Review of Grischuk and Sachin Gravitational Wave Generator via Tokamak Physics and its similarity to early universe GW from Braneworld Models

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Abstract: - Using Grischuk and Sachin (1975) amplitude for the GW generation due to plasma in a toroid, we generalize this result for Tokamak physics. We obtain evidence for strain values up to $h_{2nd-term} \sim 10^{-25} - 10^{-26}$ in a Tokamak center. These values are an order of magnitude sufficient to allow for possible detection of gravitational waves. The critical breakthrough is in utilizing a burning plasma drift current, which relies upon a thermal contribution to an electric field. The gravitational wave amplitude would be detectable in part also due to the Tokamak reaching the threshold for plasma fusion burning; the plasma fusion burning temperature obtained, of $T_{Temp} \ge 100 KeV$ is the main driver for how one could conceivably detect GW of amplitude as low as $\left[h_{2nd-term}\Big|_{T_{remp}\ge 100 KeV}\right]_{5-meters-above-Tokamak} \sim 10^{-26} - 10^{-27}$ five meters above the Tokamak center Such low strain values are extremely close to braneworld GW, and strain values in early universe cosmology

Key-Words: - Tokamak physics, confinement time (of Plasma), GW amplitude, Drift current

1 Introduction

Russian physicists Grishchuk and Sachin [1] obtained the amplitude of a Gravitational wave (GW) in a plasma as

A(amplitude-GW) = h ~
$$\frac{G}{c^4} \cdot E^2 \cdot \lambda_{GW}^2$$
 (1)

This should be compared with [2], and we can diagram the situation out as follows[3]



Fig. 1 We outline the direction of Gravitational wave "flux". If the arrow in the middle of the Tokamak ring perpendicular to the direction of the current represents the z axis, we represent where to put the GW detection device as 5 meters above the Tokamak ring along the z axis. This diagram was initially from Wesson [3]

Note that a simple model of how to provide a current in the Toroid is provided by a transformer core. This diagram is an example of how to induce the current I, used in the simple Ohms law derivation referred to in the first part of the text



Fig. 2 *Flux change provided by a transformer core,* in the simple current model first referred to in this paper. This figure is from Wesson [3]

Here, E is the electric field whereas λ_{Gw} is the gravitational wavelength for GW generated by the Tokamak in our model.

In the original Griskchuk model, we would have very small strain values, which will comment upon but which require the following relationship between GW wavelength and resultant frequency.

Note, if $\omega_{GW} \sim 10^6 Hz \Longrightarrow \lambda_{GW} \sim 300 \text{ meters}$, so we will be assuming a baseline of the order of setting $\omega_{GW} \sim 10^9 Hz \Longrightarrow \lambda_{GW} \sim .3 \text{ meters}$, as a baseline measurement for GW detection above the Tokamak. Furthermore, we will write the strain, introduced by (massive) Gravitons, as given by[4]. The precise values of the strain due solely to an Ohms law treatment of current, and the electric field will lead to , by first principles comparison of magnitude of terms using [2]

$$A(GW.amplitude) \sim h \sim \frac{G \cdot W_E \cdot V_{volume}}{c^4 \cdot \tilde{a}} (2)$$

Where

W

$$W_E = Average - energy - density$$

 $V_{volume} = Volume - Toroid$ (2a)
 $\tilde{a} = inner - radii(Toroid)$

This Eq. (2) above is due to the 1^{st} term of a two part composition of the strain, with the 2^{nd} term of the strain value significantly larger than the first term and due to ignition of the Plasma in the Tokamak. The first term of strain is largely due to what was calculated by Grishkuk [1]et. al. The second term is due to Plasma fusion burning. This plasma fusion burning contribution is due to non equilibrium contributions to Plasma ignition, which will be elaborated on in this document. Note that the first term in the strain derivation is due to the electric field within a Toroid, not Plasma fusion burning, and we will first of all discuss how to obtain the requisite strain, for the electric field contribution to the current, inside a Tokamak. making use of Ohms law.

