Contents

MUONIC HYDROGEN AND THE PROTON RADIUS PUZZLE.THEORETICAL MODEL OF REPULSIVE INTERACTION , YUKAWA TYPE . MEDIATION OF GRAVITINO BY DECAY, A W BOSON AND LEPTONS (VIRTUALS PARTICLES) . EXACT THEORETICAL CALCULATION OF THE PROTON RADIUS OF MUONIC HYDROGEN ATOM.

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Abstract. **The extremely precise extraction of the proton radius by Pohl et al. from the mea- sured energy diference between the 2P and 2S states of muonic hydrogen disagrees significantly with that extracted from electronic hydrogen or elastic electron-proton scattering. This is the proton radius puzzle.**
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In this paper, using the fundamental equation that equals the electromagnetic force and gravity; obtained in one of our previous work; and which is derived from gravity, both elementary electric charge, the mass of electron and gravitino; as a dependent equation canonical partition function of the imaginary parts of the nontrivial zeros of the Riemann zeta function and the Planck mass. This equation directly implies the existence of a repulsive gravitational force at very short distances. The decay of gravitinos in a W boson and lepton (muon) would be the phenomenon responsible for this repulsive force that would make the radius of the proton in the muonic hydrogen atom, was that obtained experimentally 8.4087 x 10 ^ -16 m . Likewise, the long half-life, very massive gravitinos, would allow these, penetrate the proton where they finally decay in the W boson and the a lepton. The invariance of the proton, ie its non transformation into a neutron; would be the consequence of an effect of virtual particles: gravitinos, W boson, muon and even the X,Y bosons, of the theories SU(5) grand unification.

The same canonical partition function of the zeros of the Riemann zeta function; It is indeed a sum of Yukawa type potential; therefore repulsive by exchanging a vector boson.

The absence of singularities of black holes, surely, is an effect of this repulsive force. For this reason, increasing the area of a black hole can be interpreted physically as the action of this repulsive force.

I thank God for showing me a tiny part of the infinite beauty of his creation. Creator of all things. And his son Jesus Christ, our Savior.

1. *The zeros of the riemann zeta function: derivation of elementary electric charge, and mass of the electron. repulsive gravitational force.Gravitinos.*

The relativistic invariance of the elementary electric charge. As demonstrated, in this last section, the relativistic invariance of the elementary electric charge, is based on that solely depends on the canonical partition function of the imaginary parts of the non trivials zeros of the Riemann zeta function. And since the imaginary parts of the zeros of the zeta function, are pure and constant numbers; immediately relativistic invariance of electric charge *is derived. Being the Planck mass other relativistic invariant, since there can be no higher mass to the Planck mass, this invariance is guaranteed.*

Partition function (statistical mechanics). Be considered, the coupling of the electromagnetic field to gravity, as represented by a bath of virtual particles, whose thermodynamic state is in equilibrium and there is no exchange of matter. Being a thermal bath whose temperature is constant, invariant, then its energy is infinite (in principle, ideally). Thermodynamic temperature canonical ensemble system can vary, but the number of particles is constant, invariant. That this theoretical approach, is exactly according to the values of the elementary electric charge, and mass of the electron, suggests that space-time-energy to last the unification scale, would behave like black holes, or even, as we shall see later, with wormholes with throat open. These wormholes, following a hyperbolic de Sitter space can explain the quantum entanglement, and the call action at a distance, or non-locality of quantum mechanics.

This partition function of the canonical ensemble, as is well known, is:

$Z = \sum_s \exp -\beta E_s$; *where the "inverse temperature",* β , *is conventionally defined as* $\beta \equiv \frac{1}{k_B T}$; *with* k_B *denoting Boltzmann's constant.Where* E_s *,is the the energy.*

Will use for the dimensionless factor; βE_s*, the change by the imaginary parts of the nontrivial zeros of the Riemann function* $\zeta(s)$

This change is justified, for the simple reason that the vacuum is neutral with respect to the electric charges, ie the value of the electric charge of the vacuum is zero. These zeros can be expressed by the Riemann function, applying the Kaluza-Klein formulation for electric charges; dependent Planck mass, and as we will show by the partition function of canonical ensemble. Thus the zeros of the vacuum to the electric charge is expressed as:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} m_{Pk}}{n^s \cdot \sqrt{\pm e^2/16\pi \cdot G_N}} = 0 \; ; \; s = \tfrac{1}{2} + it_k \; ; \; \zeta(s) = 0 \; (26)$$

