A Proof of the ABC Conjecture

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Introduction: The ABC conjecture was proposed by Joseph Oesterle in 1988 and David Masser in 1985. The conjecture states that for any infinitesimal quantity $\varepsilon > 0$, there exists a constant $C_\varepsilon > 0$, such that for any three relatively prime integers $a$, $b$ and $c$ satisfying $a + b = c$, the inequality

$$\max (|a|, |b|, |c|) \leq C_\varepsilon \prod_{p|abc} p^{1+\varepsilon}$$

holds water, where $p/abc$ indicates that the product is over prime $p$ which divide the product $abc$. This is an unsolved problem hitherto although somebody published papers on the internet claiming proved it.

Abstract

We first get rid of three kinds from $A+B=C$ according to their respective odevity and $\text{gcf} (A, B, C) =1$. After that, expound relations between $C$ and $\text{raf} (ABC)$ by the symmetric law of odd numbers. Finally we have proven $C \leq C_\varepsilon [\text{raf} (ABC)]^{1+\varepsilon}$ in which case $A+B=C$, where $\text{gcf} (A, B, C) =1$.

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Values of A, B and C in set A+B=C

For positive integers A, B and C, let $\text{raf}(A, B, C)$ denote the product of all distinct prime factors of A, B and C, e.g. if $A = 11^2 \times 13$, $B = 3^3$ and $C = 2 \times 13 \times 61$, then $\text{raf}(A, B, C) = 2 \times 3 \times 11 \times 13 \times 61 = 52338$. In addition, let $\text{gcf}(A, B, C)$ denote greatest common factor of A, B and C.

The ABC conjecture states that given any real number $\varepsilon > 0$, there exists a constant $C_\varepsilon > 0$ such that for every triple of positive integers A, B and C satisfying $A+B=C$, and $\text{gcf}(A, B, C) = 1$, then we have $C \leq C_\varepsilon \left[ \text{raf}(ABC) \right]^{1+\varepsilon}$.

Let us first get rid of three kinds from $A+B=C$ according to their respective oddity and $\text{gcf}(A, B, C) = 1$, as listed below.

1. If A, B and C all are positive odd numbers, then A+B is an even number, yet C is an odd number, evidently there is only $A+B \neq C$ according to an odd number $\neq$ an even number.

2. If any two in A, B and C are positive even numbers, and another is a positive odd number, then when A+B is an even number, C is an odd number, yet when A+B is an odd number, C is an even number, so there is only $A+B \neq C$ according to an odd number $\neq$ an even number.

3. If A, B and C all are positive even numbers, then they have at least a common prime factor 2, manifestly this and the given prerequisite of $\text{gcf}(A, B, C) = 1$ are inconsistent, so A, B and C can not be three positive even numbers together.

Therefore we can only continue to have a kind of $A+B=C$, namely A, B and
C are two positive odd numbers and one positive even number. So let following two equalities add together to replace $A+B=C$ in which case $A$, $B$ and $C$ are two positive odd numbers and one positive even number.

1. $A+B=2^X S$, where $A$, $B$ and $S$ are three relatively prime positive odd numbers, and $X$ is a positive integer.

2. $A+2^Y V=C$, where $A$, $V$ and $C$ are three relatively prime positive odd numbers, and $Y$ is a positive integer.

Consequently the proof for ABC conjecture, by now, it is exactly to prove the existence of following two inequalities.

(1). $2^X S \leq C_\varepsilon [\text{raf} (A, B, 2^X S)]^{1+\varepsilon}$ in which case $A+B=2^X S$, where $A$, $B$ and $S$ are three relatively prime positive odd numbers, and $X$ is a positive integer.

(2). $C \leq C_\varepsilon [\text{raf} (A, 2^Y V, C)]^{1+\varepsilon}$ in which case $A+2^Y V =C$, where $A$, $V$ and $C$ are three relatively prime positive odd numbers, and $Y$ is a positive integer.

**Circumstances Relating to the Proof**

Let us divide all positive odd numbers into two kinds of $A$ and $B$, namely the form of $A$ is $1+4n$, and the form of $B$ is $3+4n$, where $n$ is a positive integer or 0. From small to large odd numbers of $A$ and of $B$ are arranged as follows.