2. Comment as to the derivation of strain generated by an electric field, and small strain values in the Tokamak.

We will examine the would-be electric field, contributing to a small strain values similar in part to Ohms law .A generalized Ohm's law ties in well with Figures 1 and 2 above

$$J = \sigma \cdot E \tag{3}$$

In order to obtain a suitable electric field, to be detected via 3DSR technology [4, 5], we will use a generalized Ohm's law as given by Wesson [3](page 146), where E and B are electric and magnetic fields, and v is velocity. We should understand that this undercuts the use of Figure 2 above.

$$E = \sigma^{-1}J - \nu \times B \tag{4}$$

As discussed with Dr. Wen Hao in November, 2014, in Chongqing University, the term in Eq. given $v \times B$ deserves (4) as special commentary. If v is perpendicular to B as occurs in a simple equilibrium case, then of course, Eq. (4) would be, simply put, Ohms law, and spatial equilibrium averaging would then lead to

$$E = \sigma^{-1}J - v \times B \xrightarrow{v - perpendicular - to - B} E = \sigma^{-1}J(5)$$

What saves the contribution of Plasma burning as a contributing factor to the Tokamak generation of GW, with far larger strain values commencing is that one does not have the velocity of ions in Plasma perpendicular to B fields in the beginning of Tokamak generation. It is, fortunately for us, a non equilibrium initial process, with thermal irregularities leading to both terms in Eq. (5) contributing to the electric field values.

We will be looking for an application for radial free electric fields being applied e.g., Wesson[3] (page 120)

$$n_{j}e_{j}\cdot\left(E_{r}+v_{\perp j}B\right)=-\frac{dP_{j}}{dr}$$
(6)

Here, $n_i = \text{ion density}$, jth species, $e_i = \text{ion charge}$, jth species, E_r = radial electric field, $v_{\perp i}$ = perpendicular velocity, of jth species, B =magnetic field, and P_i = pressure, jth species. The results of Eq. (3) and Eq. (4) are

$$\frac{\mathbf{G}}{c^4} \cdot E^2 \cdot \lambda_{GW}^2 \sim \frac{\mathbf{G}}{c^4} \cdot \left[\frac{Const}{R}\right]^2 \cdot \lambda_{GW}^2 + \frac{\mathbf{G}}{c^4} \cdot \left[\frac{J_b}{n \cdot e} + v_R\right]^2 \cdot \lambda_{GW}^2 = (1^{\text{st}}) + (2^{\text{nd}})$$
(7)

Here, the 1st term is due to $\nabla \times E = 0$, and the 2nd term is due to $E_n = \frac{dP_j}{dx_n} \cdot \frac{1}{n_j \cdot e_j} - (v \times B)_n$ with the 1st term generating $h \sim 10^{-38} - 10^{-30}$ in terms of GW amplitude strain 5 meters above the Tokamak , whereas the 2nd term has an $h \sim 10^{-26}$ in terms of GW amplitude above the Tokamak. The article has contributions from amplitude from the 1st and 2nd terms separately. The second part will be tabulated

separately from the first contribution assuming a minimum temperature of $T = Temp \sim 10 KeV$ as from Wesson [3]

3.GW h strain values when the first term of Eq.(5) is used for different Tokamaks

We now look at what we can expect with the simple Ohm's law calculation for strain values. As it is, the effort lead to non usable GW amplitude values of up to $h \sim 10^{-38} - 10^{-30}$ for GW wave amplitudes 5 meters above a Tokamak, and $h \sim 10^{-36} - 10^{-28}$ in the center of a Tokamak. I.e. this would be using Ohm's law and these are sample values of the Tokamak generated GW amplitude, using the first term of Eq. (5) and obtaining the following value[1] with a change as

$$h_{First-term} \sim \frac{\mathbf{G}}{c^4} \cdot E^2 \cdot \lambda_{GW}^2 \sim \frac{\mathbf{G}}{c^4} \cdot \left[\frac{J}{\sigma}\right]^2 \cdot \lambda_{GW}^2 (\mathbf{8})$$

