The Hubble constant, length and surface

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Abstract

We review Hubble’s law formulation, we reduce $H_0$ to a combination of three fundamental constants and define the Hubble surface $\sigma_H$. 
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1 Hubble’s parameters

1.1 Hubble law

In a previous paper, Gosselin [1] showed that the electromagnetic wave transforms along distance as

\[ \lambda = \lambda_{\text{cmb}} - (\lambda_{\text{cmb}} - \lambda_0) e^{-\frac{H_0 D}{c}} \]  

(1.1)

where \( \lambda \) is the wavelength, \( \lambda_{\text{cmb}} \) the wavelength of the cosmic microwave background radiation CMB, \( \lambda_0 \) the restframe wavelength of the emitted radiation, \( D \) the cosmic distance, \( c \) vacuum speed of light and \( H_0 \) the Hubble constant. The cosmic shift is

\[ Z = \frac{\lambda - \lambda_0}{\lambda_0} \]  

(1.2)

and the cosmic shift at the cosmic microwave background is

\[ Z_{\text{cmb}} = \frac{\lambda_{\text{cmb}} - \lambda_0}{\lambda_0} \]  

(1.3)

Such source is at distance

\[ D = \frac{c}{H_0} \cdot \ln \left( \frac{\lambda_{\text{cmb}} - \lambda_0}{\lambda_{\text{cmb}} - \lambda} \right) \]  

(1.4)

\[ D = \frac{c}{H_0} \cdot \ln \left( \frac{Z_{\text{cmb}}}{Z_{\text{cmb}} - Z} \right) \]  

(1.5)

\[ D = \frac{c}{H_0} \cdot \ln \left( \frac{1}{1 - \frac{Z}{Z_{\text{cmb}}}} \right) \]  

(1.6)

a logarithmic function of the cosmic shift.

Gosselin also use the transformation of the electromagnetic wave to explain the anomalous behaviour of the Pioneer satellite and finds the value of the Hubble constant \( H_0 \)

\[ H_0 = -\frac{\ddot{\nu}}{\dot{\nu}} \cdot \frac{1}{Z_{\text{cmb}}} \]  

(1.7)

as \( 84,3 \) km/s/Mpc which is about same as the value of \( 85 \pm 5 \) km/s/Mpc found by Willick [2] from a study of the cepheids.

1.2 Construction

It is of a great interest to express a constant as a combination of fundamental ones. We express the Hubble constant as a mix of fundamental constants doing so as to cope with the units of measure and searching for a value the closest as possible to the currently accepted value. The found composition is

\[ H_0 = \frac{\alpha R_\infty^2 \left( \frac{\hbar G}{c^3} \right)^{\frac{1}{2}}}{(2\pi)^4} \]  

(1.8)
where $\alpha$ is the fine structure constant, $R_\infty$ is the Rydberg constant, $\hbar$ is the reduced Planck constant, $G$ is the universal gravitational constant and $c$ is the vacuum speed of light. Using the values of the fundamental constants as given by Codata [3] [4] also shown on table 1, we find for the Hubble constant the same value as computed previously from the Pioneer satellite $2.73193 \times 10^{-18} \text{ s}^{-1}$ or $84.3 \text{ km/s/Mpc}$.

Introducing Planck length

$$\ell_p = \frac{1}{c} \left( \frac{\hbar G}{c} \right)^{\frac{1}{2}}$$

in the previous equation, the Hubble length is

$$\ell_H = \frac{c}{H_0} = \frac{(2\pi)^4}{\alpha R_\infty^2 \ell_p}$$

We define the following reduced constants

$$\tilde{\alpha} = \frac{\alpha}{2\pi}$$

(1.11)

$$\tilde{R}_\infty = \frac{R_\infty}{2\pi}$$

(1.12)

$$\tilde{\ell}_p = \frac{\ell_p}{2\pi}$$

(1.13)

and rewrite the previous equation under a more elegant way

$$\ell_H = \left( \tilde{\alpha} \tilde{R}_\infty^2 \tilde{\ell}_p \right)^{-1}$$

(1.15)

Referring to Codata [3] [4] the values of those constants also shown in table 1, we compute the value of Hubble length as $1.09736384 \times 10^{26} \text{ meters}$.

### 1.3 Hubble surface

We observe that the digits of the Hubble constant as defined are the same as the Rydberg constant $1.097373 \times 10^7 \text{ m}^{-1}$. We define the reduced Hubble surface $\tilde{\sigma}_H$ as the ratio of Hubble length to Rydberg constant

$$\tilde{\sigma}_H = \frac{\ell_H}{R_\infty}$$

(1.16)

$$\tilde{\sigma}_H = \left( \tilde{\alpha} \tilde{R}_\infty^2 \tilde{\ell}_p \right)^{-1}$$

(1.17)

$$\tilde{\sigma}_H = 10^{19} \text{ m}^2$$

(1.18)
The corresponding Hubble surface is

\[ \sigma_H = 2\pi\tilde{\sigma}_H \]  
(1.19)
\[ \sigma_H = 2\pi 10^{19} \text{ m}^2 \]  
(1.20)

Table 2 shows the value of those three constants \( H_0, \ell_H \) et \( \sigma_H \). Table 3 gives simple geometrical equivalences to this Hubble surface. For example it is the surface of a sphere whose radius is 2 236 100 kilometers or 0.015 UA that is 3.21 times the sun radius.

