Generalized Relational Expression of Unified All Uncertainty Relations with dimensions

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We propose the generalized relational expression (GRE) to unify all the uncertainty relations (URs) with dimensions by the dimensional analysis. Here we find the general expression of URs, which product of two and *n* non-commutative physical quantities (PQs) with dimensions is equivalent to the power product of the reduced Planck constant ħ, gravitational constant G, speed of light in vacuum c, Boltzmann constant κ and elementary charge e, and find the basic relationship that any PQ with dimensions has a corresponding Planck scale. Many PQs have the same Planck scale because of same dimensions. All the Planck scales are divided into two categories, one is the basic Planck scale and derived Planck scale, and the other is Femi-Planck scale, Bose-Planck scale and Other-Planck scale. The corresponding Planck scale of any PQ is proved to be equivalent to the power product of the Planck length, Planck time, Planck mass, Planck temperature and elementary charge (or Planck charge). The GRE is found and proved that the power product of non-commutative PQs is equivalent to the ones of corresponding Planck scale. We also find the Big Bang UR concerning the temperature of Big Bang and its volume by the GRE, and the Schwarzschild black holes (SBH) UR concerning the SBH mass and its volume. These URs suggest no singularity at Big Bang and SBH with the quantum effect. We show that the GRE is generalized, interesting and significant.

1. Introduction

The Heisenberg uncertainty principle [1] made great progress in applications [2, 3], developments [4, 5] and experiments [6, 7]. These solidify its solid foundation and expand its connotation. Now there are many uncertainty relations (URs) with dimensions:

$\Delta p\Delta r \geq \hbar$ [1]; $\Delta E\Delta t \geq \hbar$ [1]; $\delta t = \beta\, \mathrm{t}_\mathrm{P}^{2/3} t^{1/3}$ [8]; $\eta / s \geq 4\pi\hbar / \kappa$ [9]; $\Delta T\Delta X \sim \mathrm{L}_\mathrm{S}^2 \sim \mathrm{L}_\mathrm{P}^2 / \mathrm{c}$ [10]; $\delta x\delta y\delta t \sim \mathrm{L}_\mathrm{P}^3 / \mathrm{c}$ [11]; $L_{\mu\nu} \sim \sqrt{\mathrm{L}_\mathrm{P} L}$ [12]; $\varepsilon(Q)\eta(P) + \varepsilon(Q)\sigma(P) + \sigma(Q)\eta(P) \geq \hbar / 2$ [7]; $(\delta t)(\delta r)^3 \geq \pi r^2 \mathrm{L}_\mathrm{P}^2 / \mathrm{c}$ [13], etc.

where Δp is the momentum fluctuation, Δr the position momentum, ħ the reduced Planck constant; ΔE the energy fluctuation, Δt the time fluctuation; δt the time fluctuation, β an order one constant, $\mathrm{t}_\mathrm{P} = \sqrt{\hbar \mathrm{G}/\mathrm{c}^5}$ Planck time, G the gravitational constant, c the speed of light in vacuum, t the time; η the ratio of shear viscosity of a given fluid perfect, s its volume density of entropy, κ the Boltzmann constant; ΔT the time-like, ΔX its space-like, L_S the string scale, $\mathrm{L}_\mathrm{P} = \sqrt{\hbar \mathrm{G}/\mathrm{c}^3}$ Planck length; δx, δy, δt are the position fluctuation and time fluctuation separately; $L_{\mu\nu}$ the transverse length, L the radial length; Q the position of a mass, $\varepsilon(Q)$ the root-mean-square error, P its momentum, $\eta(P)$ the root-mean-square disturbance, $\sigma(P)$ the standard deviation; δt and δr the sever space-time fluctuations of the constituents of the system at small scales, and r the radius of globular computer.

So there are two problems: (i) Why is there no G on the right hand of some URs? (ii) Do they have a unitive form? In this paper, we answer that G disappears because of being reduced fitly and the unitive form is the generalized relational expression (GRE). Moreover, for the origin and development of Planck length, Planck time, Planck mass $\mathrm{M}_\mathrm{P} = \sqrt{\hbar \mathrm{c}/\mathrm{G}}$, Planck energy $\mathrm{E}_\mathrm{P} = \sqrt{\hbar \mathrm{c}^5/\mathrm{G}}$ and Planck temperature $\mathrm{T}_\mathrm{P} = \sqrt{\hbar \mathrm{c}^5/\kappa^2 \mathrm{G}}$, please refer to the literature [14-18].