We summarize the results of such in our first table as given for when $\omega_{GW} \sim 10^9 Hz \Longrightarrow \lambda_{GW} \sim .3$ meters and conductivity with $\sigma(tokamak - plasma) \sim 10 \cdot m^2/\text{sec}$ and with the following provisions as to initial values. What we observe are a range of Tokamak values which are, even in the case of ITER (not yet built) beyond the reach of any technological detection devices which are conceivable in the coming decade. This table and its assuming fixed conductivity results, values σ (tokamak – plasma) ~ 10 · m²/sec as well as $\lambda_{Gw} \sim .3$ meters is why the author, after due consideration completed his derivation of results as to the 2^{nd} term of Eq. (5) which lead to even for when considering the results for the Chinese Tokamak in Hefei to have[6]

$$h_{Second-term} \sim \frac{\mathbf{G}}{c^4} \cdot E^2 \cdot \lambda_{GW}^2 \sim \frac{\mathbf{G}}{c^4} \cdot \left[\frac{J_b}{n \cdot e} + v_R\right]^2 \cdot \lambda_{GW}^2 (9)$$

or values 10,000 larger than the results in ITER due to Eq.(6).'

We summarize the results of such in our first table as given for when $\omega_{GW} \sim 10^9 Hz \Longrightarrow \lambda_{GW} \sim .3$ meters and with'

See appendix A below which has useful data

Table 1: Values of strain at center of Tokamak, and 5 meters above Tokamak:

Note that we are setting $\lambda_{Gw} \sim .3$ meters, $\sigma(tokamak - plasma) \sim 10 \cdot m^2/\text{sec}$, using Eq.6 above for Amplitude of GW.

What makes it mandatory to go the 2^{nd} term of Eq. (5) is that even in the case of ITER, 5 meters above the Tokamak ring, the GW amplitude is 1/10,000 the size of any reasonable GW detection device, and this including the new 3DSR technology (Li et al, 2009) [4,5]. Hence, we need to come up with a better estimate, which is what the 2^{nd} term of Eq.(5) is about which is derived in the next section

3 .Enhancing GW strain Amplitude via utilizing a burning Plasma drift current: Eq.(4)

The way forward is to go to Wesson, [3] (2011, page 120) and to look at the normal to surface induced electric field contribution

$$E_n = \frac{dP_j}{dx_n} \cdot \frac{1}{n_j \cdot e_j} - (v \times B)_n \tag{10}$$

If one has for v_R as the radial velocity of ions in the Tokamak from Tokamak center to its radial distance, R, from center, and B_{θ} as the direction of a magnetic field in the 'face' of a Toroid containing the Plasma, in the angular θ direction from a minimal toroid radius of R = a, with $\theta = 0$, to R = a + r with $\theta = \pi$, one has v_R for radial drift velocity of ions in the Tokamak, and B_{θ} having a net approximate value of: with B_{θ} not perpendicular to the ion velocity, so then [3]

$$(v \times B)_n \sim v_R \cdot B_\theta$$
 (11)

Also, as a first order approximation: From Wesson [3] (page 167) the spatial change in pressure denoted

$$\frac{dP_j}{dx_n} = -B_\theta \cdot j_b \tag{12}$$

Here (ibid), the drift current, using $\xi = a/R$, and drift current j_b for Plasma charges, i.e.

$$j_b \sim -\frac{\xi^{1/2}}{B_{\theta}} \cdot T_{T_{emp}} \cdot \frac{dn_{drift}}{dr}$$
(13)

Figure 3 below introduces the role of the drift current, in terms of Tokamaks[3]



Fig. 3 *Typical bootstrap currents with a shift due* to r/a where r is the radial direction of the Tokamak, and a is the inner radius of the Toroid *This figure is reproduced from Wesson* [3]

