FINITE AND INFINITE BASIS IN P AND NP

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1. Abstract

This article prove that P is not NP by using difference of bases cardinality. About NP-Complete problems, we can divide some problems to infinite disjunction of P-Complete problems. These P-Complete problems are independent of each other in disjunction. The other hand, any P-Complete problem have at most a finite number of basis of P-Complete. The reason is that each P problems have at most finite number of Least fixed point operator. Therefore, we cannot describe NP-Complete problems in P.

2. Difference of basis between P and NP

By using SAT and these verification, we show that some NP-Complete problems have infinite basis of P-Complete problems.

Definition 1. We will use the term "$v_i \in V \subset P$" as problem which verify formula with special valuation i.

If $t \in SAT$

then

$v_i(t) = \top \leftrightarrow t(i) = \top$

Theorem 2. $v_i \in P - Complete$

Proof. It is trivial that $v_i \in P$.

We show all P problems can reduce to v_i by reducing P-Complete problem $VALUE$: Formula verify problems. $VALUE(p)$ is equal $v_i \circ f(p)$ that f negate some p variables that correspond to i value. we can compute f in L.

Therefore $v_i \in P - Complete$. □

Theorem 3. *V is basis of SAT*

Proof. To think about relation between SAT and $v_i \in V$, SAT is disjunction of V.

$SAT = \bigcup V = \bigvee_{i=0}^{\infty} v_i$

Each v_i is independent of each other in disjunction because every input p have another input q that change only v_i output.

$\forall p \exists q \left(\left(v_0(p), \cdots, v_i(p), \cdots \right) = \left(v_0(q) = v_0(p), \cdots, v_i(q) = \neg v_i(p), \cdots \right) \right)$

If $v_i(p) = \top$ then $q = p \wedge (\neg i)$

else if $v_i(p) = \bot$ then $q = p \vee (i)$

That is, all v_i is necessary to compute SAT problems.

Therefore V is basis of SAT. □

From descriptive complexity, $P = FO + LFP$[1, 2, 3]. This means that every P problem have at most a finite number of LFP operators. Therefore P problem have at most a finite number of basis of P-Complete.

Theorem 4. *Any $p \in P$ have at most a finite number of basis of P-Complete.*

Proof. To prove it by using reduction to absurdity. We assume that $p \in P$ have infinite number of basis of P-Complete. These basis independent of each other and have independent LFP operators. But $P = FO + LFP$ have at most finite number of LFP operators. Therefore we cannot describe p in finite length $FO + LFP$. □

Theorem 5. $P \neq NP$

Proof. Mentioned above3, SAT have infinite basis of P-Complete. But mentioned above4, any $p \in P$ have finite basis of P. Therefore SAT is not any $p \in P$. □

References

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