This paper is organized as follows. In Sec. 2, we derive the general expression of two and *n* URs and basic relationship. In Sec. 3, we obtain the Planck scale and classify them. In Sec. 4, we prove the corresponding Planck scale of any PQ being rewritten as the power product of basic Planck scale; find and prove the GRE, and prove the URs in Sec. 1. In Sec. 5, we find the Big Bang UR and SBH UR. We conclude in Sec. 6.

2. General Expression of URs and Basic Relationship

In this section, we discover the normal form of URs, derive the general expression of two and *n* URs and basic relationship.

2.1 General expression of URs of two PQs

Observing these URs, we can discover the physical constants such as ħ, G, c and κ on the right hand and the physical quantities (PQs) on left hand. We rewrite them as

$\Delta p\Delta r \geq \hbar^1$; $\Delta E\Delta t \geq \hbar^1$; $\delta t / \beta t^{1/3} = t_P^{2/3} = \hbar^{1/3}G^{1/3}c^{-5/3}$; $\eta / 4\pi s \geq \hbar\kappa^{-1}$; $\Delta T\Delta X \sim L_S^2 \sim L_P^2 / c = \hbar G c^{-4}$; $\delta x\delta y\delta t \sim L_P^3 / c = \hbar^{3/2}G^{3/2}c^{-11/2}$; $L_{\mu\nu} / \sqrt{L} \sim \sqrt{L_P} = \hbar^{1/4}G^{1/4}c^{-3/4}$; $2[\varepsilon(Q)\eta(P) + \varepsilon(Q)\sigma(P) + \sigma(Q)\eta(P)\ ] \geq \hbar^1$; $(\delta t)(\delta r)^3 / \pi r^2 \geq L_P^2 / c = \hbar G c^{-4}$, etc.

Therefore the power product of physical constants appears on the right hand. These are their normal form. Considering two non-commutative PQs with dimensions, we obtain the general expression of URs

$$AB \sim \hbar^x G^y c^z \kappa^w e^u \tag{1}$$

where A and B are non-commutative PQs, x, y, z, w and u the unknown number, and e is the elementary charge. Applying the dimensional analysis [19] (here we use the LMtTQ units [20][1]), the dimensions of A and B are expressed as

$$[A] = [L]^{\alpha_1}[M]^{\beta_1}[t]^{\gamma_1}[T]^{\delta_1}[Q]^{\varepsilon_1}$$
$$[B] = [L]^{\alpha_2}[M]^{\beta_2}[t]^{\gamma_2}[T]^{\delta_2}[Q]^{\varepsilon_2} \tag{2}$$

where L, M, t, T and Q are the dimensions of length, mass, time, temperature and electric charge separately, α_1, α_2, β_1, β_2, γ_1, γ_2, δ_1, δ_2, ε_1 and ε_2 the known real number. The dimensions of $\hbar^x G^y c^z \kappa^w e^u$ are

$$[\hbar^x G^y c^z \kappa^w e^u] = \{[L^2][M][t^{-1}]\}^x\{[L^3][M^{-1}][t^{-2}]\}^y\{[L][t^{-1}]\}^z \times \{[L^2][M][t^{-2}][T^{-1}]\}^w\{[Q]\}^u \tag{3}$$

By the dimensional analysis, we obtain

$$[L]^{\alpha_1}[M]^{\beta_1}[t]^{\gamma_1}[T]^{\delta_1}[Q]^{\varepsilon_1}\ [L]^{\alpha_2}[M]^{\beta_2}[t]^{\gamma_2}[T]^{\delta_2}[Q]^{\varepsilon_2} = \{[L^2][M][t^{-1}]\}^x\{[L^3][M^{-1}][t^{-2}]\}^y\{[L][t^{-1}]\}^z \times \{[L^2][M][t^{-2}][T^{-1}]\}^w\{[Q]\}^u \tag{4}$$

Solving the equation (4), we gain

$$x = [(\alpha_1+\alpha_2)+(\beta_1+\beta_2)+(\gamma_1+\gamma_2)+(\delta_1+\delta_2)] / 2,$$
$$y = [(\alpha_1+\alpha_2)-(\beta_1+\beta_2)+(\gamma_1+\gamma_2)-(\delta_1+\delta_2)] / 2$$
$$z = -[3(\alpha_1+\alpha_2)-(\beta_1+\beta_2)+5(\gamma_1+\gamma_2)-5(\delta_1+\delta_2)] / 2,$$
$$w = -(\delta_1+\delta_2),\ u = (\varepsilon_1+\varepsilon_2) \tag{5}$$