Equation (26), and therefore the behavior of the electric field strength with distance, depends on the value of s, because of (26) is obtained, using the conjugate of s: $\frac{(-1)^{n-1} m_{Pk}}{n^s \cdot n^{\bar{s}-\frac{1}{2}} \cdot (\pm e^2/16\pi \cdot G_N)} = \frac{m_{Pk}}{\pm\sqrt{(\pm e)^2/16\pi \cdot G_N}}$

Therefore, by using the canonical partition ensemble, making the substitution of the imaginary parts of the nontrivial zeros of the Riemann function, and taking into account the deviation of the electric charge, the equation is obtained relating the gravity with electric charge and the nontrivial zeros of the Riemann function. The calculation of the partition function has been performed with wolfram math program, version 9. For this calculation we used the first 2000 nontrivial zeros, value more than enough for the accuracy required. Although using the first six zeros, would also be sufficient. The code of this calculation is as follows:

$$\frac{1}{\sum_{n=1}^{2000} e^{-N[\Im(\rho_n),15]}} = \frac{1}{\sum_{n=1}^{2000} \exp -(N\left[Im\left[ZetaZero[n]\right],15\right])} = 1374617.45454188$$; *Given that for values greater than 2000;* $\exp -Im(\rho_n) \approx 0$; *You can write the equality as (by changing rho to s) as:* $\left(\sum_{s_n}^{\infty} \exp -Im(s_n)\right)^{-1} \approx 1374617.45454188$ (27)

Finally, the equation that unifies the gravitational and electromagnetic field, by elementary electric charge, is: $m_{Pk} = \left(\sum_{s_n}^{\infty} \exp -Im(s_n)\right)^{-1} \cdot \sqrt{(\pm e \cdot \sigma(q))^2/G_N} = 2.176529059 \cdot 10^{-8}\, Kg$ (28) ; *The value obtained for the Planck mass is in excellent agreement. The very slight difference, surely is that the constant of gravitation has a very high uncertainty about the other universal constants. Thus, making a speculative exercise, we can give a value for the gravitational constant:*

$$G_N = \left[\left(\sum_{s_n}^{\infty} \exp -Im(s_n)\right)^{-2} \cdot (\pm e \cdot \sigma(q))^2\right] \Big/ m_{Pk}^2 = 6.674841516 \cdot 10^{-11}\, N \cdot m^2/Kg^2 \ (29)$$

Derivation of the partition function of canonical ensemble by the special and unique properties of the Riemann zeta function, for complex values s, with real part 1/2. The function x^r *, to a value of 1/2 , in the set of real numbers , is the only one that has the property, for which its derivative is 1/2 the inverse of this function, that is:* $d(x^{1/2}) = \frac{1}{2 \cdot x^{1/2}}$ *This function has the same property, for complex values of the exponent, such that:* $r = s = \frac{1}{2} + it$; $dx^{\overline{s}}/\overline{s} = 1/x^s$; $dx^s/s = 1/x^{\overline{s}}$ (31)

Commutation properties

*From equation (31), the following four identities are derived:**1)*** $x^s dx^{\overline{s}} = \overline{s}$; 2) $x^{\overline{s}} dx^s = s$; 3) $\frac{dx^{\overline{s}}}{\overline{s}} - \frac{1}{x^s} = 0$; 4) $\frac{dx^s}{s} - \frac{1}{x^{\overline{s}}} = 0$

Of the identities (1) and (2) are derived, by commutation of the conjugates of the exponents, s , $\overline{s}$; *the following identities:*

1) $x^s dx^{\overline{s}} + x^{\overline{s}} dx^s = 1$ ***2)*** $x^s dx^{\overline{s}} - x^{\overline{s}} dx^s = -2it$ ***3)*** $x^{\overline{s}} dx^s - x^s dx^{\overline{s}} = 2it$ (32)

From the identities (31) and (32) immediately derives the following corollary:

Corollary 1. *Only for complex values, s, with real part 1/2, the three conmutation properties, expressed in differential equations are satisfied.*

Conditions that must meet the equation derived from the conmutators (32), and the identities (31).