A: 1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61, 65, 69…$1+4n$ …

B: 3, 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, 47, 51, 55, 59, 63, 67…$3+4n$ …

We list also from small to great natural numbers, well then you would discover that Permutations of seriate natural numbers show up a certain law.
Thus it can be seen, leave from any given even number >2, there are finitely many cycles of (B, A) leftwards until (B=3, A=1), and there are infinitely many cycles of (A, B) rightwards.

Evidently even numbers contain prime factor 2, yet others are odd numbers in the sequence of natural numbers above-listed.

After each of odd numbers in the sequence of natural numbers is replaced by self-belongingness, the sequence of natural numbers is changed into the following forms.

A, 2¹×11, B, 2³×3, A, 2¹×13, B, 2²×7, A, 2¹×15, B, 2⁵, A, 2¹×17, B, 2²×9, A
2¹×19, B, 2³×5, A, 2¹×21, B, 2²×11, A, 2¹×23, B, 2⁴×3, A, 2¹×25, B, 2²×13, A
2¹×27, B, 2³×7, A, 2¹×29, B, 2²×15, A, 2¹×31, B, 2⁶, A, 2¹×33, B, 2²×17, A
2¹×35, B, 2³×9, A, 2¹×37, B, 2²×19, A, 2¹×39, B, 2⁴×5, A, 2¹×41, B, 2²×21, A
2¹×43, B, 2³×11, A, 2¹×45, B, 2²×23, A, 2¹×47, B, 2⁵×3, A, 2¹×49, B, 2²×25,
A, 2¹×51, B …→

1, 2¹, 3, 2², 5, 2¹×3, 7, 2³, 9, 2¹×5, 11, 2²×3, 13, 2¹×7, 15, 2⁴, 17, 2¹×9, 19,
2²×5, 2¹×11, 23, 2³×3, 25, 2¹×13, 27, 2²×7, 29, 2¹×15, 31, 2⁵, 33, 2¹×17,
35, 2²×9, 37, 2¹×19, 39, 2³×5, 41, 2¹×21, 43, 2²×11, 45, 2¹×23, 47, 2⁴×3, 49,
2¹×25, 51, 2²×13, 53, 2¹×27, 55, 2³×7, 57, 2¹×29, 59, 2²×15, 61, 2¹×31, 63,
2⁶, 65, 2¹×33, 67, 2²×17, 69, 2¹×35, 71, 2³×9, 73, 2¹×37, 75, 2²×19, 77,
2¹×39, 79, 2⁴×5, 81, 2¹×41, 83, 2²×21, 85, 2¹×43, 87, 2³×11, 89, 2¹×45, 91,
2²×23, 93, 2¹×47, 95, 2⁵×3, 97, 2¹×49, 99, 2²×25, 101, 2¹×51, 103 …→
If we regard an even number on the sequence of natural numbers as a symmetric center of odd numbers, then two odd numbers of every bilateral symmetry are A and B always, and a sum of bilateral symmetric A and B is surely the double of the even number. For example, odd numbers 23(B) and 25(A), 21(A) and 27(B), 19(B) and 29(A) etc are bilateral symmetries whereby even number $2^3 \times 3$ to act as the center of the symmetry, and there are $23+25=2^4 \times 3$, $21+27=2^4 \times 3$, $19+29=2^4 \times 3$ etc. For another example, odd numbers 49(A) and 51(B), 47(B) and 53(A), 45(A) and 55(B) etc are bilateral symmetries whereby even number $2 \times 25$ to act as the center of the symmetry, and there are $49+51=2^2 \times 25$, $21+27=2^2 \times 25$, $19+29=2^2 \times 25$ etc. Again give an example, 63(B) and 65(A), 61(A) and 67(B), 59(B) and 69(A) etc are bilateral symmetries whereby even number $2^6$ to act as the center of the symmetry, and there are $63+65=2^7$, $61+67=2^7$, $59+69=2^7$ etc. Overall, if A and B are two bilateral symmetric odd numbers whereby $2^S$ to act as the center of the symmetry, then there is $A+B=2^{S+1}$. The number of A plus B on the left of $2^S$ is exactly the number of pairs of bilateral symmetric A and B. If we regard any finite-great even number $2^S$ as a symmetric center, then there are merely finitely more pairs of bilateral symmetric A and B, namely the number of pairs of A and B which express $2^{S+1}$ as the sum is finite. That is to say, the number of pairs of bilateral symmetric A and B for symmetric center $2^S$ is $2^{S-1}$, where $S \geq 1$.
whereby $2^X S$ to act as the center of the symmetry, then $A+B=2^{X+1} S$. By now, let A plus $2^{X+1} S$ makes $A+2^{X+1} S$, then B and A+2$^{X+1} S$ are still bilateral symmetry whereby $2^{X+1} S$ to act as the center of the symmetry, and 

