal{U}} |\Psi(v)|^2}. \tag{2}$$

The fact that this probability is written in terms of a summation (or integral) suggests that some sort of accumulation process could be involved. The main purpose of this paper is to show that such an interpretation is indeed feasible, and provides a simple, plausible explanation of the hidden dynamics that give rise to quantum theories. Preliminarily, we note that our interpretation must address two issues:

- Why is probability obtained from a squared complex amplitude?
- What physically corresponds to the division in (2)?

In the following, we give what we believe to be satisfactory answers to these two questions.

We define an accumulation process as follows. Given the sequence of paths $\gamma_1, \gamma_2, \ldots$ in Γ, we define an *accumulated amplitude* $\mathcal{A}_K$ ($K \in \mathbb{Z}_+$) as:

$$\mathcal{A}_K \equiv \Sigma_{k=1}^{K} e^{iS(\gamma_k)}. \tag{3}$$

One possible interpretation of each factor $e^{iS(\gamma_k)}$ is as the phasor representation [2] of an oscillation (of unknown frequency) which depends on γ_k. The summation then corresponds to the complex amplitude of a harmonic oscillator (with the same frequency) that is successively perturbed by these oscillations.

Although we are using discrete notation, the sequence $\{\gamma_k\}$ should be thought of as a discrete approximation of a path-valued function of a continuous index, corresponding to a continuously-varying path within the space Γ of all possible paths. This variation is presumed to be governed by some (unknown) process which uniformly samples Γ over the long term. Note that as γ_k varies, the corresponding state of the universe $u_k \equiv u_{\gamma_k}$ also varies. In the process we will define, the accumulated amplitude grows to reach a fixed threshold at a particular index K, at which point u_K gives the configuration of the observable universe.

In order to obtain the probabilities (2) via this process, we impose additional conditions on the sequences $\{\gamma_k\}$ and $\{u_k\}$ as follows.

(a) There exists $N \gg 1$ and $M \gg 1$ such that $u_{kNM+1} = u_{kNM+2} = \ldots = u_{(k+1)NM}, \forall k \in \mathbb{Z}_{\geq 0}$;

(b) For each $k \in \mathbb{Z}_{\geq 0}$, the sequence $\{\gamma_{kN+1}, \gamma_{kN+2}, \ldots, \gamma_{kN}\}$ uniformly samples $\Gamma_{u_{kN}}$;

(c) The sequence $\{u_{NM}, u_{2NM}, \ldots\}$ is mixing [3] and uniformly samples $\mathcal{U}$.

These conditions correspond to a situation where $\{\gamma_k\}$ varies throughout Γ such that the sequence $\{\gamma_k\}$ uniformly samples each Γ_u that it visits before passing on to the next Γ_u. In this simple model, the dwell time within each Γ_u is the constant N: in our subsequent model, this assumption will be relaxed. The significance of M will be explained later.

Let η_k $(k = 1, 2, \ldots)$ be a sequence of independent, identically distributed (i.i.d.) complex-valued random variables with zero mean and finite variance, and define:

$$\mathcal{A}'_K = \sum_{k=1}^{K} \eta_{\lceil k/N \rceil} e^{iS(\gamma_k)}. \tag{4}$$

Finally, given $\Theta > 0$, we define the *threshold index* as the random variable:

$$K_\Theta \equiv \min(k | |\mathcal{A}'_k| < \Theta \text{ and } |\mathcal{A}'_k| \geq \Theta). \tag{5}$$

Given the above conditions and definitions, we have the following result:

Proposition: As $N, M, \theta \to \infty$, we have

$$P(u_{K_{\theta N\sqrt{M}}} = u) \to P_S(u). \tag{6}$$

In other words, the probability distribution on $\mathcal{U}$ at the stopping time defined by attaining the threshold $\theta N\sqrt{M}$ agrees with the probability distribution (2) obtained from the path-integral formalism.

