Cooperstock is wrong: The Dark Matter is necessary

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Abstract

"In a series of papers Fred Cooperstock and his collaborators showed that the application of general relativity is sufficient to explain the velocity profile of galaxies", wrote Stefan. I argue with it. First, let me say following. In Kerr spacetime is nonzero frame dragging with $g_{t\phi} = -N$, however the coordinates are not comoving. Thus, the real velocity of observer is zero, however from Cooperstock's formula it would be $V = N/r$. Thus, the larger is the frame dragging the larger is the difference between Cooperstock's theory and reality. Therefore, the large frame dragging (as it seems from calculations below) is very bad for Cooperstock.

PACS numbers:

I. HISTORY OF PROBLEM

In 2005 the Cooperstock has published in arXiv his no-Dark-Matter theory [1]. The same year came the responce from scientific community in form of critical papers [2]. In 2008 year came out work in scientific journal [3], which says, that the Cooperstock missed to put in his calculation an additional function $W = r\exp(h(r,z))$. Sadly, but this issue was not discussed in Cooperstock's 2012 paper in scientific journal [4]. But this is not the problem: first of all, the function u before dz^2 can be put to unity because of $R = R(r,z)$, $Z = Z(r,z)$ transformations. Secondly, the combinations of Einstein's equations up to G^1 precision do produce the same equations for N, which the Cooperstock uses. Even for small, but non-zero h. Note, what from axial rotation in near flat spacetime, one can conclude, that the N has $G^{1/2}$ order, whereas other small functions are of G^1 order, thus, e.g. $N \gg h$.

II. MY CONTRIBUTION

Let me explain to you, what are the curvature coordinates (t, r, z, ϕ). They are not comoving coordinates. It is those coordinates, in which the Earth and the galaxy have fixed coordinate values. A stationary observer (those coordinates are kept fixed), does not experience the centrifugal acceleration.

See arXiv:astro-ph/0507619. Take the curvature coordinates, when make the coordinate transformation $\Phi = \phi + \omega(r,z)\,t$, the new coordinates will be co-moving with matter. They will contain following undiagonal terms in metric: $g_{\Phi\, r}$, $g_{\Phi\, z}$, $g_{t\, z}$, $g_{t\, r}$, $g_{z\, r}$, which are time dependent. There is also $g_{\Phi\, t}$ Let us try to make from it the Cooperstock's Eq.(1), hereby so, that comoving feature stays. Also shall stay the axial symmetry. Therefore $f = \Phi + q(r,z)$. Latter does not change the above undiagonal elements, thus one shall transform the time: $T = t\,k(r,z)$. We have 2 unknown functions q and k. Can they eliminate all $g_{\Phi\, r}$, $g_{\Phi\, z}$, $g_{t\, z}$, $g_{t\, r}$? It is not possible, because the $g_{\Phi\, r}$ has the t dependence.

P.S. How the general form of stationary, axial-symmetric spacetime [3] was derived? Why there is only one undiagonal element?

But to really get the Eq.(1) the term $g_{\Phi\, t} \approx -N$ will get modified in process. So let us write the modification

$$N = n + X\,,$$

where n comes from centrifugal acceleration of co-movement, the X is from necessary coordinate transformations. The Cooperstock has some thing, which he believed is the star velocity in not comoving coordinates:

$$V = N/r\,.$$

Thus, the real, observable star velocity is

$$U = n/r = V - \frac{X}{r}\,.$$

If the Cooperstock were right, we have folloving velocity of the star in not-comoving system:

$$N/r\,.$$

Therefore to stay on same orbit in opposite movement one shall have velocity in comoving system

$$Y = 2N/r\,.$$

But I have calculated (see Appendix) in assuption, that $z = 0$ state is stable enough:

$$Y = N_r\,,$$

where N_r is the partial derivative $\partial N/\partial r$. Thus, the difference between results is the anomalous X!

