In that short note, I demonstrate formally the existence of a logical path implying the automatic resolution of the magnetic reconnection problem. The latter has been the motivation for numerous articles and is actually explored by the ARTEMIS mission.

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The dream of a unified nature
There is an evident reason why it should be a priori possible to find a common kinder-garden where a theory of gravity and a quantum field theory may develop together in harmony: they both permanently attempt to describe the same nature. Unfortunately the wisdom implicitly contained in these words doesn’t reveal us the way to put both children in that Eden. Between “here and now” and that wished paradise quite a century of pains and hard work in diverse directions. This document is one more essay in that challenge. It is based on recent researches and on a very personal work.

The mathematics behind the physics
Before starting an abstract fly into the unavoidable world of mathematics, let me begin at the bottom of evidences and, in some way, go back in the past, approximately until the end of the 17th century with some exceptions here and there. The quantum mechanics didn’t yet exist. Even if a few thinkers in the Greek civilization had already evocated the atomic structure of the matter (e.g.: in observing the sand on the beach), Lavoisier and others were just beginning their investigations and discovering the first molecule (H₂, O₂...). Thanks the efforts of Copernic (1473-1543), Brahe (1546-1601), Kepler (1571-1630), Galileo (1564-1642) and many others the Earth was no more the center of the universe but only one of the planets orbiting around the sun. Newton (1643-1727) told how and why they were moving in that way. As such and as all observable and known material bodies, they had a trajectory. Magellan (?-1521), Descartes (1596-1650), Leibniz (1646-1716), and others had left coordinates systems to help sailors and pedestrians to travel on see or on the floor. Among others, a little bit later, clever people like Gauss (1755-1855), Riemann (1826-1866), and Lobatchevsky (1792-1856) took care of the curvature either of these moving objects or of the ground where they were moving. And then there was the birth of quantum mechanics... This is at least what most of the modern books tell.
The Magnetic reconnection problem
Let me make a time jump and look with the help of my applications at the actual missions of the NASA. I have been very surprised, if not choked, by one of them called “ARTEMIS” for Acceleration, Reconnection, Turbulence and Electrodynamic of Moon’s Interaction with the Sun [01]. Let me isolate just one of the investigated items. What is magnetic reconnection if not the attempt to discover all solutions of the Lorentz’s law when the equations describing the local electromagnetic field are the J.C. Maxwell’s (1831-1879) ones. Let me recall that Lorentz’s law:

\[(01) \quad E + v \times B = 0\]

Where the subscript \((3)\) indicates that we are working in a three dimensional mathematical vector space, \(E\) is the electrical field, \(B\) is the magnetic field, \(\times\) denotes the exterior product which – exceptionally in these circumstances – coincides with the cross product. That’s all! The mathematical simplicity of that law, its “age” (born at the end of the 19th century) and the fact that we are still exploring it now (2015) in a closed vicinity of the Earth may illuminate the mathematicians lost in their extrapolations. The scientific community should not take this sentence as a criticism but as a warning, as a call for more pedagogy; as an invitation to be patient and to not go too quickly in the development of new theories.

The modern context
Nevertheless, that law is obviously the grandmother of the Lorentz-Einstein law (LEL) born with the theory of relativity at the beginning of the 20th century [02]. The latter incorporates the fourth dimension and diverse knowledge on the tensor calculus, on the notion of covariant derivation, on connection, etc.

\[(02) \quad \forall \alpha, \beta, \chi \in I_4 = \{0, 1, 2, 3\}: \frac{dv^\alpha}{ds} + \Gamma^\alpha_{\beta\chi} v^\beta \cdot v^\chi = k \cdot F^\alpha_{\beta} \cdot v^\beta\]

Where \(d/ds\) denotes an ordinary derivation by respect for the parameter \(s\), \((4)\) is the speed, \(\Gamma^\alpha_{\beta\chi}\) are the E.B. Christoffel’s symbols of the second kind (see in [03]), \(F^\alpha_{\beta}\) are the (up, down) components of the tensorial representation of the electromagnetic (EM) field and \(k\) is the ration: charge/mass when the latter is supposed to be not equal to zero.

Introducing extended products into the modern discussion
This law is itself the prototype example of a natural law involving an extended product. That concept allows a rewriting of the LEL in a more concise style:

\[(03) \quad \left| \frac{dv}{ds} + \bigl(\bigtriangleup \Gamma\bigr) (4) v\right| > = k \cdot F \cdot \left| (4) v \right| >\]

I have introduced here the well-known “bracket” convention (a customized way to do in quantum mechanics) and the generic symbol for the extended product built on a cube of components; here the Levi-Civita cube: \(\nabla \Gamma\). The l.h.t. is the covariant derivative of the speed whilst the r.h.t. describes the part of the force due to the EM field. The historical version (01) is recovered in annihilating the l.h.t. of (03); indeed, because of usual tensorial rules, the r.h.t. may also be written:

\[(04) \quad [F_{\alpha\beta}] \cdot \left| (4) v \right| > = \left| (4) 0 \right|>

Since the formalism of \([F_{\alpha\beta}]\) can be discovered in any good book and writes:
The theory of the (E) question

Solving formally the magnetic reconnection problem, v2, 22.01.2015


(05)

\[
\begin{bmatrix}
0 & E^1 & E^2 & E^3 \\
-E^1 & 0 & -B^3 & B^2 \\
-E^2 & B^3 & 0 & -B^1 \\
-E^3 & -B^2 & B^1 & 0
\end{bmatrix}
\begin{bmatrix}
v^0 \\
v^1 \\
v^2 \\
v^3
\end{bmatrix}
=
\begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

I can easily state that:

(06-1)

\[
< (3)E, (3)v >_{id3} = 0
\]

(06-2)

\[
(3)B \wedge (3)v + v^0 \cdot (3)E = (3)0
\]

The first relation, (06-1), tells the orthogonality of the electrical field and of the spatial speed in a three dimensional Euclidian context; the second relation, (06-2), is the relation (01) up to a scalar \(v^0\). As already known these relations describe the propagation of some EM plane wave in a very classical vacuum.