The proof of this proposition is similar to that given in [1]. Notice that (4) can be rewritten as

$$\begin{aligned}
\frac{\mathcal{A}'_{KN}}{\theta N\sqrt{M}} &= \frac{1}{\theta N\sqrt{M}} \sum_{k=1}^{K} \eta_k \left(\sum_{n=1}^{N} e^{iS(\gamma_{(k-1)N+n})} \right) \\
&\xrightarrow[N\to\infty]{} \frac{|\mathcal{U}|}{\theta|\Gamma|\sqrt{M}} \sum_{k=1}^{K} \eta_k \Psi(u_{\lceil k/M \rceil}) \qquad (7) \\
&= \frac{|\mathcal{U}|}{\theta|\Gamma|} \left(\sum_{k=1}^{\lfloor K/M \rfloor} \Psi(u_k) \Big(\frac{1}{\sqrt{M}} \sum_{m=1}^{M} \eta_{(k-1)M+m} \Big) + \Psi(u_{\lceil k/M \rceil}) \Big(\frac{1}{\sqrt{M}} \sum_{m'=M\lfloor K/M \rfloor + 1}^{K} \eta_{m'} \Big) \right). \qquad (8)
\end{aligned}$$

The proof is based on the fact that $\frac{\mathcal{A}'_{KN}}{\theta N\sqrt{M}}$ can be approximated in distribution as a Brownian motion $B(a)$ in $\mathbb{C}$ with absorbing boundary at $|z| = 1$, where $a \equiv \frac{K}{\theta^2 NM}$. For any fixed a, near the boundary the probability density of an absorbing Brownian

motion is proportional (to first order) to the distance from the boundary. This can be used to show that for any K, the probability $P(K_{\theta N\sqrt{M}} = K|u_K = u)$ is approximately proportional to $E[|\eta_K \Psi(u)|^2]$, which is proportional to $|\Psi(u)|^2$. Since for $P(u_K = u)$ is independent of u when $1 \ll K < K_{\theta N\sqrt{M}}$, it follows that $P(K_{\theta N\sqrt{M}} = K \;\&\; u_{K_{\theta N\sqrt{M}}} = u)$ is proportional to $|\Psi(u)|^2$, and summing over K gives the desired result.

Figure 1 confirms the conclusions of the above proposition. These figure show the results of simulations of the model specified by conditions (a)–(c) and equations (4)–(5). The curves show the difference between the simulated probabilities and actual probabilities for two different probability distributions $|\Psi|^2$, for different values of the threshold θ. The errors are shown on the y-axis, versus the actual probability values which are shown on the x-axis. As θ increases, the errors decrease: for $\theta = 40$, the maximum error is under 5 percent. The pattern of error apparently depends on the type of probability distribution being modeled. However, in both cases the larger probabilities are underestimated, and there is a range of intermediate probabilities that are overestimated. These phenomena may possibly enable an experimental test of the model–see Section 4 for further discussion of this issue.

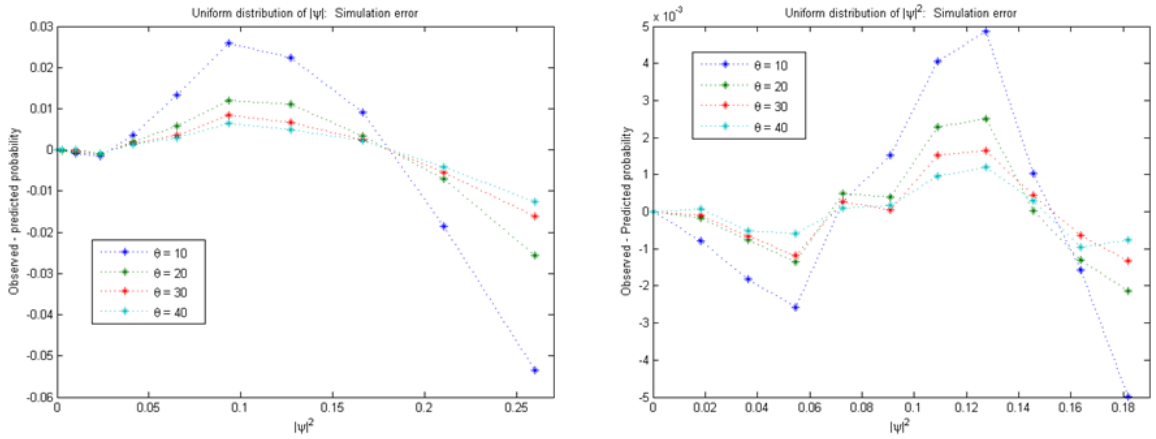

Figure 1. Deviations of computed probabilities from quantum values, for simulated preliminary accumulation model with $\theta = 10, 20, 30, 40$ and $M = 10000$, where $\{\eta_k\}$ are i.i.d. standard normal random variables. Each simulation was run 100,000 times. All simulations used 11 configurations u. For the figure on the left, the $|\psi(u_j)| \propto j$, $(j = 0, \ldots, 10)$, while for the figure on the right, $|\psi(u_j)|^2 \propto j$ $(j = 0, \ldots, 10)$.