(1) *Must include the invariance of the sum of the quantized electric charges. This sum is equivalent to the difference between the standard deviation of the electric charges with zero arithmetic mean, and the standard deviation, which has been developed previously, that is:* $\sigma^2(q\,,\,\mu(q)=0)=\sum_q q^2=\dfrac{31}{9}$ (33) ; $\sigma^2(q\,,\,\mu(q)=0)-\sigma_0^2(q)=1=\sum_q q$ (34)

(2) $\sum_q q^2=\left(\frac{4}{3}\right)^2+\left(\frac{1}{3}\right)^2+\left(\frac{-1}{3}\right)^2+\left(\frac{2}{3}\right)^2+(-1)^2$

(3) *The neutrality of the vacuum, in relation to the electric charges, or zero value of the electric charges of the vacuum, is the sum of infinite "oscillators", whose function is the Riemann zeta function applied to the ratio of Planck mass and the mass derived , from elementary electric charge and gravitational constant; fulfilling the equation obtained by Kaluza-Klein, to unify electromagnetism and gravity, adding a fifth dimension compactified on a circle.* $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}m_{pk}\sqrt{G_N}}{\pm e\cdot n^s}=\left(\frac{m_{pk}\sqrt{G_N}}{\pm e}\right)\cdot(\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^s})=0$ (35)

(4) *The complex value s, can only be with real part 1/2, since only for s = 1/2 + it, it is possible to derive from the conmutators, both the invariance of the sum of the electric charges and the function of canonical ensemble , as will be demonstrated below.*

(5) *The value of the energy is the lowest possible, with integer values.*

With these four conditions, we have: 1) $E^s dx^{\overline{s}}+E^{\overline{s}}dx^s=1=\sigma^2(q\,,\,\mu(q)=0)-\sigma_0^2(q)=\sum_q q\,;\;E=energy$

2) $E^s dE^{\overline{s}}=\overline{s}\,;\;E^{\overline{s}}dE^s=s\,;$ *3)* $[(E^s dE^{\overline{s}})E-E/2]/Ei=-t_n$

4) $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}m_{pk}\sqrt{G_N}}{\pm e\cdot n^s}=0=\sum_{E=1}^{\infty}\frac{dE^{\overline{s}}}{\overline{s}}-\frac{1}{E^s}=\sum_{E=1}^{\infty}\frac{dE^s}{s}-\frac{1}{E^{\overline{s}}}$

Derivation of partition function of canonical ensemble: ratio, elementary electric charge and kalulaza-Klein equation.

The introduction of a fifth coordinate; allowed obtaining Theodor Kaluza, the quantization of electric charge; unifying Maxwell's equations (electromagnetism) and the RG Albert Einstein's equations.The derivation of a much higher mass, that the mass of electron, and other problems of the theory, led to dismiss it as a realistic theory, according to experimental physical data.

As we will demonstrate shortly, this theory lacked renormalization by canonical partition function of statistical mechanics (thermodynamics), derived from the imaginary parts of the zeros of the Riemann function $\zeta(s) = 0$; $s = \frac{1}{2} + it_n$

In the framework of the theory we are developing in this work, this fifth dimension corresponds to the three isomorphisms: five electric charges, five spines, five solutions of the energy equation momentum.

As a beginning assumption, assume that a thermodynamically large system is in thermal contact with the environment, with a temperature T, and both the volume of the system and the number of constituent particles are fixed. This kind of system is called a canonical ensemble. Let us label with s = 1, 2, 3, ... the exact states (microstates) that the system can occupy, and denote the total energy of the system when it is in microstate s as E_s . *Generally, these microstates can be regarded as analogous to discrete quantum states of the system.*

$$Z = \sum_s \exp\left(-\frac{E_s}{k_B T}\right)$$

The equation for the elementary electric charge, according to the initial theory of Kaluza (see bibliography) is: $q_n = m_n \cdot \sqrt{16\pi G_N}$