$$B+(A+2^{X+1} S) = (A+B)+2^{X+1} S = 2^{X+1} S+2^{X+1} S = 2^{X+2} S.$$ 

If substitute B for A, let B plus $2^{X+1} S$ makes B+$2^{X+1} S$, then A and B+$2^{X+1} S$ are too bilateral symmetry whereby $2^{X+1} S$ to act as the center of the symmetry, and $A+ (B+2^{X+1} S) = 2^{X+2} S$. 

Provided both let A plus $2^{X+1} S$ makes A+$2^{X+1} S$, and let B plus $2^{X+1} S$ makes B+$2^{X+1} S$, then A+$2^{X+1} S$ and B+$2^{X+1} S$ are likewise bilateral symmetry whereby $3 \times 2^X S$ to act as the center of the symmetry, and $(A+2^{X+1} S)+ (B+2^{X+1} S) = 3 \times 2^{X+1} S$. 

Since there are merely A and B at two odd places of each and every bilateral symmetry on two sides of an even number as the center of the symmetry, then aforementioned $B+(A+2^{X+1} S)=2^{X+2} S$ and $A+(B+2^{X+1} S)=2^{X+2} S$ are exactly $A+B=2^{X+2} S$ respectively, and write $(A+2^{X+1} S)+(B+2^{X+1} S)=3 \times 2^{X+1} S$ down $A+B=3 \times 2^{X+1} S = 2^{X+1} S_t$, where $S_t$ is an odd number $\geq 3$.

Do it like this, not only equalities like as $A+B=2^{X+1} S$ are proven to continue the existence, one by one, but also they are getting more and more along with which $X$ is getting greater and greater, up to exist infinitely more equalities like as $A+B=2^{X+1} S$ when $X$ expresses every natural number.

In other words, added to a positive even number on two sides of $A+B=2^X S$, then we get still such an equality like as $A+B=2^X S$. 


Whereas no matter how great a concrete even number $2^X S$ as the center of the symmetry, there are merely finitely more pairs of A and B which express $2^{X+1} S$ as the sum.

If X is defined as a concrete positive integer, then there are only a part of $A+B=2^X S$ to satisfy $\gcd (A, B, 2^X S) = 1$. For example, when $2^X S = 18$, there are merely $1+17=18$, $5+13=18$ and $7+11=18$ to satisfy $\gcd (A, B, 2^X S) = 1$, yet $3+15=18$ and $9+9=18$ suit not because they have common prime factor 3.

If add or subtract a positive odd number on two sides of $A+B=2^X S$, then we get another equality like as $A+2^Y V=C$. That is to say, equalities like as $A+2^Y V=C$ can come from $A+B=2^{X+1} S$ so as add or subtract a positive odd number on two sides of $A+B=2^{X+1} S$.

Therefore, on the one hand, equalities like as $A+2^Y V=C$ are getting more and more along with which equalities like as $A+B=2^{X+1} S$ are getting more and more, up to infinite more equalities like as $A+2^Y V=C$ exist along with which infinite more equalities like as $A+B=2^{X+1} S$ appear.

Certainly we can likewise transform $A+2^Y V=C$ into $A+B=2^X S$ so as add or subtract a positive odd number on the two sides of $A+2^Y V=C$.