3. Adjusted model

The model we have presented above still leaves unanswered questions:

- Why should $\{u_k\}$ remain constant for intervals of size MN?
- What is the significance of the η_k's?

As to the first point, instead of supposing that $\{u_k\}$ remains constant on intervals of size N, we may suppose that $\{u_k\}$ varies slowly with k, so that

$$p(u_{k+1} \neq u_k) = \mathcal{O}\left(\frac{1}{N}\right). \tag{9}$$

Supposing that $\{\gamma_k\}_{k=1.2.3....}$ is generated by a Markov chain, it is reasonable to suppose that residence times in each u state are i.i.d. geometrical random variables. This is because under reasonable conditions, hitting times in Markov chains are asymptotically exponentially distributed [4]. (The geometrical distribution is the discrete analog of the exponential distribution.) Accordingly, we may modify the model by replacing the constant M with a geometrically-distributed random variable with the same mean.

As to the second point, we must recognize that we have failed to account for the fact that in practice we never measure the state of the entire universe, but only a subsystem. So we must take into account the effect of variations in the external system during the accumulation process. Accordingly we let Ω be the possible states of the measured subsystem, while Ω' denote the possible states of the universe external to the measured subsystem. Thus we may represent any element $u \in \mathcal{U}$ uniquely as $u = (w, w')$, where $w \in \Omega$ and $w' \in \Omega'$.

We suppose that any path in Γ can be factored into a part for Ω and a part for Ω': more precisely, that there are path spaces $\mathcal{C}$ and $\mathcal{C}'$ respectively such that any $\gamma \in \Gamma$ can be decomposed as $\gamma = (c, c')$ where $c \in \mathcal{C}, c' \in \mathcal{C}'$, and such that $u_\gamma = (w_c, w'_{c'})$. We suppose further that $|\mathcal{C}_w|$ is independent of $w \in \Omega$, so that $|\mathcal{C}_w| = |\mathcal{C}|/|\Omega|$ $\forall w$ and similarly $|\mathcal{C}'_{w'}| = |\mathcal{C}'|/|\Omega'|$ $\forall w'$. Finally, we suppose that the action S is additive: $S(\gamma_k) = S(c_k) + S(c'_k)$. From this it follows that we may write:

$$\Psi(u) = \Psi((w, w')) = \psi(w)\phi(w'), \tag{10}$$

where

$$\psi(w) \equiv \frac{|\Omega|}{|C|} \sum_{c \in C_w} e^{iS(c)}; \quad \phi(w') \equiv \frac{\Omega'}{|C'|} \sum_{c' \in C_{w'}} e^{iS(c')}. \tag{11}$$

We may also rewrite (3) as

$$\mathcal{A}_K \equiv \Sigma_{k=1}^{K} e^{iS(c_k)} e^{iS(c'_k)}. \tag{12}$$

We may now formulate a revised set of conditions as follows.

(A) There exists a $N \gg 1$ and a sequence $\{\xi_1, \xi_2, \ldots\}$ of i.i.d. geometrically-distributed random variables with $E[\xi_k] = N$, such that $w'_{\mathcal{X}_K+1} = w'_{\mathcal{X}_K+2} = \ldots = w'_{\mathcal{X}_K+\xi_k}$ $\forall K \in \mathbb{Z}_{\geq 0}$, where $\mathcal{X}_0 \equiv 0$ and $\mathcal{X}_K \equiv \sum_{k=1}^{K} \xi_k, K \geq 1$;

(B) There exists a $M \gg 1$ an a sequence $\{\zeta_1, \zeta_2, \ldots\}$ of i.i.d. geometrically-distributed random variables with $E[\zeta_k] = M$, such that $w_{\mathcal{Z}_K+1} = w_{\mathcal{Z}_K+2} = \ldots = w_{\mathcal{X}_{\mathcal{Z}_{K+1}}}$ $\forall K \in \mathbb{Z}_{\geq 0}$, where $\mathcal{Z}_0 \equiv 0$ and $\mathcal{Z}_K \equiv \sum_{k=1}^{K} \zeta_k, K \geq 1$;

(C) For each $K \in \mathbb{Z}_{\geq 0}$, the sequences $\{c'_{\mathcal{X}_K+1}, c'_{\mathcal{X}_K+2}, \ldots, c'_{\mathcal{X}_K+\xi_k}\}$ and $\{c_{\mathcal{X}_K+1}, c_{\mathcal{X}_K+2}, \ldots, c_{\mathcal{X}_K+\xi_k}\}$ uniformly sample $C'_{\mathcal{X}_{K+1}}$ and C, respectively;