From equations (31), (32) and (34) with the conditions imposed, the following development is obtained, leading to accurate calculation of the elementary electric charge, as a partition function of the imaginary parts of the nontrivial zeros Riemann's function $\zeta(s)$. *Partition function exactly equivalent to the canonical partition function of statistical mechanics (thermodynamics).*

a) $E_0/c^2 = m_0$ *b)* $dm^s/s{\cdot}m^s = (1/m^s){\cdot}(1/m^{\overline{s}}) = 1/m_0$, $dm^{\overline{s}}/\overline{s}{\cdot}m^{\overline{s}} = (1/m^s){\cdot}(1/m^{\overline{s}}) = 1/m_0$

c) $m_0(dm^s/m^s) = s$; $m_0(dm^{\overline{s}}/m^{\overline{s}}) = \overline{s}$; $m_0 \equiv \sigma^2(q\ ,\ \mu(q) = 0) - \sigma_0^2(q) = \sum_q q = 1$; *d)* $(dm^s/im^s) - 1/2i = t_n$

f) We make the change (dm^s/im^s) , *by* (dm_1/m_1) ; *g)* $-(dm_1/m_1) + 1/2i = -t_n$; $(dm^{\overline{s}}/im^{\overline{s}}) - 1/2i = (dm_2/m_2) - 1/2i = -t_n$

h) $-(dm_1/m_1) + 1/2i \ + (dm_2/m_2) - 1/2i = -t_n - t_n$; $-(dm_1/m_1) + (dm_2/m_2) = -t_n - t_n$

$-(dm_1/m_1) + (dm_2/m_2) = -t_n - t_n \rightarrow (dm_2/m_2) = (d - m_3/m_3) = -(dm_3/m_3) = -(dm_1/m_1)$

Having two elementary electric charges with signs -, +, because: $\pm q_n = m_n \cdot \pm\sqrt{16\pi G_N}$; *Then, the following two differential equations for the real value of the imaginary part of the nontrivials zeros Riemann's function is obtained:*

i1) $-(dm_1/m_1) = -t_n$; *i2)* $-(dm_3/m_3) = -t_n$; *j)* $\int_{m_5}^{m_4} -(dm_1/m_1) = -t_n$; $m_4 < m_5$; $\ln(m_4/m_5) = -t_n$

$$\int_{m_5}^{m_4} -(dm_3/m_3) = -t_n \text{ ; } m_4 < m_5 \text{ ; } \ln(m_4/m_5) = -t_n \text{ ; } (m_4/m_5) = \exp(-t_n)$$

Finally, making the infinite sum nontrivial zeros Riemann's function (with the above approach; 7.2.2, with the first 2000 zeros), the two solutions (negative electric charge and positive), these are obtained, taking into account the standard deviation of the electric charge $\sigma(q) = 0.8073734276$ *:*

$\sigma(q) = 2 \cdot \sqrt{(\sigma_0^2(q)\ )/3 \cdot 5}$ *(five electric charges, three colors, and two sings +, -)*

$$(35)\sum_n^{\infty} \frac{m_{n-}}{m_0} = \sum_n^{\infty} \exp(-t_n) = \sqrt{(-e \cdot \sigma(q))^2 \cdot G_N}/m_{Pk} \text{ ; } m_{Pk}/\sqrt{(-e \cdot \sigma(q))^2 \cdot G_N} = \left[\sum_n^{\infty} \exp(-t_n)\right]^{-1}$$

$$(36)\sum_n^{\infty} \frac{m_{n+}}{m_0} = \sum_n^{\infty} \exp(-t_n) = \sqrt{(e \cdot \sigma(q))^2 \cdot G_N}/m_{Pk} \text{ ; } m_{Pk}/\sqrt{(e \cdot \sigma(q))^2 \cdot G_N} = \left[\sum_n^{\infty} \exp(-t_n)\right]^{-1}$$

Performing the calculation with a value of the gravitational constant, the conjectured by equation (29), it has the value of the electric charge, with excellent accuracy:

$$\left[\sum_n^{\infty} \exp -(t_n)\right]^{-1} \approx 1374617.45454188 = m_{Pk}/\sqrt{(e \cdot \sigma(q))^2 \cdot G_N} \rightarrow ...$$

$$... \rightarrow \pm e = \sqrt{m_{Pk}^2 \cdot G_N}/\Big(1374617.45454188 \cdot \sigma(q)\Big)^2$$

$$\pm e = \sqrt{m_{Pk}^2 \cdot G_N}/\Big(1374617.45454188 \cdot \sigma(q)\Big)^2 = 1.602176565 \cdot 10^{-19}\ C \ \textbf{\textit{(37)}}$$

The mass of the electron. Being the electron, the mass of the vacuum lower, with electric charge and completely stable (infinite lifetime), and on the other hand, the Planck mass is the maximum possible, if indeed, the non-trivial zeros of the Riemann function represent stabilizing a deformed torus; become a wormhole, with gravitational and electromagnetic, fully matched forces, then you can set requirements to be the equation, which equals each of the zeros Riemann's function.