The idea sketching the solution of the magnetic reconnection problem

The problem of the magnetic reconnection is the search for the solutions of (01) or (06-2) when the J. C. Maxwell’s laws hold true. Since the theory of relativity generates the LEL as law of motion for charged particles immersed in a gravitational field and since the LEL has the solutions:

(07)

\[
< (4)u, (4)u >_{g} = \text{invariant}
\]

the following schema applies:

Theory of relativity
\[\downarrow\]
LEL: (03)
\[\downarrow\]
(07)

Schema n°1

Now, if by the virtue of some intelligent introspection into the mysterious world of mathematics, it would be possible to find a logical path such that, e.g.:

Theory of relativity \(\rightarrow\) J.C. Maxwell’s laws for EM fields in vacuum

Schema n°2

Then we would have the certitude that in a common domain of definition, if the Einstein’s theory applies, then automatically the J. C. Maxwell’s laws for the EM fields in vacuum and the LEL apply simultaneously and the magnetic reconnection problem – as part of the whole- would be solved.

But there is a distance between what the head thinks and what the hand can do. If the schema n°1 is actually pure evidence, the logical path validating the schema n°2 has to be discovered.
The strategic E. B. Christoffel's work

The path in fact exists. It lies on a deep re-analyze of the E. B. Christoffel’s work [03]. If it is actually admitted that that work represents the corner stone of the A. Einstein’s work [02] it is less well-known that the same work can be used for antisymmetric bilinear homogeneous differential forms. This is exactly the way I have explored. It allows two types of considerations.

The first considerations lies on a trivial application of the Heisenberg’s uncertainty principle (HUP) [04] to the pair (energy, time) and involves the E. B. Christoffel's work via a simplified version of the LEL preserving the Planck’s constant, h (up to a constant scalar). It follows (07) in a natural way, as mathematical consequence of that type of analysis.

\[
\text{LEL: (03)} \\
\downarrow \\
\text{Simplified LEL} \\
\downarrow \\
\text{Work which is needed to move the difference of realization of the simplified LEL} \\
\equiv \\
\text{HUP (energy, time)} \\
\downarrow \\
\text{Analysis based on a comparison with a Taylorization} \\
\downarrow \\
\text{Set of three relations of coherence} \\
\downarrow \\
\text{One of them is (07)} \\
\]

One of them is a new formalism for the (up, down) components of the tensorial representation of the EM field:

\[
F_{\alpha \beta} = ... \\
\]

The third one has actually only a mathematical role for the coherence

\[\text{Schema n°3}\]

Look at the schema n°1 and compare it with the schema n°3. The latter obviously offers an alternative path connecting the LEL and (07). That path, although a little bit longer and being the occasion of polemics into the scientific community\(^1\), also proposes a totally different vision on the nature. Furthermore, because of the intervention of the HUP into the demonstration, the solutions of the theory of relativity now appear to be in some way embedded into the quantum mechanics. In my opinion, the polemic around and about the presumed limitations of the HUP can disappear as soon as a change of paradigm is accepted; namely: 1°) what is called “vacuum” is –despite the appearances- a totally unstable energetic place when the ad hoc conditions are realized; 2°) all particles are ephemeral phenomenon going soon or late back to an energetic vacuum-state.

The second considerations are developed around the concept of Poisson’s brackets. It can be demonstrated that J. C. Maxwell’s laws in vacuum are recovered in applying the E. B. works to permanently vanishing antisymmetric bilinear homogeneous differential forms; the latter being

\(^1\) because of the fact that it is believed that the version of the HUP involved into that way of thinking only concerns the creation-annihilation of virtual particles in vacuum.
constructed on \([F_{\alpha\beta}]\). Furthermore the theory proposes a third connection based on the partial derivatives of \((\text{up, down})\) components of the tensorial representation of the EM field.

\[
\begin{align*}
&\text{(03)} \\
\downarrow
&\text{Schema n°3} \\
\downarrow
&(08) \rightarrow [F_{\alpha\beta}] \rightarrow \{\cdot\cdot\} \equiv \partial_{\beta} F_{\alpha\beta} \rightarrow \text{J. C. Maxwell’s laws in vacuum}
\end{align*}
\]

So far, combining the schemas n°1 and n°4, I find the schema n°2 which was the missing part to formally solve the magnetic reconnection problem.

**Bibliography**

[01] Informations are available for the public at (under your own responsibility – no commercial use): [http://www.nasa.gov/mission_pages/artemis/]


[03] Über die Transformationen der homogenen Differenzialausdrücke zweites Grades: E.B. Christoffel, in Journal für die reine und angewandte Mathematik, (pp. 46 - 70), 1869. An electronic copy is available at the Göttingen State and University Library (no commercial use).