(D) The sequences $\{c'_{\mathcal{X}_K+1}, c'_{\mathcal{X}_K+2}, \ldots, c'_{\mathcal{X}_K+\xi_k}\}$ and $\{c_{\mathcal{X}_K+1}, c_{\mathcal{X}_K+2}, \ldots, c_{\mathcal{X}_K+\xi_k}\}$ are statistically independent;

(E) The sequences $\{w'_{\mathcal{X}_1}, w'_{\mathcal{X}_2}, \ldots\}$ and $\{w_{\mathcal{X}_{\mathcal{Z}_1}}, w_{\mathcal{X}_{\mathcal{Z}_2}}, \ldots\}$ are mixing, and uniformly sample Ω and Ω' respectively.

Assumptions (A) and (B) reflect the assertion that the variation of the path within path space is described by a Markov process in which the observable state of the universe varies slowly. The state of the measured subsystem varies more slowly than the external state, because the external state is much larger and there are many more possibilities for variation. The other assumptions have similar justifications as were given for assumptions (b)–(c) in the simplified model.

Following these assumptions, we may compute:

$$\begin{aligned}
\frac{\mathcal{A}'_{\mathcal{Z}_K}}{\theta N\sqrt{M}} &= \frac{1}{\theta N\sqrt{M}} \sum_{k=0}^{K-1} \sum_{m=\mathcal{Z}_k}^{\mathcal{Z}_{k+1}-1} \sum_{n=\mathcal{X}_m+1}^{\mathcal{X}_{m+1}} e^{iS(c_n)+S(c'_n)} \\
&\approx \frac{|\Omega||\Omega'|}{\theta\sqrt{M}|C||C'|} \sum_{k=0}^{K-1} \sum_{m=\mathcal{Z}_k}^{\mathcal{Z}_{k+1}-1} \frac{\xi_{m+1}}{N} \psi(w_{\mathcal{X}_{m+1}}) \phi(w'_{\mathcal{X}_{m+1}}) \qquad (13) \\
&= \frac{|\Omega||\Omega'|}{\theta|C||C'|} \sum_{k=0}^{K-1} \left(\psi(w_{\mathcal{X}_{\mathcal{Z}_{k+1}}}) \cdot \frac{1}{\sqrt{M}} \sum_{m=\mathcal{Z}_k}^{\mathcal{Z}_{k+1}-1} \frac{\xi_{m+1}}{N} \phi(w'_{\mathcal{X}_{m+1}}) \right) \\
&= \frac{|\Omega||\Omega'|}{\theta|C||C'|} \sum_{k=1}^{K} \left(\psi(w_{\mathcal{X}_{\mathcal{Z}_k}}) \cdot \frac{1}{\sqrt{\zeta_k}} \sum_{m=1}^{\zeta_k} \eta_{m,k} \right), \qquad (14)
\end{aligned}$$

where the approximation holds for large N and

$$\eta_{m,k} \equiv \sqrt{\frac{\zeta_k}{M}} \left(\frac{\xi_{\mathcal{Z}_{k-1}+m}}{N} \right) \phi\left(w'_{\mathcal{X}_{\mathcal{Z}_{k-1}+m}} \right). \qquad (15)$$

Notice the similarity between (8) and (14). Instead of a summation over M, there is a summation over ζ_k, which has expectation M. Within this summation, instead of the mean-zero i.i.d. random variables $\{\eta_k\}$, we now have $\{\eta_{k,m}\}$ given by the complicated expression (15). By assumption, both $\frac{\zeta_k}{M}$ and $\frac{\xi_{\mathcal{Z}_{k-1}+m}}{N}$ are independent, and have expectation 1; while the additional complex factor $\phi\left(w'_{\mathcal{X}_{\mathcal{Z}_{k-1}+m}} \right)$ will vary randomly with mean zero as the process evolves. If we assume that $\{\eta_{k,m}\}$ are (approximately) i.i.d. mean-zero random variables, then (14) and (8) are virtually identical, except that ζ_k in (14) replaces M in (8). However, $E[\zeta_k] = M$; and conditioning on the different possible values of ζ, we may obtain the same result that the probability density for w_{K_Θ} is given by $|\psi(w)|^2$.