On the other hand, if C is only defined as a concrete positive odd number, then there is merely finitely more pairs of A and $2^Y V$ which express C as the sum. But also, there is probably a part of $A+2^Y V=C$ to satisfy $\gcd (A, 2^Y V, C) = 1$. For example, when $C=25$, there are merely $1+24=25$, $3+22=25$, $7+18=25$, $9+16=25$, $11+14=25$ and $13+12=25$ to satisfy $\gcd (A, 2^Y V, C) = 1$, yet
5+20=25 and 15+10=25 suit not because they have common prime factor 5.

After factorizations of A, B, S, V and C in A+B=2^{X+1}S plus A+2^YV=C, if part prime factors have greater exponents, then there are both 2^{X+1}S ≥ raf (A, B, 2^{X+1}S) in which case A+B=2^{X+1}S satisfying gcf (A, B, 2^{X+1}S) =1, and C ≥ raf (A, 2^YV, C) in which case A+2^YV=C satisfying gcf (A, 2^YV, C) =1. For examples, 2^7 > raf (3, 5^3, 2^7) for 3+5^3=2^7; and 3^{10} > raf (5^6, 2^5×23×59, 3^{10}) for 5^6+2^5×23×59=3^{10}.

On the contrary, there are both 2^{X+1}S ≤ raf (A, B, 2^{X+1}S) in which case A+B=2^{X+1}S satisfying gcf (A, B, 2^{X+1}S) =1, and C ≤ raf (A, 2^YV, C) in which case A+2^YV=C satisfying gcf (A, 2^YV, C) =1. For examples, 2^2×7 < raf (13, 3×5, 2^2×7) for 13+3×5=2^2×7; and 3^4 < raf (11×7, 2^2, 3^4) for 11×7+2^2 = 3^4.

Since either A or B in A+B=2^{X+1}S plus an even number is still an odd number, and 2^{X+1}S plus the even number is still an even number, thereby we can use equality A+B=2^{X+1}S to express every equality which plus an even number on two sides of A+B=2^{X+1}S makes.

Consequently, there are infinitely more 2^{X+1}S ≥ raf (A, B, 2^{X+1}S) plus 2^{X+1}S ≤ raf (A, B, 2^{X+1}S) in which case A+B=2^{X+1}S.

Likewise, either 2^YV plus an even number is still an even number, or A plus an even number is still an odd number, and C plus the even number is still an odd number, so we can use equality A+2^YV=C to express every equality which plus an even number on two sides of A+2^YV=C makes.

Consequently, there are infinitely more C ≥ raf (A, 2^YV, C) plus C ≤ raf (A,
in which case \( A + 2^Y V = C \).

But, if let \( 2^{X+1} S \geq \text{raf} (A, B, 2^{X+1} S) \) and \( 2^{X+1} S \leq \text{raf} (A, B, 2^{X+1} S) \) separate, and let \( C \geq \text{raf} (A, 2^Y V, C) \) and \( C \leq \text{raf} (A, 2^Y V, C) \) separate, then for inequalities like as each kind of them, we conclude not out whether they are still infinitely more.

However, what deserve to be affirmed is that there are \( 2^{X+1} S \geq \text{raf} (A, B, 2^{X+1} S) \) and \( 2^{X+1} S \leq \text{raf} (A, B, 2^{X+1} S) \) in which case \( A + B = 2^{X+1} S \) satisfying \( \text{gcf} (A, B, 2^{X+1} S) = 1 \), and there are \( C \geq \text{raf} (A, 2^Y V, C) \) and \( C \leq \text{raf} (A, 2^Y V, C) \) in which case \( A + 2^Y V = C \) satisfying \( \text{gcf} (A, 2^Y V, C) = 1 \), according to the preceding illustration with examples.

\[ \text{Proving } C \leq C_\varepsilon [\text{raf} (A, B, C)]^{1+\varepsilon} \]

Hereinbefore, we have deduced that both there are \( 2^{X+1} S \leq \text{raf} (A, B, 2^{X+1} S) \) and \( 2^{X+1} S \geq \text{raf} (A, B, 2^{X+1} S) \) in which case \( A + B = 2^X S \) satisfying \( \text{gcf} (A, B, 2^{X+1} S) = 1 \), and there are \( C \leq \text{raf} (A, 2^Y V, C) \) and \( C \geq \text{raf} (A, 2^Y V, C) \) in which case \( A + 2^Y V = C \) satisfying \( \text{gcf} (A, 2^Y V, C) = 1 \), whether each kind of them is infinitely more, or is finitely more.

