Cooperstock is wrong: The Dark Matter is necessary

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(Dated: February 22, 2015)

Abstract

"In a series of papers Fred Cooperstock and his collaborators showed that the application of general relativity is sufficient to explain the velocity profile of galaxies", wrote Stefan. I argue with it.

PACS numbers:
I. HISTORY OF PROBLEM

Dear readers, the picture of Physics lefts you in confusion. The prime example is refutation of black holes in 2014, Phys.Lett.B 738, 617 by Laura, a Professor. I have arguments against her paper, but perhaps I am the only one, who is worried. They keep bringing things forward, which are thought to be refuted and over refuted. Another example of mind blowing is the Dr. Cooperstock. First his attempt was to deny the Standards of Metrology (within ”Energy Localization hypothesis”). I have arguments against his idea. Then he came up with another mind abuse: absence of long detected Dark Matter. In the following I am defending the Dark Matter from the nihilistic aggression of Dr. Cooperstock. Speaking of nihilism, the most grim picture is in Quantum Mechanics of Niels Bohr. In 2015 they have ”proved” in elitist ”Nature”, that Schrödinger’s Cat is real. Thus, the world does not exist: a thing can not both be and not be. It is very convenient now: if even a grain of sand is crazy hallucination (like the ”proven” ”reality” of undead cat), then this non-existent grain needs no divine (loved, but more often hated) Creator. The reason of delusion: they have missed an intelligent factors, e.g. evil spirits, which very often act on the measuring device. Recall the wrong alarms in atomic armies.

Quantum Bohr tells: no nature, until you look at it. But how can I look at nature, if there is no nature in the first place? Respect Quantum theory of David Bohm.

In 2005 the Cooperstock has published in arXiv his no-Dark-Matter theory [1]. The same year came the responce from scientific community in form of critical papers [2]. In 2008 year came out work in scientific journal [3], which says, that the Cooperstock missed to put in his calculation an additional function \( W = r \exp(h(r, z)) \). Sadly, but this issue was not discussed in Cooperstock’s 2012 paper in scientific journal [4]. But this is not the problem: first of all, the function \( u \) before \( dz^2 \) can be put to unity because of \( R = R(r, z) \), \( Z = Z(r, z) \) transformations. Secondly, the combinations of Einstein’s equations up to \( G^1 \) precision do produce the same equations for \( N \), which the Cooperstock uses. Even for small, but non-zero \( h \). Note, what from axial rotation in near flat spacetime, one can conclude, that the \( N \) has \( G^{1/2} \) order, whereas other small functions are of \( G^1 \) order, thus, e.g. \( N \gg h \).
II. SINGULAR DISK

Cooperstock has axially rotating cloud of pressure-free dust. Thus, the stationary solution is the dust, which has felt into the rotating plane \( z = 0 \). The stationary solution (even in co-moving coordinates) does not have dust outside the attracting \( z = 0 \) plane. But because dust-particles are considered point-like, this plane is singular (see also [2]). Singularity violates the weak fields approximation of Cooperstock.

The issue of wrong metric (1) (in [1]) (see the arguments below) has caused the paper [2], which tells, that the energy conditions for this thin plane of matter are violated: there is no dust, but the exotic matter. Stars are not exotic matter. I thought so.

III. SIMPLIFIED PROOF, THAT COOPERSTOCK IS INCOMPLETE

In Kerr spacetime is nonzero frame dragging with \( g_{t\phi} = -N \), however the coordinates are not comoving. Thus, the real velocity of observer is zero, however from Cooperstock’s formula it would be \( V = N/r \). Thus, the larger is the frame dragging the larger is the difference between Cooperstock’s theory and reality. Therefore, the large frame dragging (as it seems from calculations below) is very bad for Cooperstock. P.S. Kerr’s B.-L. coordinates are not co-moving, because observer, which has certain B.-L. coordinates is stationary (as seen from a distant observer).

Now take not the Kerr, but the rotating Milky Way in Cooperstock approach. In coordinates of not-comoving observer, ”curvature coordinates” there is \( g_{t\phi} = X \).

What sign has \( X \)? The near circular orbits have following connection between 4-velocity components

\[
1 = g_{tt}(t_s)^2 - g_{\phi\phi}(\phi_s)^2 - g_{zz}(z_s)^2 + 2g_{t\phi}t_s \phi_s,
\]

where \( g_{tt} > 0, g_{\phi\phi} > 0, g_{zz} > 0 \). Let us divide it by \((t_s)^2\), and put \( \Omega = f_s/t_s \)

\[
1/t_s^2 = g_{tt} - g_{\phi\phi}\Omega^2 - g_{zz}(z_s/t_s)^2 + 2g_{t\phi}\Omega
\]

Suppose now \( g_{t\phi} > 0 \). To have second solution \( \Omega_2 < 0 \) one must have \( 1/t_s^2 < g_{tt} - g_{zz}(z_s/t_s)^2 \).