Figures 2 and 3 shows the deviations from quantum theory of computations using the adjusted model specified in (A)-(E), as a function of the parameters M and θ. As expected, increasing values of M and θ lead to convergence to quantum theoretical values. Figure 2 shows that as in the preliminary model (see Figure 1), deviations

associated with smaller values of θ are positive for midrange values of $|\psi|^2$, and become negative for larger values of $|\psi|^2$. Figure 3 shows that finite values of M also introduce an error that is positive for small values of $|\psi|^2$, and negative for larger values.

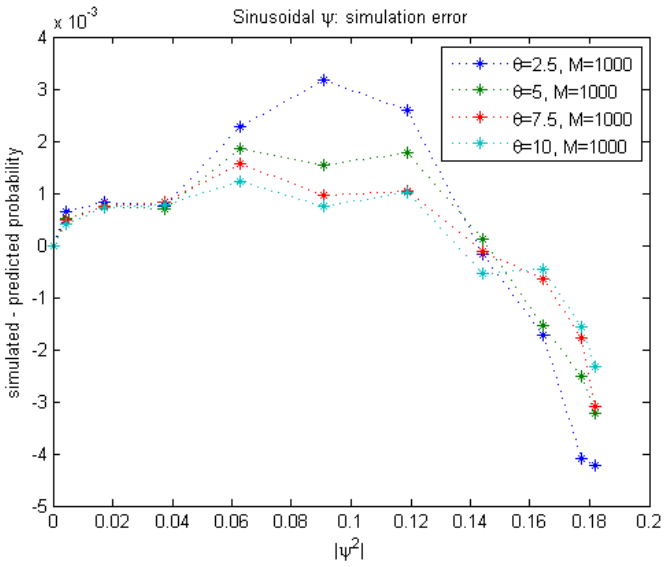

Figure 2. Deviations in computed probability distribution for the simulated adjusted accumulation model, where $\{\eta_{n,k}\}$ are i.i.d. standard normal random variables. Each simulation was run 200,000 times. All simulations used 11 configurations u. The distribution of $|\psi|^2$ values corresponds to a sinusoidal wavefunction, $|\psi(u_j)|^2 \propto \sin^2\left(\frac{j\pi}{20}\right)$. The figure shows the effect of varying the threshold parameter θ.

4. Experimental Verification

In the above model, quantum probabilities are generated by an accumulative process which essentially performs a stochastic approximation to the quantum path integrals. If this model is physically accurate, we may expect that measured probabilities should deviate slightly from theoretical quantum values due to stochastic fluctuations. In the previous sections, we showed that finite values of θ and M introduced deviations from quantum-theoretical probabilities. In particular, both finite M and finite θ introduce deviations that are positive for small probabilities, but negative for large probabilities. Another possible source of numerical error, which we did not model in the simulation, results from the approximation

$$\frac{1}{\xi_m} \sum_{n=\mathcal{X}_m+1}^{\mathcal{X}_m+\xi_m} e^{iS(c_n)} \approx \psi(w_{\mathcal{X}_{m+1}}), \tag{16}$$

which was used in (13). If we suppose there is a random error of constant variance ϵ^2 in this approximation, then by carrying through the computations it can be shown that probabilities turn out to be proportional to $|\psi(w)|^2 + \epsilon^2$ rather than $|\psi(w)|^2$.