Then one has

$$B_{\theta}^{2} \cdot \left(j_{b}/n_{j} \cdot e_{j}\right)^{2} \sim \frac{B_{\theta}^{2}}{e_{j}^{2}} \cdot \frac{\xi^{1/4}}{B_{\theta}^{2}} \cdot \left[\frac{1}{n_{drift}} \cdot \frac{dn_{drift}}{dr}\right]^{2}$$
(14)
$$\sim \frac{\xi^{1/4}}{e_{j}^{2}} \cdot \left[\frac{1}{n_{drift}} \cdot \frac{dn_{drift}}{dr}\right]^{2}$$

Now, the behavior of the numerical density of ions, can be given as follows, namely growing in the radial direction, then[3]

$$n_{drift} = n_{drift} \Big|_{initial} \cdot \exp\left[\tilde{\alpha} \cdot r\right]$$
(15)

This exponential behavior then will lead to the 2nd term in Eq.(5) having in the center of the Tokamak, for an ignition temperature of $T_{Temp} \ge 10 KeV$ a value of

$$h_{2nd-term} \sim \frac{G}{c^4} \cdot B_{\theta}^2 \cdot \left(j_b / n_j \cdot e_j\right)^2 \cdot \lambda_{GW}^2$$
(16)
$$\sim \frac{G}{c^4} \cdot \frac{\xi^{1/4} \tilde{\alpha}^2 T_{Temp}^2}{e_j^2} \cdot \lambda_{GW}^2 \sim 10^{-25}$$

As shown in **Fig. 4** (copied from Wesson 2011), [3] there is a critical ignition temperature at its lowest point of the curve in the having $T_{Temp} \ge 30 KeV$ as an optimum value of the Tokamak ignition temperature for $n_{ion} \sim 10^{20} m^{-3}$, with a still permissible temperature value of $T_{Temp}\Big|_{safe-upper-bound} \approx 100 KeV$ with a value of $n_{ion} \sim 10^{20} m^{-3}$, due to from page 11, [3] the relationship of Eq.(16), where τ_E is a Tokamak confinement of plasma time of about 1-3 seconds, at least due to [3]

$$n_{ion} \cdot \tau_E > .5 \times 10^{20} \cdot m^{-3} \cdot \text{sec}$$
(17)



Fig. 4 The value of $n\tau_E$ required to obtain ignition, as a function of temperature. Figure reproduced from Wesson [3]

Also, as shown in Fig. 4, $T_{T_{emp}}\Big|_{safe-upper-bound} \approx 100 KeV$, then one could have at the Tokamak center, i.e. even the Hefei based Tokamak[3,6]

$$h_{2nd-term}\Big|_{T_{remp} \ge 100 KeV} \sim \frac{G}{c^4} \cdot \frac{\xi^{1/4} \tilde{\alpha}^2 T_{Temp}^2}{e_i^2} \cdot \lambda_{GW}^2 \sim 10^{-25} - 10^{-26}$$
(18)

This would lead to, for a GW reading 5 meters above the Tokamak, then lead to for then the Hefei PRC Tokamak[3,6]

$$\begin{bmatrix} h_{2nd-term} \Big|_{T_{temp} \ge 100 \, KeV} \end{bmatrix}_{5-meters-above-Tokamak}$$

$$\sim \frac{G}{c^4} \cdot \frac{\xi^{1/4} \tilde{\alpha}^2 T_{temp}^2}{e_j^2} \cdot \lambda_{GW}^2 \sim 10^{-27}$$
(19)

Note that the support for up to 100 KeV for temperature can yield more stability in terms of thermal Plasma confinement as give in **Fig. 5** below, namely from [3] we have



Fig 5 Illustrating how increase in temperature can lead to the H mode region, in Tokamak physics where the designated equilibrium point, in Fig. 5 is a known way to balance conduction loss with alpha particle power, which is a known way to increase τ_E i.e. Tokamak confinement of plasma time[2]