Thus we find the general expression of URs of two PQs

$$AB \sim [\hbar^{((\alpha_1+\alpha_2)+(\beta_1+\beta_2)+(\gamma_1+\gamma_2)+(\delta_1+\delta_2))}]^{\frac{1}{2}} \times [G^{((\alpha_1+\alpha_2)-(\beta_1+\beta_2)+(\gamma_1+\gamma_2)-(\delta_1+\delta_2))}]^{\frac{1}{2}} \times [c^{-(3(\alpha_1+\alpha_2)-(\beta_1+\beta_2)+5(\gamma_1+\gamma_2)-5(\delta_1+\delta_2))}]^{\frac{1}{2}} \times \kappa^{-(\delta_1+\delta_2)}e^{(\varepsilon_1+\varepsilon_2)} \tag{6}$$

It shows that the product of two non-commutative PQs with dimensions is equivalent to the power product of the reduced Planck constant, gravitational constant, speed of light in vacuum, Boltzmann constant and elementary charge.

2.2 Basic relationship

Ordering $\alpha_1 = \alpha_2 = \alpha$, $\beta_1 = \beta_2 = \beta$, $\gamma_1 = \gamma_2 = \gamma$, $\delta_1 = \delta_2 = \delta$, and $\varepsilon_1 = \varepsilon_2 = \varepsilon$ in the general expression of URs (6), that is A and B having the same dimensions

$$[A] = [B] = [L]^{\alpha}[M]^{\beta}[t]^{\gamma}[T]^{\delta}[Q]^{\varepsilon} \tag{7}$$

We obtain

$$\hbar^{(\alpha+\beta+\gamma+\delta)}G^{(\alpha-\beta+\gamma-\delta)}c^{-(3\alpha-\beta+5\gamma-5\delta)}\kappa^{-2\delta}e^{2\varepsilon} = A_P B_P = A_P^2 = B_P^2 \tag{8}$$

where A_P and B_P are the corresponding Planck scale of A and B separately. Extracting the square root, we find the basic relationship

$$A \sim A_P = [\hbar^{(\alpha+\beta+\gamma+\delta)}G^{(\alpha-\beta+\gamma-\delta)}c^{-(3\alpha-\beta+5\gamma-5\delta)}\kappa^{-2\delta}e^{2\varepsilon}]^{\frac{1}{2}} \tag{9}$$

The above relationship shows that any PQ with dimensions has a corresponding Planck scale which is equivalent to the power product of ħ, G, c, κ and e.

2.3 General expression of URs of n PQs

Similarly considering n non-commutative PQs with dimensions, we have

$$\prod_{i=1}^{n} A_i \sim \hbar^x G^y c^z \kappa^w e^u,\ i = 1, 2, 3\ldots n \tag{10}$$

where A_i is the PQ, A_i and A_{i+1} are non-commutative. The dimensions of $\prod_{i=1}^{n} A_i$ are

$$[\prod_{i=1}^{n} A_i] = [L]^{\sum_i^n \alpha_i}[M]^{\sum_i^n \beta_i}[t]^{\sum_i^n \gamma_i}[T]^{\sum_i^n \delta_i}[Q]^{\sum_i^n \varepsilon_i} \tag{11}$$

where α_i, β_i, γ_i, δ_i and ε_i are known real number. By the dimensional analysis also, we find the general expression of URs of n PQs

$$\prod_{i=1}^{n} A_i \sim [\hbar^{((\sum_i^n \alpha_i)+(\sum_i^n \beta_i)+(\sum_i^n \gamma_i)+(\sum_i^n \delta_i))}]^{\frac{1}{2}} \times [G^{((\sum_i^n \alpha_i)-(\sum_i^n \beta_i)+(\sum_i^n \gamma_i)-(\sum_i^n \delta_i))}]^{\frac{1}{2}} \times [c^{-(3(\sum_i^n \alpha_i)-(\sum_i^n \beta_i)+5(\sum_i^n \gamma_i)-5(\sum_i^n \delta_i))}]^{\frac{1}{2}} \times \kappa^{-(\sum_i^n \delta_i)}e^{(\sum_i^n \varepsilon_i)} \tag{12}$$