Now it is clear, that first solution \( \Omega_1 > 0 \) is larger, than second one: \( \Omega_1 > |\Omega_2| \): it will be more difficult to nullify positive term \( g_{tt} - g_{zz}(z_s/t_s)^2 - 1/t_s^2 \) when this part \( 2g_{t\phi}\Omega > 0 \) does not help.
Conclusion: $X > 0$.

Now make "local" coordinate transformation into "moving" coordinates: $\Phi = \phi - \Omega t$, where $\Omega > 0$ is step-function: within $|z| < \epsilon, r_1 < r < r_1 + \epsilon, \epsilon \ll 1$ the $\Omega = \text{const}$, otherwise the $\Omega$ is zero. Thus, you will get $g_{t,\Phi} = X - r^2 \Omega$. Now the Cooperstock has $g_{t,\Phi} = -N$, where $N = \omega r^2 > 0$. Thus, the observable rotation is $\Omega = \omega + X/r^2 > \omega$. Therefore, the stronger the Frame Dragging ($X \to \infty$), the more Cooperstock has missed the Truth: $\Omega/\omega \neq 1$.

From the text below it turns out, that $X/r = 2N r - |N_r|$ Thus, the observable velocity is

$$U = V + X/r = 3N r - |N_r|,$$

Thus, Cooperstocks $V = N/r$ formula should be replaced with formula for $U$. I think, his beautiful Figure 1 will remain, but the density profile, and, thus, the Milky Way mass, will be changed 4 times.

A. Why this is over-simplified derivation?

The local transformation with a step function will not produce from curvature coordinates the Cooperstock’s co-moving metric (1). Thus, the application of General Relativity will not be justified for the step-function talk. Therefore the above formula for $U$ is not correct. See in the following the alternative formula for $U$, which could tell, that Dark Matter is necessary.

IV. MY CONTRIBUTION

Let me explain to you, what are the curvature coordinates $(t, r, z, \phi)$. They are not comoving coordinates. It is those coordinates, in which the Earth and the galaxy have fixed coordinate values. A stationary observer (those coordinates are kept fixed), does not experience the centrifugal acceleration.

See arXiv:astro-ph/0507619. Take the curvature coordinates, when make the coordinate transformation $\Phi = \phi - \omega(r, z) t$, the new coordinates will be co-moving with matter. They
will contain following undiagonal terms in metric: \( g_{\Phi r}, g_{\Phi z}, g_{t z}, g_{t r}, g_{z r} \), which are time dependent. There is also \( g_{\Phi t} \). Let us try to make from it the Cooperstock’s Eq.(1), hereby so, that comoving feature stays. Also shall stay the axial symmetry. Therefore \( f = \Phi + q(r, z) \). Latter does not change the above undiagonal elements, thus one shall transform the time: \( T = t k(r, z) \). We have 2 unknown functions \( q \) and \( k \). Can they eliminate all \( g_{\Phi r}, g_{\Phi z}, g_{t z}, g_{t r} \)? It is not possible, because the \( g_{\Phi r} \) has the \( t \) dependence.

P.S. How the general form of stationary, axial-symmetric spacetime [3] was derived? Why there is only one undiagonal element?

But to really get the Eq.(1) the term \( g_{\Phi t} \approx -N \) will get modified in process. So let us write the modification

\[
N = n + X,
\]

where \( n \) comes from centrifugal acceleration of co-movement, the \( X \) is from necessary coordinate transformations. The Cooperstock has some thing, which he believed is the star velocity in not comoving coordinates:

\[
V = N/r.
\]

Thus, the real, observable star velocity is

\[
U = n/r = V - \frac{X}{r}.
\]

If the Cooperstock were right, we have following velocity of the star in not-comoving system:

\[
N/r.
\]

Therefore to stay on same orbit in opposite movement one shall have velocity in comoving system

\[
Y = 2N/r.
\]

But I have calculated (see Appendix) in assumption, that \( z = 0 \) state is stable enough:

\[
Y = |N_r|,
\]

where \( N_r \) is the partial derivative \( \partial N/\partial r \). Thus, the difference between results is the anomalous \( X \)!

\[
X/r = 2N/r - |N_r|.
\]
Thus, the
\[ U = |N_r| - \frac{N}{r}. \]

One can come to the same conclusion, but different way. The star has velocity \( V = N/r \) in not-comoving coordinates. The star emits test-particle in opposite direction, it found to have \( v = |N_r| \) of velocity. Thus, using the simple rule of velocities, one have, that velocity of test-particle in not-comoving coordinates is \( W = v - V \). To find anomaly, which we have called "frame dragging", we compare the velocities of clockwise movement and counterclockwise movement: \( X/r = V - W = 2N/r - |N_r| \). Thus, we have found the same formula.