1. Details of the model in terms of terms of adding impurities to the Plasma to get a longer confinement time (possibly to improve the chances of GW detection). We add this detail in, due to a question raised by Dr. Li who wished for longer confinement times for the Plasma in order to allegedly improve the chances of GW detection for a detector 5 meters above the Tokamak in Hefei. Wesson [3] (2011) stated that the confinement time may be made proportional to the numerical density of argon/ neon seeded to the plasma [3](page 180). This depends upon the nature of the Tokamak, but it is a known technique, and is suitable for analysis, depending upon the specifics of the Tokamak. I.e. this is a detail Dr. Li can raise with his co workers in Hefei, PRC in 2014 [6].

2. Restating the energy density and power which would be in the Hefei Tokamak, using the formalism of Eq.(2) directly

$$W_{E} \sim \frac{\tilde{\alpha} \cdot \lambda_{GW}^{2}}{V_{volume}} \cdot \frac{\xi^{1/4} \tilde{\alpha}^{2} T_{Temp}^{2}}{e_{j}^{2}}$$

$$W_{E} \cdot V_{volume} \sim \tilde{\alpha} \cdot \lambda_{GW}^{2} \cdot \frac{\xi^{1/4} \tilde{\alpha}^{2} T_{Temp-plasma-fusion-burning}^{2}}{e_{j}^{2}}$$
(20)

The temperature for Plasma fusion burning, is then about between 30 to 100 KeV, as given by Wesson []

The corresponding power as given by Wesson is then for the Tokamak[3]

$$P_{\Omega} = E \cdot J \le \frac{E}{\mu_0} \cdot \frac{B_{\phi}}{R}$$
(21)

The tie in with Eq.(19) by Eq. (21) can be seen by first of all setting the E field as related to the B field, via E (electrostatic) ~ $10^{12}Vm^{-1}$ as equivalent to a magnetic field B ~ $10^4T(Torr)$ as given by [2]. In a one second interval, if we use the input power as an experimentally supplied quantity, then the effective E field is

$$E_{applied} \sim \frac{\xi^{1/8} \cdot \tilde{\alpha}}{e_j} \times T_{\text{Tokamak-temperature}}$$
(22)

What is found is, that if Eq. (21) and Eq.(22) hold. Then by Wesson[3], pp. 242-243, if $Z_{eff} \sim 1.5, q_a q_0 \sim 1.5, (R/\tilde{a}) \approx 3$ Then the temperature of a Tokamak, to good approximation would be between 30 to 100 KeV, and then one has[3]

$$B_{\phi}^{4/5} \sim .87 \cdot \left(\tilde{T} = T_{Tokamak-temperature}\right)$$
(23)

Then the power for the Tokamak is

$$P_{\Omega}\big|_{Tokamak-toroid} \leq \frac{\xi^{1/8} \cdot \tilde{\alpha}}{\mu_0 \cdot e_j \cdot R} \times \frac{\left(T_{Tokamak-temperature}\right)^{9/4}}{\left(.87\right)^{5/4}} (24)$$

Then, per second, the author derived the following rate of production per second of a $10^{-34} eV$ graviton, as given by, if $\tilde{a} = R/3$

$$n|_{massive-gravitons/sec ond} Tokamak$$

$$\propto \frac{3 \cdot \hbar \cdot e_{j}}{\mu_{0} \cdot R^{2} \cdot \xi^{1/8} \cdot \tilde{\alpha}} \times \frac{\left(T_{\text{Tokamak-temperature}}\right)^{1/4}}{\lambda_{Graviton}^{2} \cdot m_{graviton} \cdot c^{2} \cdot (.87)^{5/4}} (25)$$

$$\sim 1/\lambda_{Graviton}^{2} scaling$$

If there is a fixed mass for a massive graviton, the above means that as the wavelength decreases, that the number of gravitons produced between plasma burning temperatures of 30 to 100 KeV changes dramatically. The change in graviton number is not nearly so sensitive as to Plasma fusion burning as for 30 to 100 KeV temperature variation.