Certainly when $n = 2$, it become the general expression of URs (6). Ordering $\alpha_i = \alpha_{i+1} = \alpha$, $\beta_i = \beta_{i+1} = \beta$, $\gamma_i = \gamma_{i+1} = \gamma$, $\delta_i = \delta_{i+1} = \delta$ and $\varepsilon_i = \varepsilon_{i+1} = \varepsilon$ in (12), A_i and A_{i+1} having the same dimensions, we obtain

$$[\hbar^{n(\alpha+\beta+\gamma+\delta)}]^{\frac{1}{2}}[G^{n(\alpha-\beta+\gamma-\delta)}]^{\frac{1}{2}}[c^{-n(3\alpha-\beta+5\gamma-5\delta)}]^{\frac{1}{2}}\kappa^{-n\delta}e^{n\varepsilon} = A_P^n \tag{13}$$

Extracting the nth-root, we gain (9) again.

3. Planck scale

In this section, we obtain the Planck scale, and classify them.

3.1 Basic Planck scale

Ordering $\alpha = 1$, $\beta = \gamma = \delta = \varepsilon = 0$ in (7) and (9), we obtain Planck length immediately

$$L_P = \sqrt{\hbar G/c^3}$$

Instructing $\gamma = 1$, $\alpha = \beta = \delta = \varepsilon = 0$, obtain Planck time

$$t_P = \sqrt{\hbar G/c^5}$$

Ordering $\beta = 1$, $\alpha = \gamma = \delta = \varepsilon = 0$, obtain Planck mass

[1] Chien Wei-Zang used L, M, T, θ and Q indicated the dimension of length, mass, time, temperature and electric charge separately in [20].

$$M_P = \sqrt{\hbar c/G}$$

Instructing $\delta = 1$, $\alpha = \beta = \gamma = \varepsilon = 0$, obtain Planck temperature

$$T_P = \sqrt{\hbar c^5/\kappa^2 G}$$

Ordering $\varepsilon = 1$, $\alpha = \beta = \gamma = \delta = 0$, obtain elementary charge

$$Q_e = e$$

If using $[Q]^2 = [L]^3[M][T]^{-2}$, obtain Planck electric charge

$$Q_P = \sqrt{\hbar c} \sim e$$

These are the basic Planck scale [14].

3.2 Derived Planck scale

From (7) and (9), we gain the derived Planck scale [14] except for the basic one. For example

Planck energy E_P

$$[E_P] = [L^2][M][T^{-2}],\ E_P = \sqrt{\hbar c^5/G}$$

Planck momentum P_P

$$[P_P] = [L][M][T^{-1}],\ P_P = \sqrt{\hbar c^3/G}$$

Planck curvature tensor $R_{\mu\nu P}$

$$[R_{\mu\nu P}] = [L^{-2}],\ R_{\mu\nu P} = c^3 / \hbar G$$

Because many PQs have the same dimensions, they have the same Planck scale, for example

Planck energy density ρ_P

$$[\rho_P] = [L^{-1}][M][T^{-2}],\ \rho_P = c^7 / \hbar G^2$$

Planck pressure p_P

$$[p_P] = [L^{-1}][M][T^{-2}],\ p_P = c^7 / \hbar G^2$$

Planck force per unit area f_P

$$[f_P] = [L^{-1}][M][T^{-2}],\ f_P = c^7 / \hbar G^2$$

Planck energy- momentum tensor $T_{\mu\nu P}$

$$[T_{\mu\nu P}] = [L^{-1}][M][T^{-2}],\ T_{\mu\nu P} = c^7 / \hbar G^2$$

Etc.

3.3 Classifications

All the Planck scales can be divided into two categories. One is the basic Planck scale and derived Planck scale [14]. The other is the Femi-Planck scale its exponent is half integer such as L_P, t_P, M_P, T_P, E_P, P_P, and so on, the Bose-Planck scale whose exponents are integer such as Q_e, ρ_P, p_P, f_P, $R_{\mu\nu P}$, $T_{\mu\nu P}$, etc, and Other-Planck scale such as the Planck wave function ψ_P.

$$[\psi_P] = [L^{-3/2}],\ \psi_P = (\hbar G / c^3)^{-3/4}$$

4. GRE

In this section, we prove that basic relationship (9) is rewritten as the power product of basic Planck scale; find and prove the GRE, and prove the URs in Sec. 1,.

