Generalized Relational Expression of Unified All Uncertainty Relations with dimensions

Yong Bao

Postbox 777, 100 Renmin South Road, Luoding 527200, Guangdong, China
E-mail: baoyong9803@163.com

We propose the generalized relational expression (GRE) to unify all the uncertainty relations (URs) with dimensions by the dimensional analysis. Here we find the general expression of URs, which product of two and \( n \) non-commutative physical quantities (PQs) with dimensions is equivalent to the power product of the reduced Planck constant \( \hbar \), gravitational constant \( G \), speed of light in vacuum \( c \), Boltzmann constant \( k \) and elementary charge \( e \), and find the basic relationship that any PQ with dimensions has a corresponding Planck scale. Many PQs have the same Planck scale because of same dimensions. All the Planck scales are divided into two categories, one is the basic Planck scale and derived Planck scale, and the other is Femi-Planck scale, Bose-Planck scale and Other-Planck scale. The corresponding Planck scale of any PQ is proved to be equivalent to the power product of the Planck length, Planck time, Planck mass, Planck temperature and elementary charge (or Planck charge). The GRE is found and proved that the product power of non-commutative PQs is equivalent to the ones of corresponding Planck scale. We also find the Big Bang UR concerning the temperature of Big Bang and its volume by the GRE, and the Schwarzschild black holes (SBH) UR concerning the SBH mass and its volume. These URs suggest no singularity at Big Bang and SBH with the quantum effect. We show that the GRE is generalized, interesting and significant.

1. Introduction

The Heisenberg uncertainty principle [1] made great progress in applications [2, 3], developments [4, 5] and experiments [6, 7]. These solidify its solid foundation and expand its connotation. Now there are many uncertainty relations (URs) with dimensions:

\[ \Delta p \Delta r \geq h [1]; \Delta E \Delta t \geq h [1]; \delta t = \beta L_p^{2/3} t^{1/3} [8]; \eta / s \geq 4 \pi \hbar / k [9]; \Delta T \sim L_p^2 \sim L_p / c [10]; \delta x / \delta t \sim L_p / c [11]; L_{\mu \nu} \sim \sqrt{4 \pi \hbar / G} [12]; \varepsilon (Q)(\eta (P) + \sigma (Q)\eta (P) + \sigma (Q)\sigma (P) \geq h / 2 [7]; (\delta \eta / \delta r)^3 \geq \pi r^2 L_p^2 / c ^ [13], \text{ etc.} \]

where \( \Delta p \) is the momentum fluctuation, \( \Delta r \) the position momentum, \( h \) the reduced Planck constant; \( \Delta E \) the energy fluctuation, \( \Delta t \) the time fluctuation; \( \delta t \) the time fluctuation, \( \eta \) an order one constant, \( t_0 = \sqrt{\hbar G / c^3} \) Planck time, \( G \) the gravitational constant, \( c \) the speed of light in vacuum, \( t \) the time; \( \eta \) the ratio of shear viscosity of a given fluid perfect, \( s \) its volume density of entropy, \( k \) the Boltzmann constant; \( \Delta X \) the time-like, \( \Delta X \) its space-like, \( L_p \) the string scale, \( L_p = \sqrt{\hbar G / c^3} \) Planck length; \( \delta x \), \( \delta y \), \( \delta t \) are the position fluctuation and time fluctuation separately; \( L_{\mu \nu} \) the transverse length, \( L \) the radial length; \( Q \) the position of a mass, \( \varepsilon (Q) \) the root-mean-square error, \( P \) its momentum, \( \eta (P) \) the root-mean-square disturbance, \( \sigma (P) \) the standard deviation; \( \delta t \) and \( \delta r \) the seven space-time fluctuations of the constituents of the system at small scales, and \( r \) the radius of globular computer.

So there are two problems: (i) Why is there no G on the right hand of some URs? (ii) Do they have a unitive form? In this paper, we answer that \( G \) disappears because of being reduced fitly and the unitive form is the generalized relational expression (GRE). Moreover, for the origin and development of Planck length, Planck time, Planck mass, Planck temperature and elementary charge (or Planck charge). The GRE is found and proved that the product power of non-commutative PQs is equivalent to the ones of corresponding Planck scale. We also find the Big Bang UR concerning the temperature of Big Bang and its volume by the GRE, and the Schwarzschild black holes (SBH) UR concerning the SBH mass and its volume. These URs suggest no singularity at Big Bang and SBH with the quantum effect. We show that the GRE is generalized, interesting and significant.