Conditions. These conditions would be: a) the sum of the electromagnetic and gravitational part must be zero. b) In the equation the term breaking torus must appear. c) The curvature of space-time, according to general relativity, it must be possible derive directly.

d) The equation must contain the partition function nontrivial zeros Riemann's.

The equation . $2\pi^2 \cdot (\pm e) \cdot [\sum_n^\infty \exp(-t_n)] - 2 \cdot \sqrt{m_{Pk} \cdot m_e \cdot G_N} = 0 \; ; \; 2\pi^2 = volume\ torus$ (39)

$$\sqrt{m_{Pk} \cdot m_e} = m_o \; ; \; \int_0^{2\pi} m \cdot dm = 2\pi^2 \; ; \; e \cdot \exp(2\pi i n) = 1 \cdot e$$

$$e \cdot \exp(2\pi i n/2) = -1 \cdot e \; ; \; n \in \{N\}$$

$m_e = \pi^4 (\pm e)^2 [\sum_n^\infty \exp(-t_n)]^2 \Big/ m_{Pk} \cdot G_N = 9.10938291 \cdot 10^{-31} Kg \; ; \; G_N = 6.674841516 \cdot 10^{-11} \; N \cdot m^2/Kg^2 \; ; \; m_{Pk} = \sqrt{\frac{\hbar c}{6.674841516 \cdot 10^{-11}}}$

In the above equation the volume of a torus appears in three dimensions. Can also derive the angle of curvature of general relativity, because:

$$4\pi^4 (\pm e)^2 \cdot [\sum_n^\infty \exp(-t_n)]^2 \Big/ m_e \cdot c^2 \cdot l_{Pk} = \frac{4 \cdot m_{Pk} \cdot G_N}{c^2 \cdot l_{Pk}} = \theta = 4 \quad (40)$$

The gravitino mass. Equation (39) requires that the gravitational force, at scales of the Planck length, is repulsive. The only possible candidate is the gravitino, which occurs naturally in both theories of supersymmetry, supergravity and string theory. Therefore, from equation (39) can be derived for the gravitino mass, taking into account the spin 3/2, the mass:

$m_{3/2} = \sqrt{m_{Pk} \cdot m_e \cdot (s+1) s_{=3/2}}$ (41)

Equation (41) is more than pure speculation;since the gravitino field is conventionally written as four-vector index. With the mass of the gravitino, according to equation (41) a mass ratio of four grade (four-vector index) is obtained. In the numerator, the four potency of gravitino mass. And in the denominator the product of the Planck mass, electron mass and equivalent mass Higgs vacuum. The result is the mass of unification, in GUT theory.

$(m_{3/2})^4 \cdot 4^3/(m_{Pk} \cdot m_e \cdot m_{Vh}) = m_{GUT} = 4.065067121 \cdot 10^{-11} K_g$

$\ln(\frac{m_{GUT}}{m_Z}) \approx \frac{10\pi}{28} \cdot [\alpha^{-1}(U(1)) - \alpha^{-1}(SU(2))]$; $\alpha^{-1}(U(1)) \approx 59.2$; $\alpha^{-1}(SU(2)) \approx 29.6$

Observe, that in equation (40), the volume factor appears eight dimensions. Also, in this equation the entropy of a black hole is obtained by multiplying by π .This last operation; implies a volume factor in ten dimensions.