4.1 Proof of basic relationship

The basic relationship (9) can be rewritten as

$$A_P = L_P^\alpha M_P^\beta t_P^\gamma T_P^\delta Q_e^\varepsilon \tag{14}$$

From (9), we obtain

$$\begin{aligned} A_P &= [\hbar^\alpha G^\alpha c^{-3\alpha}]^{\frac{1}{2}}[\hbar^\beta G^{-\beta} c^\beta]^{\frac{1}{2}}[\hbar^\gamma G^\gamma c^{-5\gamma}]^{\frac{1}{2}}[\hbar^\delta G^{-\delta} c^{5\delta}]^{\frac{1}{2}}\kappa^{-\delta} e^\varepsilon \\ &= [\sqrt{\hbar G/c^3}]^\alpha [\sqrt{\hbar c/G}]^\beta [\sqrt{\hbar G/c^5}]^\gamma [\sqrt{\hbar c^5/\kappa^2 G}]^\delta e^\varepsilon \\ &= L_P^\alpha M_P^\beta t_P^\gamma T_P^\delta Q_e^\varepsilon \end{aligned}$$

Therefore the corresponding Planck scale of any PQ is equivalent to the power product of Planck length, Planck mass, Planck time, Planck temperature and elementary charge.

4.2 GRE

Considering all the non-commutative PQs, we find the GRE

$$\prod_{i=1}^n A_i^{a_i} \sim \prod_{i=1}^n A_{iP}^{a_i};\quad i = 1, 2, 3\ldots n \tag{15}$$

where A_i is the PQ, A_i and A_{i+1} are non-commutative, a_i is the real number, and A_{iP} is the corresponding Planck scale of A_i. It shows that the power product of non-commutative PQs is equivalent to the ones of corresponding Planck scale.

4.3 Proving GRE

We prove the GRE by the same way in 2.3. Considering *n* non-commutative PQs with a_i power, we have

$$\prod_{i=1}^n A_i^{a_i} \sim \hbar^x G^y c^z \kappa^w e^u \tag{16}$$

The dimensions of $\prod_{i=1}^n A_i^{a_i}$ are

$$[\prod_{i=1}^n A_i^{a_i}] = [L]^{\sum_i^n a_i\alpha_i}[M]^{\sum_i^n a_i\beta_i}[t]^{\sum_i^n a_i\gamma_i}[T]^{\sum_i^n a_i\delta_i}[Q]^{\sum_i^n a_i\varepsilon_i} \tag{17}$$

Using the dimensional analysis too, we gain the general expression of URs of *n* PQs with a_i power

$$\begin{aligned} \prod_{i=1}^n A_i^{a_i} &\sim [\hbar^{\left((\sum_i^n a_i\alpha_i)+(\sum_i^n a_i\beta_i)+(\sum_i^n a_i\gamma_i)+(\sum_i^n a_i\delta_i)\right)}]^{\frac{1}{2}} \\ &\times [G^{\left((\sum_i^n a_i\alpha_i)-(\sum_i^n a_i\beta_i)+(\sum_i^n a_i\gamma_i)-(\sum_i^n a_i\delta_i)\right)}]^{\frac{1}{2}} \\ &\times [c^{-\left(3(\sum_i^n a_i\alpha_i)-(\sum_i^n a_i\beta_i)+5(\sum_i^n a_i\gamma_i)-5(\sum_i^n a_i\delta_i)\right)}]^{\frac{1}{2}} \\ &\times \kappa^{-(\sum_i^n a_i\delta_i)} e^{(\sum_i^n a_i\varepsilon_i)} \\ &= [\sqrt{\hbar G/c^3}]^{\sum_i^n a_i\alpha_i}[\sqrt{\hbar c/G}]^{\sum_i^n a_i\beta_i}[\sqrt{\hbar G/c^5}]^{\sum_i^n a_i\gamma_i} \\ &\times [\sqrt{\hbar c^5/\kappa^2 G}]^{\sum_i^n a_i\delta_i} e^{\sum_i^n a_i\varepsilon_i} \\ &= L_P^{\sum_i^n a_i\alpha_i} M_P^{\sum_i^n a_i\beta_i} t_P^{\sum_i^n a_i\gamma_i} T_P^{\sum_i^n a_i\delta_i} Q_e^{\sum_i^n a_i\varepsilon_i} \\ &= \prod_{i=1}^n L_P^{a_i\alpha_i} M_P^{a_i\beta_i} t_P^{a_i\gamma_i} T_P^{a_i\delta_i} Q_e^{a_i\varepsilon_i} = \prod_{i=1}^n A_{iP}^{a_i} \end{aligned} \tag{18}$$

where $A_{iP} = L_P^{\alpha_i} M_P^{\beta_i} t_P^{\gamma_i} T_P^{\delta_i} Q_e^{\varepsilon_i}$.