$$X/r = 2N/r - N_r\,.$$

Thus, the

$$u = N_r - \frac{N}{r}\,.$$

One can come to the same conclusion, but different way. The star has velocity $V = N/r$ in not-comoving coordinates. The star emits test-particle in opposite direction, it found to have $v = N_r$ of velocity. Thus, using the simple rule of velocities, one have, that velocity of test-particle in not-comoving coordinates is $W = v - V$. To find anomaly, which we have called "frame dragging", we compare the velocities of clockwise movement and counterclockwise movement: $X/r = V - W = 2N/r - N_r$. Thus, we have found the same formula.

Let us check. In the $V = const$ state holds roughly $N = b\,r$, thus $u = 0$. That is much less than V, thus, there is no flat platoe in Cooperstock's theory. In the $V \sim r$ regime holds roughly $N = k\,r^2$, thus $u = k\,r$. This corresponds to the linear law near the core of a galaxy.

III. APPENDIX

Take circular orbits of a massive test particles. We have metric (15) in file cooper.pdf. Is taken $w = 0$. Because metric is ϕ, t-independent, then from the use of Killing's vectors we have

$$u_t = E = const\,, \qquad u_\phi = L_z = const.$$

Hopefully, these constants you can get from following equations

$$u_\nu u^\nu = 1\,,$$

$$\frac{dz}{ds} = 0\,, \qquad \frac{d^p\, z}{ds^p} = 0\,,$$

and

$$\frac{dr}{ds} = 0\,, \qquad \frac{d^p\, r}{ds^p} = 0\,.$$

For all $p = 1, 2, \ldots$. The higher derivatives are required because the motion shall be stable.

After that you can find the angular velocity as

$$\Omega = \frac{u^\phi}{u^t}\,.$$

Then extract by Cooperstock's "local transformation" the ω and compare the two angular velocities in not comoving coordiantes: $|\omega{+}0|$ and $|\omega{+}\Omega|$, the clockwise and counterclockwise respectfully. The difference is the Frame Dragging.

I have got: $\Omega = -(N_r)/r$ and $\omega = N/r^2$. The difference $\omega - |\Omega + \omega| = 2\omega + \Omega = 2N/r^2 - (N_r)/r$. There are two $G^{1/2}$ quantities, the difference can be G^2 small quantity. See: $G^2 = G^{1/2} - G^{1/2}$. The $\Omega = -(N_r)/r$ is gotten in assumption, that equatorial movement $z = 0$ holds for long enough. But in 2012 paper is talk, that there is asymmetry in relation to $z = 0$ plane, if I understood it correctly. Thus, a test-particle would change its $z = 0$ state. If holds $2\,N/r^2 - (N_r)/r = 0$ then $N(r) = k\,r^2$, where k is constant. Velocity $V = N/r = k\,r$, which is good form for Milky Way core (see linear law in Fig.1, 2005 paper). Cooperstock has also $N_{rr} + N_{zz} = N_r/r$. Its solution in form $N = k\,r^2 \exp(-z)$ is good in the core: for small r.

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[1] F. I. Cooperstock, S. Tieu, 2005. General Relativity Resolves Galactic Rotation Without Exotic Dark Matter, arXiv:astro-ph/0507619

[2] Mikolaj Korzynski, 2005. Singular disk of matter in the Cooperstock-Tieu galaxy model, arXiv:astro-ph/0508377; D. Vogt, P. S. Letelier, 2005. Presence of exotic matter in the Cooperstock and Tieu galaxy model, arXiv:astro-ph/0510750

[3] Aleksandar Rakic, Dominik J. Schwarz, General Relativistic Rotation Curves in a Post-Newtonian Light, Proceedings of Science, 2008:096,2008 (arXiv:0811.1478).

[4] J.D. Carrick, F.I. Cooperstock, General relativistic dynamics applied to the rotation curves of galaxies, Astrophys Space Sci (2012) 337:321329