2. General Expression of URs and Basic Relationship

In this section, we discover the normal form of URs, derive the general expression of two and \( n \) URs and basic relationship.

2.1 General expression of URs of two PQs

Observing these URs, we can discover the physical constants.
such as h, G, c and k on the right hand and the physical quantities (PQs) on left hand. We rewrite them as 
\[ \Delta p \Delta t \geq h^2; \quad \Delta E \Delta t \geq h^2; \quad \delta t / \beta \leq \gamma; \quad \gamma = \frac{\sqrt{4x}}{2} = \frac{c}{hGc^4}; \quad \eta / 4xs \geq \delta c^{-4}; \quad \Delta T \Delta x \approx \frac{L}{P} = hGc^{-4} \delta \lambda y \delta t = \frac{L}{2}; \quad \gamma = \frac{3}{2}G \gamma^2 \cosec (\gamma); \quad \Delta L / \gamma^2 = \frac{1}{hGc^4}, etc.

Therefore the power product of physical constants appears on the right hand. These are their normal form. Considering two non-commutative PQs with dimensions, we obtain the general expression of URs

\[ AB \sim h^2G^\gamma c^\alpha w^\delta e^\epsilon \]  

(1)

where A and B are non-commutative PQs, x, y, z, w and u the unknown number, and e is the elementary charge. Applying the dimensional analysis [19] (here we use the LMTQ units [20]), the dimensions of A and B are expressed as


(2)


where L, M, t, T and Q are the dimensions of length, mass, time, temperature and electric charge separately, \( \alpha, \beta, \gamma, \delta, \epsilon, \zeta \) and \( \text{e1} \) and \( \text{e2} \) the known real number. The dimensions of \( h^2G^\gamma c^\alpha w^\delta e^\epsilon \) are

\[ [h^2G^\gamma c^\alpha w^\delta e^\epsilon] = [L^2][M][T^-1][T^{-2}][T^{-1}]^2 \times [L^2][M][T^{-2}][T^{-1}]^2 \times [L][T]^2 \times [T]^2 \times [Q]^1 \]  

By the dimensional analysis, we obtain


(3)

Solving the equation (4), we gain

\[ x = \frac{\alpha_1 + \alpha_2 + \beta_1 + \beta_2 + \gamma_1 + \gamma_2 + \delta_1 + \delta_2}{2}; \quad y = \frac{\alpha_1 + \alpha_2 - \beta_1 - \beta_2 + \gamma_1 + \gamma_2 - \delta_1 - \delta_2}{2}; \quad z = \frac{-3(\alpha_1 + \alpha_2 - \beta_1 - \beta_2 + 5\gamma_1 + \gamma_2 - 5\delta_1 + \delta_2)}{2}; \quad \]  

(5)

Thus we find the general expression of URs of two PQs

\[ AB \sim [h^{(\alpha_1 + \alpha_2 + \beta_1 + \beta_2 + \gamma_1 + \gamma_2 + \delta_1 + \delta_2)} \times [G^{(\alpha_1 + \alpha_2 - \beta_1 - \beta_2 + \gamma_1 + \gamma_2 - \delta_1 - \delta_2)} \times [c^{-(3 \alpha_1 + \alpha_2 + \beta_1 + \beta_2 + 5 \gamma_1 + \gamma_2 - 5 \delta_1 + \delta_2)} \times [k^{-\delta_1 - \delta_2}]e^{(\text{e1} + \text{e2})}]  

(6)

It shows that the product of two non-commutative PQs with dimensions is equivalent to the power product of the reduced Planck constant, gravitational constant, speed of light in vacuum, Boltzmann constant and elementary charge.

2.2 Basic relationship

Ordering \( \alpha_1 = \alpha_2 = \alpha; \beta_1 = \beta_2 = \beta; \gamma_1 = \gamma_2 = \gamma; \delta_1 = \delta_2 = \delta \). \(^1\)

\(^1\)Chen Wei-Zeng used L, M, T, 0 and Q indicated the dimension of length, mass, time, temperature and electric charge separately in [20].

\[\text{L}_p = \sqrt{hGc^2} \]

Instructing \( \gamma = 1, \alpha = \beta = \delta = \epsilon = 0 \), obtain Planck time

\[\text{t}_p = \sqrt{hGc^2}\]