4.4 Proving URs

Applying the GRE (15), we can prove the URs in Sec.1.

$\Delta p\Delta r \sim P_P L_P = \sqrt{\hbar c^3/G}\sqrt{\hbar G/c^3} = \hbar$; $\Delta E\Delta t \sim E_P t_P = \sqrt{\hbar c^5/G}\sqrt{\hbar G/c^5} = \hbar$; $\delta t / t^{1/3} \sim t_P / t_P^{1/3} = t_P^{2/3}$; $\eta / s \sim \eta_P / s_P = \sqrt{c^9/\hbar G^3} / \sqrt{c^9\kappa^2/\hbar^3 G^3} = \hbar / \kappa$; $\Delta T\Delta X \sim t_P L_P \sim \hbar G / c^4 = L_P^2 / c \sim L_S^2$; $\delta x\delta y\delta t \sim L_P^2 t_P = L_P^3 / c$; $L_{\mu\nu} / \sqrt{L} \sim L_P / \sqrt{L_P} = \sqrt{L_P}$; $\varepsilon(Q)\eta(P) + \varepsilon(Q)\sigma(P) + \sigma(Q)\eta(P) \sim \sqrt{\hbar G/c^3}\sqrt{\hbar c^3/G} = \hbar$; $(\delta t)(\delta r)^3 / r^2 \sim t_P L_P^3 / L_P^2 = L_P^2 / c$, etc.

where $\eta_P = \sqrt{c^9/\hbar G^3}$ is the Planck ratio of shear viscosity of a given fluid perfect, and $s_P = \sqrt{c^9\kappa^2/\hbar^3 G^3}$ its Planck volume density of entropy (from basic relationship (9)). Thus we find that there is no G on some URs right hand because it is reduced fitly.

5. No Singularity at Big Bang and SBH

In this section, we find the Big Bang UR and SBH UR by the GRE.

5.1 Big Bang UR

S.W. Hawking and R. Penrose proved that the universe originated the Big Bang singularity [21]. Many literatures discussed no singularity at the Big Bang and black holes with the quantum effect, please refer to [18] [22-25]. The one of the characteristic of Big Bang singularity is zero volume and limitless high temperature.

Then we can find the relationship of Big Bang temperature and its volume by the GRE (15)

$$T_B V_B \sim \mathrm{T_P V_P} = \mathrm{T_P L_P^3} = \hbar^2 \mathrm{G} / \kappa \mathrm{c}^2 \quad (19)$$

where T_B is the Big Bang temperature, V_B its volume, and $\mathrm{V_P} = \mathrm{L_P^3}$ the Planck volume. This is the Big Bang UR. It shows that it is impossible to measure the Big Bang temperature and its volume simultaneously. When $\hbar \to 0$, we obtain

$$T_B V_B \sim 0 \quad (20)$$

Because $T_B > 0$, we gain $V_B \sim 0$, the Big Bang volume is zero, thus the Big Bang singularity appears without the quantum effect. We suggest no singularity at the Big Bang with quantum effect. Substituting $a = \mathrm{c}\kappa T / 2\pi\hbar$ [26] into (19), we obtain

$$a_B V_B \sim a_\mathrm{p} \mathrm{V_p} = \hbar \mathrm{G} / 2\pi \mathrm{c} \quad (21)$$

where a_B is the Big Bang acceleration, and $a_\mathrm{p} = \sqrt{\mathrm{c}^7/\hbar \mathrm{G}}$ the Planck acceleration. It is the UR for Big Bang acceleration and its volume.