Numerical inputs into Eq. (25) have indicated that there are roughly 1000 gravitons per second generated by Plasma fusion burning, with a strain value of $h \sim 10^{27}$ 5 meters above the center of the Hefei Tokamak. If so, then the long confinement time of the Hefei Tokamak ,for plasmas, would indicate a chance that a detector may be able to obtain a graviton signal. That depends upon if $h \sim 10^{27}$ is, with the equipment available actually detectable. If so, then the next task is the extremely time consuming process of experimental verification of the measurements , and answering questions as to the reliability of the obtained data sets.

6 Conclusion. GW generation due to the Thermal output of Plasma burning

Further elaboration of this matter in the experimental detection of experimental data sets for massive gravity lies in the viability of the expression derived , namely Eq. (19)

• $h \sim 10^{27}$ for a GW detected 5 meters above a Tokamak represents the extreme limits of what could be detected, but it is within the design specifications of what Dr. Li et al. (2009)[4,5] presented for PRD readership. The challenge, as frankly brought up in discussions in Chongqing University is to push development of 3DSR hardware to its limits, and use the Hefei Tokamak configuration as a test bed for the new technology embodied in the Plasma fusion burning generation of Gravitation waves.

The importance of the formulation is in the explicit importance of temperature. i.e. a of temperature range at least $10KeV \le T_{Temp} \le 100KeV$. In making this range for Eq.(25), care must also be taken to obtain a sufficiently long confinement time for the fusion plasma in the Tokamak of at least 1 second or longer, and this is a matter of applied engineering dependent upon the instrumentation of the Tokamak in Hefei, PRC.

Furthermore,..Wen, Li, and Fang, proved in [7] the likelihood of braneworld generation of HFGW which are close to the values of strain and frequency which could be generated by the Tokamak described above.

The author hopes that in 2015, there will be the beginning of confirmation of this process so that some studies may commence. If so, then the next question will be finding if the instrumentation of Li and colleagues [3,4] can be utilized and developed. This is expected to be extremely difficult, but the Tokamak fusion process may allow for falsifiable testing and eventual verification.

If the Tokamak can be seen, in Hefei[6], to give strain and HFGW values commensurate

with [7], then there is good reason to utilize the hardware for early universe GW detection. This is the hope of the Chongqing University GW team as of 2015.

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Appendix A

Table 1: Values of strain at center of Tokamak, and 5 meters above Tokamak if only using square of E field contribution to strain equation. This table neglects using Eq.(19), which allows for $h \sim 10^{27}$: In Appendix A, only Eq. (8) is utilized for the strain value which is woefully inadequate

 $\lambda_{Gw} \sim .3 \text{ meters}$, $\sigma(tokamak - plasma) \sim 10 \cdot m^2/\text{sec}$, using Eq.6 above for Amplitude of GW.

Experiment	Site/ location	Plasma current, in (Mega-Amps) MA	Strain, h, in center of the Tokamak	Strain, h, 5 meters above the center of the Tokamak
JET	Culham, Oxfordshire (UK)	8 = 3.2 MA (circular plasma) + 4.8 MA (D- shape plasma)	$h \sim 10^{-31}$	$h \sim 10^{-33}$
ASDEX	Garching (GER)	5	$h \sim 10^{-32}$	$h \sim 10^{-34}$
DIII-D	San Diego (USA)	3-3.5	$h \sim 10^{-32}$	$h \sim 10^{-34}$
HL-2A	Chengdu (PRC)	.48	$h \sim 10^{-34}$	$h \sim 10^{-36}$
HT-7U	Hefei (PRC)	1.0	$h \sim 10^{-33}$	$h \sim 10^{-34}$
ITER(planned)	Saint Paul Les- Durance (FR)	15	$h \sim 10^{-28} - 10^{-29}$	$h \sim 10^{-30} - 10^{-31}$