Ordering \( \beta = 1, \alpha = \gamma = \delta = \epsilon = 0 \), obtain Planck mass

\[|A| = |B| = [L]^{\alpha_1}[M]^\delta[\tau]^\delta[Q]^\epsilon \]  

(7)

We obtain

\[ h(x + y + \delta)(x + y + \delta - (3\alpha - \beta + 5\gamma - 5\delta))e^{-(3\alpha - \beta + 5\gamma - 5\delta)} \]

(8)

where \( \text{A}_p \) and \( \text{B}_p \) are the corresponding Planck scale of A and B separately. Extracting the square root, we find the basic relationship

\[ A \sim \text{A}_p = h(x + y + \delta)q^{-(3\alpha - \beta + 5\gamma - 5\delta)}e^{-2\delta - 2\epsilon - x} \]  

(9)

The above relationship shows that any PQ with dimensions has a corresponding Planck scale which is equivalent to the power product of h, G, c, k and e.

2.3 General expression of URs of n PQs

Similarly considering n non-commutative PQs with dimensions, we have

\[ \prod_{i=1}^{n} |A_i| \sim h^nG^n c^n w^n e^n, i = 1, 2, 3, \ldots n \]  

(10)

where A1 is the PQ, A1 and A1+1 are non-commutative. The dimensions of \( \prod_{i=1}^{n} |A_i| \) are

\[ |\prod_{i=1}^{n} A_i| = |L|^\sum \alpha_i[M]^\sum \beta_i[T]^\sum \gamma_i[G]^\sum \delta_i[Q]^\sum \epsilon_i \]  

(11)

where \( \alpha_i, \beta_i, \gamma_i, \delta_i \) and \( \epsilon_i \) are known real number. By the dimensional analysis also, we find the general expression of URs of n PQs

\[ \prod_{i=1}^{n} A_i \sim h^n(A_{\alpha_1} + (\Sigma \beta_i)(\Sigma \gamma_i)(\Sigma \delta_i))^{\frac{1}{2}} \times [G^{(\Sigma \alpha_i - \Sigma \beta_i - \Sigma \gamma_i - \Sigma \delta_i)} \times [c^{-(3 \Sigma \alpha_i - \Sigma \beta_i + 5 \Sigma \gamma_i - 5 \Sigma \delta_i)} \times [k^{-\Sigma \delta_i}]e^{(\Sigma \epsilon_i)}]  

(12)

Certainly when \( n = 2 \), it become the general expression of URs (6). Ordering \( \alpha_i = \alpha_{i+1} = \alpha; \beta_i = \beta_{i+1} = \beta; \gamma_i = \gamma_{i+1} = \gamma; \delta_i = \delta_{i+1} = \delta \) and \( \epsilon_i = \epsilon_{i+1} = \epsilon \) in (12), A1 and A1+1 having the same dimensions, we obtain

\[ h^n(x + y + \delta)(x + y + \delta - (3\alpha - \beta + 5\gamma - 5\delta))e^{-n \delta \text{e}^{\text{e}}x} \]

(13)

Extracting the nth-root, we gain (9) again.

3. Planck scale

In this section, we obtain the Planck scale, and classify them.

3.1 Basic Planck scale

Ordering \( \alpha = 1, \beta = \gamma = \delta = \epsilon = 0 \) in (7) and (9), we obtain Planck length immediately

\[\text{L}_p = \sqrt{hGc^2} \]

Ordering \( \beta = 1, \alpha = \gamma = \delta = \epsilon = 0 \), obtain Planck mass
1. The Planck scale of any PQ is equivalent to the power product of Planck length, Planck mass, Planck time, Planck temperature and elementary charge.

2. The power product of non-commutative PQs is equivalent to the one of corresponding Planck scale.

3. Because many PQs have the same dimensions, they have the same Planck scale, for example.

4. Etc.

5. All the Planck scales can be divided into two categories. One is the basic Planck scale and derived Planck scale [14]. The other is the Fermi-Planck scale of its exponent is half integer such as \( L_p, T_p, M_p, P_p, \) and so on, the Bose-Planck scale whose exponents are integer such as \( Q_e, \rho_p, p, \) \( R_{\rho p}, T_{\rho p}, \) etc. and other-Planck scale such as the Planck wave function \( \psi_p \).