Let us check. In the \( V = \text{const} \) state holds roughly \( N = br \), thus \( U = 0 \). That is much less than \( V \), thus, there is no flat plateau in Cooperstock’s theory. In the \( V \sim r \) regime holds roughly \( N = kr^2 \), thus \( U = kr \). This corresponds to the linear law near the core of a galaxy.

V. APPENDIX

Take circular orbits of a massive test particles. We have metric (15) in file cooper.pdf. Is taken \( w = 0 \). Because metric is \( \phi, t \)-independent, then from the use of Killing’s vectors we have
\[ u_t = E = \text{const}, \quad u_\phi = L_z = \text{const}. \]

Hopefully, these constants you can get from following equations
\[ u_\nu u^\nu = 1, \]
\[ \frac{dz}{ds} = 0, \quad \frac{dp_z}{ds^p} = 0, \]
and
\[ \frac{dr}{ds} = 0, \quad \frac{dp_r}{ds^p} = 0. \]
For all \( p = 1, 2, \ldots \). The higher derivatives are required because the motion shall be stable.

After that you can find the angular velocity as
\[ \Omega = \frac{u^\phi}{u^t}. \]

Then extract by Cooperstock’s "local transformation" the \( \omega \) and compare the two angular velocities in not comoving coordiantes: \( |\omega + 0| \) and \( |\omega + \Omega| \), the clockwise and counterclockwise respectfully. The difference is the Frame Dragging.
I have got: $\Omega = -|N_r|/r$ and $\omega = N/r^2$. The difference $\omega - |\Omega + \omega| = 2\omega + \Omega = 2N/r^2 - |N_r|/r$. There are two $G^{1/2}$ quantities, the difference can be $G^2$ small quantity. See: $G^2 = G^{1/2} - G^{1/2}$. The $\Omega = -|N_r|/r$ is gotten in assumption, that equatorial movement $z = 0$ holds for long enough. But in 2012 paper is talk, that there is asymmetry in relation to $z = 0$ plane, if I understood it correctly. Thus, a test-particle would change its $z = 0$ state. If holds $2N/r^2 - |N_r|/r = 0$ then $N(r) = k r^2$, where $k$ is constant. Velocity $V = N/r = k r$, which is good form for Milky Way core (see linear law in Fig.1, 2005 paper). Cooperstock has also $N_{rr} + N_{zz} = N_r/r$. Its solution in form $N = k r^2 \exp(-z)$ is good in the core: for small $r$.

VI. POSSIBLE SOURCE OF COOPERSTOCK’S DELUSION

Theorem: The metric in curvature coordinates $(t, r, \phi)$ can be stationary and axial-symmetric. But in co-moving coordinates $(T, R, \Phi)$ it can not.

Proof: For simplicity let us consider $(1+2)$ Gravity, which flat spacetime metric is

$$ds^2 = dt^2 - r^2 d\phi^2 - dr^2.$$

The transformation, which conserves co-movement ($u^\Phi = 0$), and absence of radial motion ($u^R = 0$) and axial symmetry can only be following

$$\Phi = \phi + h(r) - w(r) t, \quad R = r, \quad T = t/x(r) + y(r) + \phi$$

Turns out, that $x(r) = -1/w(r)$ and, thus, the $T$ is not changing for given star. Therefore it is not the time. Thus, the transformation in (Time+Space) Gravity is impossible.

VII. THE PAPER OF FAROOK ET AL [5]

It could be of value for Cooperstock: they use unmodified General Relativity with rotation curves and they do need Dark Matter. This paper greatly supports my book: [6].

VIII. VAN STOCKUM DUST

In Wikipedia the metric is $z$-independent. Thus, it is the solution has infinite mass: it is everywhere on $z$-axis. The rigid rotation would imply, that there is superluminal velocity at
some distance: $V = \omega r > c$. But the space is curved, so the allowed velocity can be. Can the solution be attached to vacuum outside, at surface $r = R$? Note, that transformation $\Phi = \phi + \omega t$, where rigid rotation has $\omega = const$ does not vanish the $g_{t\Phi}$. But transformation with $\omega(r)$ does not correspond to reality. Therefore, the larger the frame dragging the more Cooperstock missed the Truth.


