Generalized Relation of Unifying All Uncertainty Relations with Dimensions

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We propose the generalized relation to unify all the uncertainty relations (URs) with dimensions by the dimensional analysis. From normal forms of URs, an assumption is proposed which physical quantities have the formal symmetry with physical constants. Here we find the basic relation. Any physical quantity with dimension has a corresponding Planck scale and many physical quantities have the same Planck scales because of same dimensions. All Planck scales can be classified by two methods, one is the basic Planck scale and derived Planck scale, and another is Femi-Planck scale, Bose-Planck scale and Other-Planck scale. The basic relation can be rewritten as the ones of corresponding Planck scales. We find the generalized relation which the power products of physical quantities are equivalent to the ones of corresponding Planck scales. We also find the Big Bang UR between its temperature and volume by the generalized relation, and the Schwarzschild black holes (SBH) UR between its mass and volume. These suggest no singularity at Big Bang and in SBH with the quantum effect. We show that the generalized relation is generalized, interesting and significant.

1. Introduction

The Heisenberg uncertainty principle's [1] application [2, 3], development [4, 5] and experiment [6, 7] made great progress. These founded the firm foundation for it and extended its connotation. Now there are many uncertainty relations (URs) with dimensions:

$\Delta p\Delta r \geq \hbar$ [1]; $\Delta E\Delta t \geq \hbar$ [1]; $\delta t = \beta\, t_P^{2/3} t^{1/3}$ [8]; $\eta / s \geq 4\pi\hbar / \kappa$ [9]; $\Delta T\Delta X \sim L_S^2 \sim L_P^2 / c$ [10]; $\delta x\delta y\delta t \sim L_P^3 / c$ [11]; $L_{\mu\nu} \sim \sqrt{L_P L}$ [12]; $\varepsilon(Q)\eta(P) + \varepsilon(Q)\sigma(P) + \sigma(Q)\eta(P) \geq \hbar / 2$ [7]; $(\delta t)(\delta r)^3 \geq \pi r^2 L_P^2 / c$ [13], etc.

where Δp is the momentum fluctuation, Δr is the position momentum, ħ is the reduced Planck constant; ΔE is the energy fluctuation, Δt is the time fluctuation; δt is the time fluctuation, β is an order one constant, $t_P = \sqrt{\hbar G/c^5}$ is Planck time, G is the gravitational constant, c is the speed of light, t is the time; η is the ratio of shear viscosity of a given fluid perfect, s is its volume density of entropy, κ is the Boltzmann constant; ΔT is the time-like, ΔX is its space-like, L_S is the string scale, $L_P = \sqrt{\hbar G/c^3}$ is Planck length; δx, δy, δt are the position fluctuation and time fluctuation separately; $L_{\mu\nu}$ is the transverse length, L is the radial length; Q is the position of a mass, $\varepsilon(Q)$ is the root-mean-square error, P is its momentum, $\eta(P)$ is the root-mean-square disturbance, $\sigma(P)$ is the standard deviation; δt and δr are the sever space-time fluctuations of the constituents of the system at small scales, and r is the radius of globular computer.

So there are two problems: (i) Why hasn't G on some formulas right hand? (ii) Whether has the unitive form for them? In this paper, we solve that G disappears because of being reduced fitly and the unitive form is the generalized relation. Moreover, for the origin and development of Planck length, Planck time, Planck mass $M_P = \sqrt{\hbar c/G}$, Planck energy $E_P = \sqrt{\hbar c^5/G}$ and Planck temperature $T_P = \sqrt{\hbar c^5/\kappa^2 G}$, please refer to the literature [14-18].

This paper is organized as follows. In Sec. 2, we propose an assumption, and derive the basic relation. In Sec. 3, we obtain the Planck scales and classify them. In Sec. 4, we prove the basic relation being rewritten as the one of corresponding Planck scales, find the generalized relation, and prove the URs in Sec. 1. In Sec. 5, we find the Big Bang UR and SBH UR. We conclude in Sec. 6.