6. From (9), we obtain

\[
A_p = [h]^{a_i a_i} c^{-3 a_i} [\hbar]^{\beta_i} \gamma_i - 4 \delta_i \gamma_i - \beta_i \gamma_i \bar{c}^{\rho_i} \gamma_i c^{\gamma_i - 5 \gamma_i} [\hbar]^{\epsilon_i - 1} \kappa_i \hbar c^\epsilon
\]

\[
= \left[ [\hbar G/c]^\omega \right] \left[ \hbar G/c^\gamma \right] \left[ [\hbar G/c]^\gamma \right] \left[ \hbar G/c^\epsilon \right] \left[ \hbar G/c^\omega \right] \left[ \hbar G/c^\gamma \right] \left[ \hbar G/c^\epsilon \right] = L_p^2 M_p^2 T_p^2 Q_e^2
\]

The corresponding Planck scale of any PQ is equivalent to the power product of Planck length, Planck mass, Planck time, Planck temperature and elementary charge.

4.2 GRE

Considering all the non-commutative PQs, we find the GRE

\[
\Pi_{i=1}^n A_i^{a_i} \sim \Pi_{i=1}^n A_i^{a_i} \quad i = 1, 2, 3, \ldots n
\]

where \( A_i \) is the PQ, \( A_i \) and \( A_i' \) are non-commutative, \( a_i \) is the real number, and \( A_i \) is the corresponding Planck scale of \( A_i \). It shows that the power product of non-commutative PQs is equivalent to the one of corresponding Planck scale.

4.3 Proving GRE

We prove the GRE by the same way in 2.3. Considering \( n \) non-commutative PQs with \( a_i \) power, we have

\[
\Pi_{i=1}^n A_i^{a_i} \sim h^2 G^x c^y \kappa^z e^u
\]

The dimensions of \( \Pi_{i=1}^n A_i^{a_i} \) are

\[
\left( \Pi_{i=1}^n A_i^{a_i} \right) = \left( [\hbar G/c]^\omega \right) \left( [\hbar G/c]^\gamma \right) \left( [\hbar G/c]^\gamma \right) \left( [\hbar G/c]^\epsilon \right) \left( [\hbar G/c] \right) = L_p^0 M_p^0 T_p^0 Q_e^0
\]

Using the dimensional analysis too, we gain the general expression of URs of \( n \) PQs with \( a_i \) power

\[
\Pi_{i=1}^n A_i^{a_i} \sim \Pi_{i=1}^n A_i^{a_i} \quad i = 1, 2 \ldots n
\]

\[
\Pi_{i=1}^n A_i^{a_i} = L_p^0 M_p^0 T_p^0 Q_e^0
\]

4.4 Proving URs

Applying the GRE (15), we can prove the URs in Sec. 1.

\[
\Delta s \sim \sqrt{s} - t \sim s \sim \eta \sim s \sim \theta \sim s \sim \eta \sim s \sim \theta \sim s \sim \eta \sim s \sim \theta
\]

where \( A_i \) is the Planck scale of any given fluid perfect, and \( s_p = \sqrt{c^2 / G H G} = G \) its Planck volume density of entropy (from basic relationship (9)). Thus we find that there is no G on some URs right hand because it is reduced fitly.
5. No Singularity at Big Bang and SBH

In this section, we find the Big Bang UR and SBH UR by the GRE.

5.1 Big Bang UR

S.W. Hawking and R. Penrose proved that the universe originated the Big Bang singularity [21]. Many literatures discussed no singularity at the Big Bang and black holes with the quantum effect, please refer to [18] [22-25]. The one of the characteristic of Big Bang singularity is zero volume and limitless high temperature.

Then we can find the relationship of Big Bang temperature and its volume by the GRE (15)

\[ T_B V_B \sim T_P V_P = T_P L_P^3 = \hbar^2 G / \kappa c^2 \]  

(19)

where \( T_B \) is the Big Bang temperature, \( V_B \) its volume, and \( V_P = L_P^3 \) the Planck volume. This is the Big Bang UR. It shows that it is impossible to measure the Big Bang temperature and its volume simultaneously. When \( h \rightarrow 0 \), we obtain

\[ T_B V_B \sim 0 \]  

(20)

Because \( T_B > 0 \), we gain \( V_B \sim 0 \), the Big Bang volume is zero, thus the Big Bang singularity appears without the quantum effect. We suggest no singularity at the Big Bang with quantum effect. Substituting \( a = c \kappa T / 2 \hbar \) [26] into (19), we obtain

\[ a_B V_B \sim a_P V_P = hG / 2c \]  

(21)

where \( a_B \) is the Big Bang acceleration, and \( a_P = \sqrt{c^2 / hG} \) the Planck acceleration. It is the UR for Big Bang acceleration and its volume.