## 2 Tables

<table>
<thead>
<tr>
<th>Constant</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum light speed</td>
<td>c</td>
<td>2,997 924 58 \times 10^8</td>
<td>\text{m} \cdot \text{s}^{-1}</td>
</tr>
<tr>
<td>Gravitational</td>
<td>G</td>
<td>6,673 84(80) \times 10^{-11}</td>
<td>\text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}</td>
</tr>
<tr>
<td>Planck</td>
<td>h</td>
<td>6,626 069 57(29) \times 10^{-34}</td>
<td>\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-1}</td>
</tr>
<tr>
<td>Reduced Planck</td>
<td>\tilde{h}</td>
<td>1,054 571 726(47) \times 10^{-34}</td>
<td>\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-1}</td>
</tr>
<tr>
<td>Fine structure</td>
<td>\alpha</td>
<td>7,297 352 5698(24) \times 10^{-3}</td>
<td></td>
</tr>
<tr>
<td>Reduced fine structure</td>
<td>\tilde{\alpha}</td>
<td>1,161 409 733 \times 10^{-3}</td>
<td></td>
</tr>
<tr>
<td>Rydberg</td>
<td>\RyRd</td>
<td>1,097 373 156 8539(55) \times 10^7</td>
<td>\text{m}^{-1}</td>
</tr>
<tr>
<td>Reduced Rydberg</td>
<td>\RyRd</td>
<td>1,746 523 62 \times 10^6</td>
<td>\text{m}^{-1}</td>
</tr>
<tr>
<td>Plank length</td>
<td>\ell_p</td>
<td>1,616 199(97) \times 10^{-35}</td>
<td>\text{m}</td>
</tr>
<tr>
<td>Reduced Planck length</td>
<td>\ell_p</td>
<td>2,572 260 59 \times 10^{-36}</td>
<td>\text{m}</td>
</tr>
<tr>
<td>Lyman ( \alpha )</td>
<td>\La</td>
<td>9,112 670 51 \times 10^{-8}</td>
<td>\text{m}</td>
</tr>
<tr>
<td>Astronomical unit</td>
<td>UA</td>
<td>1,495 978 707 00(3) \times 10^{11}</td>
<td>\text{m}</td>
</tr>
</tbody>
</table>

Table 1: Fundamental constants

<table>
<thead>
<tr>
<th>Constant</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hubble constant</td>
<td>\H0</td>
<td>2,731 93 \times 10^{-18}</td>
<td>\text{s}^{-1}</td>
</tr>
<tr>
<td>Hubble length</td>
<td>\ellH</td>
<td>1,097 37 \times 10^{26}</td>
<td>\text{m}</td>
</tr>
<tr>
<td>Hubble surface</td>
<td>\sigmaH</td>
<td>2\pi 10^{19}</td>
<td>\text{m}^2</td>
</tr>
</tbody>
</table>

Table 2: New constants

5
<table>
<thead>
<tr>
<th>Unit of measure</th>
<th>Symbol</th>
<th>Value (meters)</th>
<th>Square (side)</th>
<th>Disc (radius)</th>
<th>Sphere (radius)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meters</td>
<td>m</td>
<td>1</td>
<td>7,9266 × 10^9</td>
<td>4,4721 × 10^9</td>
<td>2,2361 × 10^9</td>
</tr>
<tr>
<td>Earth-Moon</td>
<td>EM</td>
<td>3,84399 × 10^8</td>
<td>20,62</td>
<td>11,63</td>
<td>5,82</td>
</tr>
<tr>
<td>Sun radius</td>
<td>SR</td>
<td>6,959 9(7) × 10^8</td>
<td>11,39</td>
<td>6,43</td>
<td>3,21</td>
</tr>
<tr>
<td>Astronomical unit</td>
<td>AU</td>
<td>1,495 978 921(1) × 10^11</td>
<td>5,3 × 10^{-2}</td>
<td>3,0 × 10^{-2}</td>
<td>1,5 × 10^{-2}</td>
</tr>
</tbody>
</table>

Table 3: Equivalent surfaces

3 Conclusion

We redefined the Hubble constant as a function of three fundamental constants of nature. The computed value is identical to the one previously obtained through the Pioneer satellite that is 84,3 km/sec/Mega Parsec. The corresponding Hubble length brought us to define a new constant, the Hubble surface whose value is \(2\pi 10^{19} \text{ m}^2\).
References