2. An Assumption and Basic relation

In this section, we propose an assumption, and derive the basic relation.

2.1 An assumption and basic relation

Observing these URs, we can discover the physical constants such as ħ, G, c and κ on the right hand and the physical quantities on left hand. We rewrite them as

$\Delta p\Delta r \geq \hbar^1$; $\Delta E\Delta t \geq \hbar^1$; $\delta t / \beta t^{1/3} = t_P^{2/3} = \hbar^{1/3}G^{1/3}c^{-5/3}$; $\eta / 4\pi s \geq \hbar\kappa^{-1}$; $\Delta T\Delta X \sim L_S^2 \sim L_P^2 / c = \hbar G c^{-4}$; $\delta x\delta y\delta t \sim L_P^3 / c = \hbar^{3/2}G^{3/2}c^{-11/2}$; $L_{\mu\nu} / \sqrt{L} \sim \sqrt{L_P} = \hbar^{1/4}G^{1/4}c^{-3/4}$; $2[\varepsilon(Q)\eta(P) + \varepsilon(Q)\sigma(P) + \sigma(Q)\eta(P)] \geq \hbar^1$; $(\delta t)(\delta r)^3 / \pi r^2 \geq L_P^2 / c = \hbar G c^{-4}$, etc.

Therefore the physical constants appear power products on the right hand. These are their normal form. Applying the π law [19], any physical quantity can be expressed as the power products of basic ones. Using the five units, we obtain

$$A = r^{\alpha} m^{\beta} t^{\gamma} T^{\delta} Q^{\varepsilon} \quad (1)$$

where *A* is any physical quantity, *r*, *m*, *t*, *T* and *Q* are the length, mass, time, temperature and electric charge separately, α, β, γ, δ and ε are the real number. From the normal form of above URs, we can assume

$$r^{\alpha} m^{\beta} t^{\gamma} T^{\delta} Q^{\varepsilon} \sim \hbar^{x} G^{y} c^{z} \kappa^{w} e^{u} \quad (2)$$

where *x*, *y*, *z*, *w* and *u* are the unknown number, and e is the elementary charge. (2) shows that the physical quantities have the beautiful formal symmetry with the physical constants. By the dimensional analysis [19], we obtain

$$[L]^{\alpha}[M]^{\beta}[t]^{\gamma}[T]^{\delta}[Q]^{\varepsilon} \sim \{[L^2][M][t^{-1}]\}^{x}\{[L^3][M^{-1}][t^{-2}]\}^{y}\{[L][t^{-1}]\}^{z} \{[L^2][M][t^{-2}][T^{-1}]\}^{w}\{[Q]\}^{u} \quad (3)$$

where L, M, t, T and Q are the dimensions of length, mass, time, temperature and electric charge separately. Solving (3) we gain

$x = (\alpha+\beta+\gamma+\delta) / 2$, $y = (\alpha-\beta+\gamma-\delta) / 2$,

$z = -(3\alpha-\beta+5\gamma-5\delta) / 2$, $w = -\delta$, $u = \varepsilon$

Thus we find the basic relation.

$$r^{\alpha} m^{\beta} t^{\gamma} T^{\delta} Q^{\varepsilon} \sim [\hbar^{(\alpha+\beta+\gamma+\delta)} G^{(\alpha-\beta+\gamma-\delta)} c^{-(3\alpha-\beta+5\gamma-5\delta)} \kappa^{-2\delta} e^{2\varepsilon}]^{1/2} \quad (4)$$

It shows that the power products of the length, mass, time, temperature and electric charge which express any physical quantity are equivalent to the one of ħ, G, c, κ and e.

3. Planck Scales

In this section, we obtain the Planck scales, and classify them.