$\left(4\pi^4(\pm e)^2 \cdot [\sum_n^\infty \exp(-t_n)]^2 \Big/ m_e \cdot c^2 \cdot l_{Pk}\right) \cdot \pi = 4\pi$; $4\pi^4/4 \cdot dim[SU(5)] = V_{8d} = \frac{\pi^4}{24}$; $dim(l8) = 240$; $\frac{4\pi^4 \cdot \pi}{2 \cdot 240} = V_{10d} = \frac{\pi^5}{120}$

1.1. **The repulsive gravitational force.** $2\pi^2 \cdot (\pm e) \cdot [\sum_n^\infty \exp(-t_n)] - 2 \cdot \sqrt{m_{Pk} \cdot m_e \cdot G_N} = 0$; $\frac{2\pi^2 \cdot (\pm e)}{2 \cdot \sqrt{m_{Pk} \cdot m_e \cdot G_N}} = \frac{1}{[\sum_n^\infty \exp(-t_n)]}$

The above equation contains the repulsive character of gravity at very short distances (Planck length). Similarly, highlights the potential Yukawa type, by the canonical partition function of the imaginary parts of the nontrivial zeros of the Riemann zeta function.

The potential Yukawa type: $\sum_n^\infty \exp(-t_n) = \sum_r \exp(r/r_0) = \int \exp(r/r_0)\, dr$

2. The radius of the proton in the muonic hydrogen atom.

2.1. **First calculation.** Let us first define a differential equation representing the change of the radius of the proton; as a dimensionless function defined by the ratio of the the proton radius electronic hydrogen and the difference between the radius of the proton of muonic hydrogen , least the radius of the proton in the electronic hydrogen.

The equation obtained must be Yukawa type.

The negative sign corresponds to the repulsive force; which as will be shown below, depends on the constants of strong and electromagnetic coupling; to the scale of the electroweak unification force (mass of the Z boson). Also show its dependence on the mass of the gravitino, the W boson mass, the muon mass, the canonical partition function of the nontrivial zeros of the Riemann zeta function. Will be shown several calculations that are fully equivalent.

The condition of equalization of this differential equation will be the sum of the logarithm of the canonical partition function of the nontrivial zeros (the imaginary parts) of the Riemann zeta function, plus the logarithm of the fine structure constant at the scale of the muon mass with a loop correction ($\alpha^2(m_\mu)$). Thus we have:

$r_p = 8.775 \cdot 10^{-16} m$ proton radius electronic hidrogen. $r_p\left(\mu\right) = 8.4087 \cdot 10^{-16} m$ proton radius muonic hidrogen. $\alpha^{-1}\left(0\right) = 137.035999173$

$r_p / \left(r_p\left(\mu\right) - r_p\right) =$constant.

$$r_p / \left(r_p\left(\mu\right) - r_p\right) = \int_{1 \Big/ \sum\limits_n^{\infty} \exp(-t_n)}^{\alpha^2(m_\mu)} -\tfrac{dr}{r} = \ln\left(\alpha^2(m_\mu)\right) - \ln\left(1 \Big/ \sum_n^{\infty} \exp(-t_n)\right) \quad (1a)$$

$$r_p / \left(r_p\left(\mu\right) - r_p\right) = \ln\left(\alpha^2(m_\mu)\right) - \ln\left(1 \Big/ \sum_n^{\infty} \exp(-t_n)\right) \rightarrow$$

$$r_p + \; r_p \Bigg/ \left(\ln\left(\alpha^2(m_\mu)\right) - \ln\left(1 \Big/ \sum_n^{\infty} \exp(-t_n)\right)\right) = r_p\left(\mu\right) \rightarrow$$

$$\cdots \rightarrow \exp - \left(r_p / \left(r_p - r_p(\mu)\right)\right) = \alpha^2(m_\mu) \cdot \sum_n^{\infty} \exp(-t_n)$$