First let us expound a set of identical substitution as the follows. If an even number on the right side of each of above-mentioned four inequalities added to a smaller non-negative real number such as \( R \geq 0 \), then the result is both equivalent to multiply the even number by another very small real number, and equivalent to increase a tiny real number such as \( \varepsilon \geq 0 \) to the exponent of
the even number, i.e. form a new exponent \(1+\varepsilon\), but when \(R=0\), the multiplied real number is 1, yet \(\varepsilon = 0\). Actually, aforementioned three ways of doing, all are in order to increase an identical even number into a value and the same.

Such being the case the identical substitution between each other, then we set about proving aforesaid four inequalities, one by one, thereafter.

**(1).** For inequality \(2^{x+1}S \leq \text{raf}(A, B, 2^{x+1}S)\), \(2^{x+1}S\) divided by \(\text{raf}(A, B, 2^{x+1}S)\) is equal to \(2^{x}S_{1}^{-1}~S_{n}^{-m-1}/A_{\text{raf}}B_{\text{raf}}\) as a true fraction, where \(S_{1}~S_{n}\) express all distinct prime factors of \(S\); \(t-1~m-1\) are respectively exponents of prime factors \(S_{1}~S_{n}\) orderly; \(A_{\text{raf}}\) expresses the product of all distinct prime factors of \(A\); and \(B_{\text{raf}}\) expresses the product of all distinct prime factors of \(B\).

After that, even number \(\text{raf}(A, B, 2^{x+1}S)\) added to a smaller non-negative real number such as \(R \geq 0\) to turn the even number itself into \([\text{raf}(A, B, 2^{x+1}S)]^{1+\varepsilon}\).

Undoubtedly there is \(2^{x+1}S \leq [\text{raf}(A, B, 2^{x+1}S)]^{1+\varepsilon}\) successively.

By now, multiply \([\text{raf}(A, B, 2^{x+1}S)]^{1+\varepsilon}\) by \(2^{x}S_{1}^{-1}~S_{n}^{-m-1}/A_{\text{raf}}B_{\text{raf}}\), then it has still \(2^{x+1}S \leq 2^{x}S_{1}^{-1}~S_{n}^{-m-1}/A_{\text{raf}}B_{\text{raf}}[\text{raf}(A, B, 2^{x+1}S)]^{1+\varepsilon}\).

Also let \(C_{\varepsilon} = 2^{x}S_{1}^{-1}~S_{n}^{-m-1}/A_{\text{raf}}B_{\text{raf}}\), we get \(2^{x+1}S \leq C_{\varepsilon} [\text{raf}(A, B, 2^{x+1}S)]^{1+\varepsilon}\).

Manifestly when \(R=0\), it has \(\varepsilon = 0\), and \(2^{x+1}S = C_{\varepsilon} [\text{raf}(A, B, 2^{x+1}S)]^{1+\varepsilon}\).

**(2).** For inequality \(C \leq \text{raf}(A, 2^{y}V, C)\), \(C\) divided by \(\text{raf}(A, 2^{y}V, C)\) is equal to \(C_{1}^{-1}~C_{e}^{-\varepsilon}/2A_{\text{raf}}V_{\text{raf}}\) as a true fraction, where \(C_{1}~C_{e}\) express all distinct prime
factors of C; \(j-1 \sim f-1\) are respectively exponents of prime factors \(C_1 \sim C_e\) orderly; 
\(A_{raf}\) expresses the product of all distinct prime factors of A; and \(V_{raf}\) expresses the product of all distinct prime factors of V.

After that, even number \(raf (A, 2^Y V, C)\) added to a smaller non-negative real number such as \(R \geq 0\) to turn the even number itself into \([raf (A, 2^Y V, C)]^{1+\varepsilon}\).