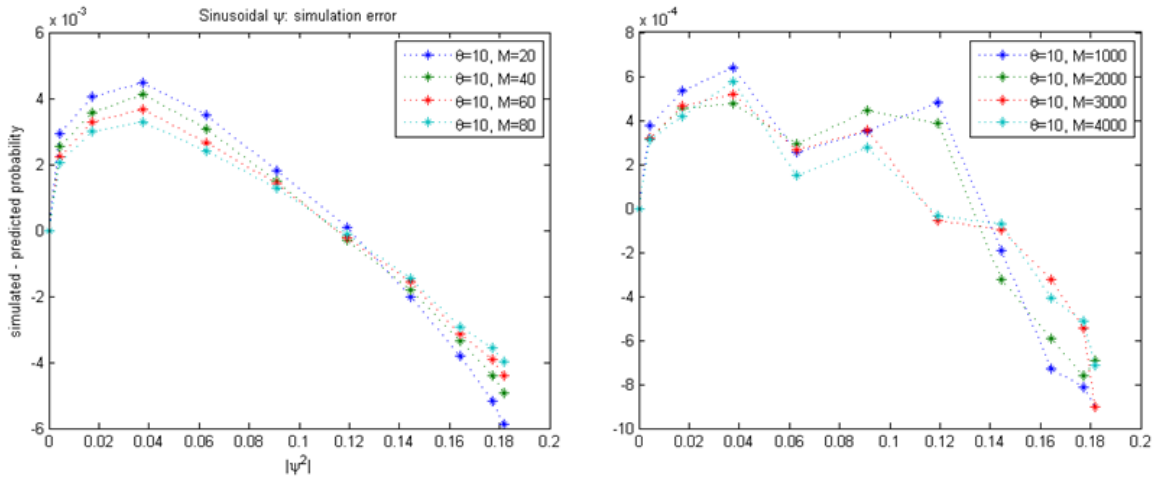

Figure 3. Deviations of computed probabilities from quantum values, for simulated adjusted accumulation model, for different values of the accumulation length M. The figure on the left uses small values of M, while the figure on the right uses larger values. Other simulation parameters are as in Figure 2, except each simulation was run 1 million times.

This produces a deviation from theoretical probabilities that decreases linearly with increasing probability density. Unfortunately, since the parameters of the process are not directly accessible, it is not possible to predict the size of the effects described above. We may conclude that numerical approximation effects should introduce a deviation from quantum-theoretic probabilities that decreases roughly linearly with increasing probability density.

5. Discussion

This construction provides a conceptually simple solution to the conundrums of quantum theory. It accounts for all quantum paradoxes, since it yields the same predictions as quantum theory (to a close approximation). It also has many advantages compared to other interpretations of quantum mechanics. It avoids the agnosticism of the Copenhagen interpretation; it circumvents the complicated branching and enormous configuration space required by the many-worlds interpretation; and it requires no internal guidance system for particles as does Bohmian quantum mechanics. Unlike these other interpretations, the probabilistic nature of the wavefunctions is intrinsic to the model.

According to the model, the universe is not at all what it appears. It indicates that causality is an illusion, and that apparent "cause and effect" relationships are correlations in the outcome of an inaccessible process that occurs outside of spacetime. The Big Bang is not the "origin" of the universe, because the Big Bang is also part of the outcome of a non-chronological process which produces past, present, and future

together as an entirety. The vacuum is not a "boiling sea of virtual particles and antiparticles,"[5] as quantum field theories seem to imply, but only appears to be because of the accumulation process through which the observable universe is actualized.

If this picture of the nature of the universe proves to accurate, it has profound implications for how we may regard the world around us, and how we regard ourselves.

6. Simulation Code

The following code was used for the simulation in Figure 2. The code can be easily modified to generate the other figures.

```
% Parameters
nsim = 200000;                                  % # simulations
nconfig=4;                                      % # configs simulated
Theta_fac = 2.5;                                % Theta increment
Theta_fac0=Theta_fac;                           % Orig. theta
Ncfg = 11;                                      % Number of internal configs
n_acc_mean = 1000;                              % M interval
acc_mean0 = n_acc_mean;                         % Orig. M
Theta = Theta_fac*sqrt(n_acc_mean);             % Rescaled threshold

% Arrays to store results
Counts = zeros(Ncfg,1);
Q = [];

% Create measurable configurations
Psi = cos((0:.1:1)*pi/2)';                      % creation of "wavefunction"
[Tmp,TmpI] = sort(abs(Psi));                    % sort for convenience
Psi = Psi(TmpI);
Prob = abs(Psi).^2;
Prob = Prob / sum(Prob);                        %Normalized probabilities

% Computations
for jj = 1:1:nconfig                            % Loop over configurations
for ii = 1:1:nsim                               % Perform simulations
    A = 0;                                      % Accumulate:
    while abs(A) < Theta                        % Until threshold is attained

        this_cfg = randi([1,Ncfg]);             % Choose current w
        this_acc = round(-1*n_acc_mean*log(rand()));% Dwell time (exponential)
        Rtmp = randn(this_acc,1)+1i*randn(this_acc,1);% generate eta's
        Ptmp = Psi(this_cfg);                   % Amplitude
        for jj = 1:1:this_acc                   % Accumulate for this w
```

```
            A = A + Rtmp(jj)*Ptmp;
            if abs(A)>Theta                    % Break if pass threshold
                break
            end
        end
    end
    Counts(this_cfg) = Counts(this_cfg)+1;     % record w
end
Q = [Q Counts/sum(Counts)]                     % Summary results for this config
%n_acc_mean = acc_mean0 + n_acc_mean;          % Increment M
Theta_fac = Theta_fac + Theta_fac0;            %Increment theta
Theta = Theta_fac*sqrt(n_acc_mean);
end
plot(Prob,Q - Prob*ones(1,nconfig),'*:');
```

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