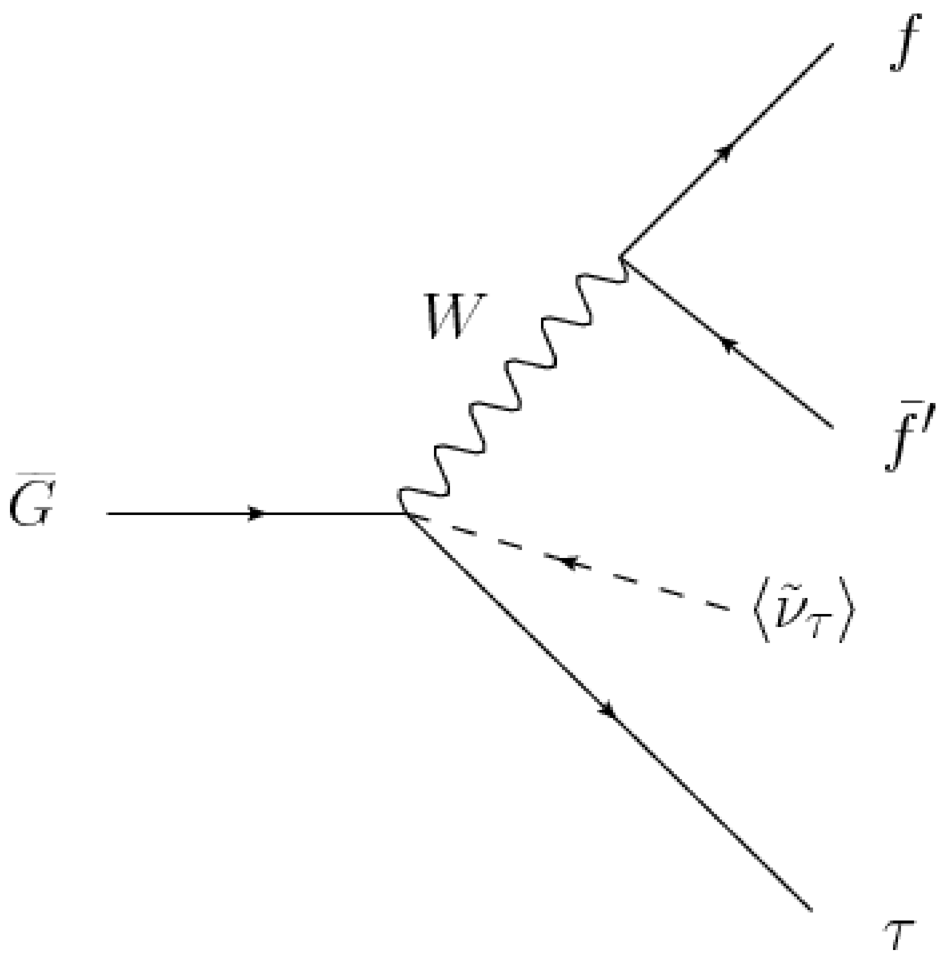
$$\alpha\left(m_\mu\right) = \left\{\alpha\left(0\right) / \left[1 + \left(\alpha\left(0\right)/3\pi\right) \cdot \ln(m_e^2/m_\mu^2)\right]\right\} - \left(\tfrac{\alpha^3(0)}{4\pi^2}\right) \cdot \ln(m_e^2/m_\mu^2) = (135.9026602)^{-1}$$

$r_p / \left(r_p - r_p(\mu)\right) = -\ln[(135.9026602)^{-2}] - \ln\left(1/1374617.45454188\right) = 9.823877791 + 14.13368604 = 23.95756383$

$r_p(\mu) = r_p - (r_p/23.95756383) = 8.775 \cdot 10^{-16} m - (8.775 \cdot 10^{-16} m/23.9575638) = 8.775 \cdot 10^{-16} m - 3.662726337 \cdot 10^{-17} m = 8.408727366 \cdot 10^{-16} m \; (1b)$

As can verify the estimate obtained by (1b), is in excellent agreement with the experimentally measured value.

2.2. **Proton radius of muonic hydrogen atom as a function of wave lengths Comptom virtual W boson and the virtual $m_{3/2}$ gravitino.** Is considered a linear combination Compton wavelength. According to a Feynman diagram, in which the main contribution is the gravitino (virtual) decay in a W boson (virtual), and muon (real interaction), tau and / or electron (virtuals). The diagram would be:



The linear combination of Comptom wavelengths must satisfy the conservation of momentum; so the difference between the radius of the proton of the electronic atom, minus the radius of the proton in the muonic atom is written with the following linear equation of Comptom wavelengths of the gravitino and W boson; both virtual particles. The contributions of the masses of the three leptons (tau, muon and electron) will be taken into account.

$$r_p - r_p(\mu) = (h/m_W \cdot c \cdot \sin_{eff}(\theta_W) \cdot \cos_{eff}(\theta_W)) + \left(h \cdot [m_\tau + m_\mu + m_e/m_e] / m_{3/2} \cdot c \cdot \sin_{eff}(\theta_W)\right) \quad (2a)$$