5.2 SBH UR

Similarly considering the SBH mass and its volume, we find

\[ M_{BH} V_{BH} \sim M_P V_P = M_P L_P^3 = h^2 G / c^4 \]  

(22)

where \( M_{BH} \) is the SBH mass, and \( V_{BH} \) its volume. It is the SBH UR. Also it is impossible to measure the SBH mass and volume simultaneously. When \( h \rightarrow 0 \), we obtain

\[ M_{BH} V_{BH} \sim 0 \]  

(23)

Because \( M_{BH} > 0 \), we have \( V_{BH} \sim 0 \), the volume is zero, the SBH singularity appears without quantum effect also. We also suggest no singularity in SBH with quantum effect. Taking \( M = \rho V \) to (22), we gain

\[ M_{BH}^3 / \rho_{BH} \sim h^2 G / c^4, \rho_{BH} V_{BH}^2 \sim h^2 G / c^4 \]  

(24)

where \( \rho_{BH} \) is the mass density of SBH. These are the URs for the mass density of SBH and its mass or volume.

6. Conclusion

In this paper, we investigate the URs with dimensions by the dimensional analysis. We find the following results.

1) The normal form of URs is discovered. The PQs are on the left hand of URs, and the physical constants such as the reduced Planck constant \( \hbar \), gravitational constant \( G \), speed of light in vacuum \( c \) and Boltzmann constant \( \kappa \) are on right hand. These power products of physical constants which are rewritten appear.

2) The general expression of URs is found. It shows that the product of two and \( n \) non-commutative PQs with dimensions is equivalent to the power product of \( h, G, c, \kappa \) and elementary charge \( e \).

3) The basic relationship is found. Any PQ with dimensions has a corresponding Planck scale, which is equivalent to the power product of \( h, G, c, \kappa \) and \( e \).

4) The Planck length \( L_P \), Planck time \( t_P \), Planck mass \( M_P \), Planck temperature \( T_P \), elementary charge \( Q_e \) (or Planck charge), Planck energy \( E_P \), Planck momentum \( P_P \), Planck curvature tensor \( R_{\mu \nu \rho} \), Planck energy density \( P \), Planck pressure \( P_P \), Planck force per unit area \( F_P \), Planck energy-momentum tensor \( T_{\mu \nu \rho} \) etc are obtained again. Many PQs have the same Planck scale because of the same dimensions such as \( \rho_P, P_P, f_P \) and \( T_{\mu \nu \rho} \).

5) All the Planck scales are divided into two categories. One is the basic Planck scale including \( L_P, t_P, M_P, T_P \) and \( Q_e \), and derived Planck scale such as \( E_P, P_P, \rho_P, f_P, R_{\mu \nu \rho}, T_{\mu \nu \rho} \). Planck wave function \( \psi_P \) and so on. The other is the Fermi-Planck scale its exponent is half integer such as \( L_q, t_q, M_q, T_q, E_q, P_q \) etc, the Bose-Planck scale whose exponents are integers such as \( Q_e, \rho_P, P_P, f_P, R_{\mu \nu \rho}, T_{\mu \nu \rho} \) etc, and Other-Planck scale such as Planck wave function \( \psi_P \).

6) The corresponding Planck scale of any PQ is proved to be equivalent to the power product of \( L_q, t_q, M_q, T_q \) and \( Q_e \).

7) The GRE is found and proved. It shows that the power product of the non-commutative PQs is equivalent to ones of corresponding Planck scale. The URs in Sec. 1 are proved by the GRE. \( G \) disappears on some URs right hand because of being reduced fifty.

8) The Big Bang UR concerning the temperature \( T_B \) of Big Bang and its volume \( V_B \) is found by the GRE. It suggests no singularity at the Big Bang with the quantum effect. The UR concerning Big Bang acceleration \( a_B \) and \( V_B \) is obtained. Similarly the SBH UR concerning the SBH mass \( M_{BH} \) and its volume \( V_{BH} \) is found; also no singularity is in SBH with quantum effect. The URs concerning the mass density \( \rho_{BH} \) of SBH and \( M_{BH} \) or \( V_{BH} \) is gained.

9) The GRE unifies all URs with dimensions. It is generalized, interesting and significant; any UR is its special case. Because depends on the dimension, GRE can’t obtain the factor and relation without dimensions.

References