3.1 Basic Planck scale

Ordering $\alpha = 1$, $\beta = \gamma = \delta = \varepsilon = 0$ in (4), we obtain Planck length immediately

$$r_P = L_P = \sqrt{\hbar G/c^3}$$

Instructing $\gamma = 1$, $\alpha = \beta = \delta = \varepsilon = 0$, obtain Planck time

$$t_P = \sqrt{\hbar G/c^5}$$

Ordering $\beta = 1$, $\alpha = \gamma = \delta = \varepsilon = 0$, obtain Planck mass

$$m_P = M_P = \sqrt{\hbar c/G}$$

Instructing $\delta = 1$, $\alpha = \beta = \gamma = \varepsilon = 0$, obtain Planck temperature

$$T_P = \sqrt{\hbar c^5/\kappa^2 G}$$

Ordering $\varepsilon = 1$, $\alpha = \beta = \gamma = \delta = 0$, obtain elementary charge (or Planck charge)

$$Q_P = Q_e = e$$

These are the basic Planck scale.

3.2 Derived Planck scales

From (4), the corresponding Planck scale A_P of *A* is

$$A_P = [\hbar^{(\alpha+\beta+\gamma+\delta)} G^{(\alpha-\beta+\gamma-\delta)} c^{-(3\alpha-\beta+5\gamma-5\delta)} \kappa^{-2\delta} e^{2\varepsilon}]^{1/2} \quad (5)$$

Consequently any physical quantity with dimension has a corresponding Planck scale.

$$A \sim A_P \quad (6)$$

For example

Planck energy E_P

$$[E_P] = [L^2][M][T^{-2}],\ E_P = \sqrt{\hbar c^5/G}$$

Planck momentum P_P

$$[P_P] = [L][M][T^{-1}],\ P_P = \sqrt{\hbar c^3/G}$$

Planck curvature tensor $R_{\mu\nu P}$

$$[R_{\mu\nu P}] = [L^{-2}],\ R_{\mu\nu P} = c^3 / \hbar G$$

Because many physical quantities have the same dimensions, they have the same Planck scales, for example

Planck energy density ρ_P

$$[\rho_P] = [L^{-1}][M][T^{-2}],\ \rho_P = c^7 / \hbar G^2$$

Planck pressure p_P

$$[p_P] = [L^{-1}][M][T^{-2}],\ p_P = c^7 / \hbar G^2$$

Planck force per unit area f_P

$$[f_P] = [L^{-1}][M][T^{-2}],\ f_P = c^7 / \hbar G^2$$

Planck energy- momentum tensor $T_{\mu\nu P}$

$$[T_{\mu\nu P}] = [L^{-1}][M][T^{-2}],\ T_{\mu\nu P} = c^7 / \hbar G^2$$

Etc. These are belonging to the derived Planck scale.

3.3 Classifications

We can classify all the Planck scales by two methods. First are basic Planck scale and derived Planck scale. Second are that One's power is the half integer, call it Femi-Planck scale, such as L_P, t_P, M_P, T_P, E_P, P_P, etc; another is the integer, call it Bose-Planck scale, such as Q_e, ρ_P, p_P, f_P, $R_{\mu\nu P}$, $T_{\mu\nu P}$, etc; others call Other-Planck scale, such as the Planck wave function ψ_P

$$[\psi_P] = [L^{-3/2}],\ \psi_P = (\hbar G / c^3)^{-3/4}$$

4. Generalized Relation

In this section, we prove that (4) can be rewritten as the one of corresponding Planck scales, find the generalized relation, and prove the URs in Sec. 1.

4.1 Proof

The basic relation (4) can be rewritten as the one of corresponding Planck scales

$$r^{\alpha} m^{\beta} t^{\gamma} T^{\delta} Q^{\varepsilon} \sim L_P^{\alpha} M_P^{\beta} t_P^{\gamma} T_P^{\delta} Q_e^{\varepsilon} \quad (7)$$

We prove (7) now. From (4), we obtain

$$r^{\alpha} m^{\beta} t^{\gamma} T^{\delta} Q^{\varepsilon} \sim [\hbar^{\alpha} G^{\alpha} c^{-3\alpha}]^{1/2}[\hbar^{\beta} G^{-\beta} c^{\beta}]^{1/2}[\hbar^{\gamma} G^{\gamma} c^{-5\gamma}]^{1/2}[\hbar^{\delta} G^{-\delta} c^{5\delta}]^{1/2} \kappa^{-\delta} e^{\varepsilon}$$

$$= [\hbar G / c^3]^{\alpha/2}[\hbar c / G]^{\beta/2}[\hbar G / c^5]^{\gamma/2}[\hbar c^5 / \kappa^2 G]^{\delta/2} e^{\varepsilon}$$

$$= L_P^{\alpha} M_P^{\beta} t_P^{\gamma} T_P^{\delta} Q_e^{\varepsilon}$$

Thus the basic relation is equivalent to the one of corresponding Planck scales.