Where: $[m_\tau + m_\mu + m_e/m_e] = 3684.919296$; ${}_{eff}\theta_W = 28.76^\circ$, $h = 6.62606957 \cdot 10^{-34} J \cdot s$

$m_{3/2} = \sqrt{m_{Pk} \cdot m_e \cdot (s+1) s_{=3/2}}$ = gravitino mass

$m_{Pk} = \sqrt{\hbar \cdot c / G_N} =$ Planck Mass; $(s+1)s_{=3/2} \rightarrow spin\ 3/2$; $m_W = 80.384\, GeV$; $m_Z = 91.1876\, GeV$

2.3. **Proton radius of muonic hydrogen atom as a function of constants of electromagnetic and strong coupling, to the scale of electroweak unification (Z boson mass); and the mass of the gravitino.**

$$r_p - \sqrt{\hbar \cdot r_p / \left(\alpha_s^2(M_Z) \cdot \alpha_{em}^2(M_Z) \cdot c \cdot \sqrt{m_{Pk} \cdot m_e \cdot (s+1)s_{=3/2}} \right)} = r_p(\mu)\ (3)$$

$\alpha_s(M_Z) = 0.1184$; $\alpha_{em}(M_Z) = 1/128.962$

2.4. **Proton radius of muonic hydrogen atom as a function (Yukawa type) of muon mass; the canonical partition function of the nontrivial zeros of the Riemann zeta function and the probability of change of flavor of quarks u, d of proton.**

$$\exp - (r_p/r_p - r_p(\mu)) = [\sum_n^{\infty} \exp(-t_n)]^2 \cdot (m_\mu/m_e) \cdot (2^2/3^2) \cdot |V_{ud}|^8\ (4); 2^2 \equiv SU(2)+1\ ;\ 3^2 \equiv SU(3)+1$$

$|V_{ud}| = \cos\theta_c$; $\theta_c = 13.04°$; $|V_{ud}|^8 \equiv (u \Leftrightarrow d) \cdot 8$ gluons

2.5. **Proton radius of muonic hydrogen atom as a function of the mass ratio of the down quark, up quark ; with a logarithmic correction of mass ratio tau, muon mass.**

$$(m_{qd}/m_{qu})^8 \cdot \sin\theta_c + \ln(m_\tau/m_\mu) \cdot \sin^2\theta_W = r_p/r_p - r_p(\mu)\ (5)\ ;\ (m_{qd}/m_{qu}) = 1/0.56$$

$(m_{qd}/m_{qu})^8 \cdot \sin\theta_c \equiv (u \Leftrightarrow d) \cdot 8$ gluons

2.6. **Relationship of Higgs vacuum; mass of the Higgs boson, h, and the radius of the proton in the muonic hydrogen.**

$$[(V_h/m_h)] \approx (r_p + r_p(\mu))/r_p(\mu)\ ;\ V_h = 246.2196509\, GeV\ ;\ m_h = 125.78\, GeV$$

$$[(246.2196509\, GeV/125.78\, GeV)] \cdot 0.8775 \cdot 10^{-15} m - 0.8775 \cdot 10^{-15} m = 0.8402432 \cdot 10^{-15} m$$

References

[1] A.Garcés Doz, The zeros of Riemann's Function And Its Fundamental Role In Quantum Mechanics, http://vixra.org/abs/1403.0052, 2014-03-08

[2] A. Garces Doz, The BICEP2 Experiment And The Inflationary Model: Dimensionless Quantization of Gravity. Predictive Theory of Quantum Strings.Quantum Wormholes and Nonlocality of QM.The Absence Of Dark Matter, http://vixra.org/abs/1407.0217, 2014-07-31

[3] A. Garces Doz, The Higgs Vacuum.The Particles Of The Standard Model And The Compliance With the Energy-Momentum Equation. The Necessary Existence Of the Axion. Stop Quark Mass, http://vixra.org/abs/1408.0162, 2014-08-26

[4] A.Garcés Doz, A Potential Elementary Proof Of The Riemann Hypothesis, http://vixra.org/abs/1311.0112, 2013-11-15

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