5.2 SBH UR

Similarly considering the SBH mass and its volume, we find

$$M_H V_H \sim \mathrm{M_P V_P} = \mathrm{M_P L_P^3} = \hbar^2 \mathrm{G} / \mathrm{c}^4 \quad (22)$$

where M_H is the SBH mass, and V_H its volume. It is the SBH UR. Also it is impossible to measure the SBH mass and volume simultaneously. When $\hbar \to 0$, we obtain

$$M_H V_H \sim 0 \quad (23)$$

Because $M_H > 0$, we have $V_H \sim 0$, the volume is zero, the SBH singularity appears without quantum effect also. We also suggest no singularity in SBH with quantum effect. Taking $M = \rho V$ to (22), we gain

$$M_H^2 / \rho_H \sim \hbar^2 \mathrm{G} / \mathrm{c}^4, \ \rho_H V_H^2 \sim \hbar^2 \mathrm{G} / \mathrm{c}^4 \quad (24)$$

where ρ_H is the mass density of SBH. These are the URs for the mass density of SBH and its mass or volume.

6. Conclusion

In this paper, we investigate the URs with dimensions by the dimensional analysis. We find the following results.

1) The normal form of URs is discovered. The PQs are on the left hand of URs, and the physical constants such as the reduced Planck constant ħ, gravitational constant G, speed of light in vacuum c and Boltzmann constant κ are on right hand. These power products of physical constants which are rewritten appear.

2) The general expression of URs is found. It shows that the product of two and *n* non-commutative PQs with dimensions is equivalent to the power product of ħ, G, c, κ and elementary charge e.

3) The basic relationship is found. Any PQ with dimensions has a corresponding Planck scale, which is equivalent to the power product of ħ, G, c, κ and e.

4) The Planck length $\mathrm{L_P}$, Planck time $\mathrm{t_P}$, Planck mass $\mathrm{M_P}$, Planck temperature $\mathrm{T_P}$, elementary charge $\mathrm{Q_e}$ (or Planck charge), Planck energy $\mathrm{E_P}$, Planck momentum $\mathrm{P_P}$, Planck curvature tensor $\mathrm{R_{\mu\nu P}}$, Planck energy density ρ_P, Planck pressure $\mathrm{p_P}$, Planck force per unit area $\mathrm{f_P}$, Planck energy-momentum tensor $\mathrm{T_{\mu\nu P}}$ etc are obtained again. Many PQs have the same Planck scale because of the same dimensions such as ρ_P, $\mathrm{p_P}$, $\mathrm{f_P}$ and $\mathrm{T_{\mu\nu P}}$.

5) All the Planck scales are divided into two categories. One is the basic Planck scale including $\mathrm{L_P}$, $\mathrm{t_P}$, $\mathrm{M_P}$, $\mathrm{T_P}$ and $\mathrm{Q_e}$, and derived Planck scale such as $\mathrm{E_P}$, $\mathrm{P_P}$, ρ_P, $\mathrm{p_P}$, $\mathrm{f_P}$, $\mathrm{R_{\mu\nu P}}$, $\mathrm{T_{\mu\nu P}}$, Planck wave function ψ_P and so on. The other is the Femi-Planck scale its exponent is half integer such as $\mathrm{L_P}$, $\mathrm{t_P}$, $\mathrm{M_P}$, $\mathrm{T_P}$, $\mathrm{E_P}$, $\mathrm{P_P}$, etc, the Bose-Planck scale whose exponents are integers such as $\mathrm{Q_e}$, ρ_P, $\mathrm{p_P}$, $\mathrm{f_P}$, $\mathrm{R_{\mu\nu P}}$, $\mathrm{T_{\mu\nu P}}$, etc, and Other-Planck scale such as Planck wave function ψ_P.

6) The corresponding Planck scale of any PQ is proved to be equivalent to the power product of $\mathrm{L_P}$, $\mathrm{t_P}$, $\mathrm{M_P}$, $\mathrm{T_P}$ and $\mathrm{Q_e}$.

7) The GRE is found and proved. It shows that the power product of the non-commutative PQs is equivalent to ones of corresponding Planck scale. The URs in Sec. 1 are proved by the GRE. G disappears on some URs right hand because of being reduced fitly.

8) The Big Bang UR concerning the temperature T_B of Big Bang and its volume V_B is found by the GRE. It suggests no singularity at the Big Bang with the quantum effect. The UR concerning Big Bang acceleration a_B and V_B is obtained. Similarly the SBH UR concerning the SBH mass M_H and its volume V_H is found; also no singularity is in SBH with quantum effect. The URs concerning the mass density ρ_H of SBH and M_H or V_H is gained.

9) The GRE unifies all URs with dimensions. It is generalized, interesting and significant; any UR is its special case. Because depends on the dimension, GRE can't obtain the factor and relation without dimensions.

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