4.2 Generalized relation

Considering all the physical quantities, we find

$$\prod_{i=1}^{n} A_i^{\alpha_i} \sim \prod_{i=1}^{n} A_{iP}^{\alpha_i}; \quad i = 1, 2, 3 \ldots n \qquad (8)$$

where A_i is the physical quantity, α_i is the real number, and A_{iP} is the corresponding Planck scale. This is the generalized relation. It shows that the power products of physical quantities are equivalent to the ones of corresponding Planck scales.

4.3 Proving URs

Applying the generalized relation (8), we can prove the URs in Sec.1.

$\Delta p \Delta r \sim P_P L_P = \sqrt{\hbar c^3/G}\sqrt{\hbar G/c^3} = \hbar$; $\Delta E \Delta t \sim E_P t_P = \sqrt{\hbar c^5/G}\sqrt{\hbar G/c^5} = \hbar$; $\delta t / t^{1/3} \sim t_P / t_P^{1/3} = t_P^{2/3}$; $\eta / s \sim \eta_P / s_P = \sqrt{c^9/\hbar G^3} / \sqrt{c^9\kappa^2/\hbar^3 G^3} = \hbar / \kappa$; $\Delta T \Delta X \sim t_P L_P \sim \hbar G / c^4 = L_P^2 / c \sim L_S^2$; $\delta x \delta y \delta t \sim L_P^2 t_P = L_P^3 / c$; $L_{\mu\nu} / \sqrt{L} \sim L_P / \sqrt{L_P} = \sqrt{L_P}$; $\varepsilon(Q)\eta(P) + \varepsilon(Q)\sigma(P) + \sigma(Q)\eta(P) \sim \sqrt{\hbar G/c^3}\sqrt{\hbar c^3/G} = \hbar$; $(\delta t)(\delta r)^3 / r^2 \sim t_P L_P^3 / L_P^2 = L_P^2 / c$, etc.

where $\eta_P = \sqrt{c^9/\hbar G^3}$ is the Planck ratio of shear viscosity of a given fluid perfect, and $s_P = \sqrt{c^9\kappa^2/\hbar^3 G^3}$ is its Planck volume density of entropy (from formula (5)). Thus we find that there hasn't G on some formulas right hand because it is reduced fitly.

5. No singularity at Big Bang and SBH

In this section, we find the Big Bang UR and SBH UR by the generalized relation.

5.1 Big Bang UR

S.W. Hawking and R. Penrose proved that the universe originated the Big Bang singularity [20]. Many literatures discussed no singularity at the Big Bang and black holes with the quantum effect, please refer to [18] [21-24]. The one of the characteristic of Big Bang singularity is zero volume and limitless high temperature.

Then we can find the relation of Big Bang temperature and its volume by the generalized relation

$$T_B V_B \sim T_P V_P = T_P L_P^3 = \hbar^2 G / \kappa c^2 \qquad (9)$$

where T_B is the Big Bang temperature, V_B is its volume, and $V_P = L_P^3$ is the Planck volume. This is the Big Bang UR. It shows that it is impossible to measure the Big Bang temperature and its volume simultaneously. When $\hbar \to 0$, we obtain

$$T_B V_B \sim 0 \qquad (10)$$

Because $T_B > 0$, we gain $V_B \sim 0$, the Big Bang volume is zero, thus the Big Bang singularity appears without the quantum effect. We suggest no singularity at the Big Bang with quantum effect. Substituting $a = c\kappa T / 2\pi\hbar$ [25] into (9), we obtain

$$a_B V_B \sim a_p V_p = \hbar G / 2\pi c \qquad (11)$$

where a_B is the Big Bang acceleration, and $a_p = \sqrt{c^7/\hbar G}$ is the Planck acceleration. It is the UR between Big Bang acceleration and its volume.

5.2 SBH UR

Similarly considering the mass and volume of SBH, we find

$$M_H V_H \sim M_P V_P = M_P L_P^3 = \hbar^2 G / c^4 \qquad (12)$$

where M_H is the SBH mass, and V_H is its volume. It is the SBH UR. Also it is impossible to measure the SBH mass and volume simultaneously. When $\hbar \to 0$, we obtain

$$M_H V_H \sim 0 \qquad (13)$$

Because $M_H > 0$, we have $V_H \sim 0$, the volume is zero, the SBH singularity appears without quantum effect also. We also suggest no singularity in SBH with quantum effect. Taking $M = \rho V$ to (12), we gain

$$M_H^2 / \rho_H \sim \hbar^2 G / c^4, \; \rho_H V_H^2 \sim \hbar^2 G / c^4 \qquad (14)$$

where ρ_H is the mass density of SBH. These are the URs between the mass density of SBH and its mass or volume.

6. Conclusion

In this paper, we investigate the relations between the physical quantities and the physical constants by the dimensional analysis. We find the following results.

1) The basic relation is found. The power products of the length *r*, mass *m*, time *t*, temperature *T* and electric charge *Q* which express any physical quantity are equivalent to the one of the reduced Planck constant ħ, gravitational constant G, speed of light c, Boltzmann constant κ and elementary charge e.

2) Any physical quantity with dimension has a corresponding Planck scale. The Planck length L_P, Planck time t_P, Planck mass M_P, Planck temperature T_P, elementary charge Q_e (or Planck charge), Planck energy E_P, Planck momentum P_P, Planck curvature tensor $R_{\mu\nu P}$, Planck energy density ρ_P, Planck pressure p_P, Planck force per unit area f_P, Planck energy-momentum tensor $T_{\mu\nu P}$ etc are found. Many physical quantities have the same Planck scales because of the same dimensions.

3) All the Planck scales are classified by two methods. First are the basic Planck scales including L_P, t_P, M_P, T_P and Q_e and derived Planck scales such as E_P, P_P, ρ_P, p_P, f_P, $R_{\mu\nu P}$, $T_{\mu\nu P}$, Planck wave function ψ_P etc. Second are the Femi-Planck scale which power is the half integer such as L_P, t_P, M_P, T_P, E_P, P_P, etc, the Bose-Planck scale which power is the integer such as Q_e, ρ_P, p_P, f_P, $R_{\mu\nu P}$, $T_{\mu\nu P}$, etc and the Other-Planck scale which power is others such as ψ_P.

4) The basic relation can be rewritten as the one of corresponding Planck scales. The power products of *r*, *m*, *t*, *T* and

Q are equivalent to the one of L_P, t_P, M_P, T_P and Q_e.

5) The generalized relation is found. It shows that the power products of the physical quantities are equivalent to the ones of corresponding Planck scales. The URs in Sec. 1 are proved by the generalized relation. G disappears on some URs because of being reduced fitly.

6) The Big Bang UR between its temperature T_B and volume V_B is found by the generalized relation. It suggests no singularity at the Big Bang with the quantum effect. The UR between Big Bang acceleration a_B and its volume V_B is obtained. Similarly the SBH UR between its mass M_H and volume V_H is found; also no singularity is in SBH with quantum effect. The URs between the mass density ρ_H of SBH and it's M_H or V_H is gained.

7) The generalized relation unifies all URs with dimensions. It is generalized, interesting and significant; any UR is its special case. Generalized relation includes the quantum gravity such as the Big Bang UR and SBH UR. No prerequisite for these relations, they are better than other theories to remove the singularity of Big Bang and black hole. Because depends on the dimensions, generalized relation can't obtain the factor and relation without dimensions.

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