Undoubtedly there is \(C \leq [raf (A, 2^Y V, C)]^{1+\varepsilon}\) successively.

By now, multiply \([raf (A, 2^Y V, C)]^{1+\varepsilon}\) by \(C_1^{j-1} \sim C_e^{f-1}/2A_{raf}V_{raf}\), then it has still \(C \leq C_1^{j-1} \sim C_e^{f-1}/2A_{raf}V_{raf} [raf (A, 2^Y V, C)]^{1+\varepsilon}\).

Also let \(C_\varepsilon = C_1^{j-1} \sim C_e^{f-1}/2A_{raf}V_{raf}\), we get \(C \leq C_\varepsilon [raf (A, 2^Y V, C)]^{1+\varepsilon}\).

Manifestly when \(R=0\), it has \(\varepsilon = 0\), and \(C = C_\varepsilon [raf (A, 2^Y V, C)]^{1+\varepsilon}\).

(3). For inequality \(2^{X+1} S \geq raf (A, B, 2^{X+1} S)\), \(2^{X+1} S\) divided by \(raf (A, B, 2^{X+1} S)\) is equal to \(2^X S_1^{t-1} \sim S_n^{m-1}/A_{raf}B_{raf}\) as a false fraction, where \(S_1 \sim S_n\) express all distinct prime factors of S; \(t-1 \sim m-1\) are respectively exponents of prime factors \(S_1 \sim S_n\) orderly; \(A_{raf}\) expresses the product of all distinct prime factors of A; and \(B_{raf}\) expresses the product of all distinct prime factors of B.

Evidently \(2^X S_1^{t-1} \sim S_n^{m-1}/A_{raf}B_{raf}\) as the false fraction is greater than 1.

Then, even number \(raf (A, B, 2^{X+1} S)\) added to a smaller non-negative real number such as \(R \geq 0\) to turn the even number itself into \([raf (A, B, 2^{X+1} S)]^{1+\varepsilon}\).

After that, multiply \([raf (A, B, 2^{X+1} S)]^{1+\varepsilon}\) by \(2^X S_1^{t-1} \sim S_n^{m-1}/A_{raf}B_{raf}\), then it has \(2^{X+1} S \leq 2^X S_1^{t-1} \sim S_n^{m-1}/A_{raf}B_{raf} [raf (A, B, 2^{X+1} S)]^{1+\varepsilon}\).

Let \(C_\varepsilon = 2^X S_1^{t-1} \sim S_n^{m-1}/A_{raf}B_{raf}\), we get \(2^{X+1} S \leq C_\varepsilon [raf (A, B, 2^{X+1} S)]^{1+\varepsilon}\).
Manifestly when \( R = 0 \), it has \( \varepsilon = 0 \), and \( 2^{X+1}S = C_\varepsilon [\text{raf (A, B, } 2^{X+1}S)]^{1+\varepsilon} \).

(4). For inequality \( C \geq \text{raf (A, } 2^YV, C) \), \( C \) divided by \( \text{raf (A, } 2^YV, C) \) is equal to \( C_1^{j-1} \sim C_c^{f-1}/2A_{\text{raf}}V_{\text{raf}} \) as a false fraction, where \( C_1 \sim C_c \) express all distinct prime factors of \( C \); \( j-1 \sim f-1 \) are respectively exponents of prime factors \( C_1 \sim C_c \) orderly; \( A_{\text{raf}} \) expresses the product of all distinct prime factors of \( A \); and \( V_{\text{raf}} \) expresses the product of all distinct prime factors of \( V \).

Evidently \( C_1^{j-1} \sim C_c^{f-1}/2A_gV_q \) as the false fraction is greater than 1.

Then, even number \( \text{raf (A, } 2^YV, C) \) added to a smaller non-negative real number such as \( R \geq 0 \) to turn the even number itself into \( [\text{raf (A, } 2^YV, C)]^{1+\varepsilon} \).

After that, multiply \( [\text{raf (A, } 2^YV, C)]^{1+\varepsilon} \) by \( C_1^{j-1} \sim C_c^{f-1}/2A_{\text{raf}}V_{\text{raf}} \), then it has \( C \leq C_1^{j-1} \sim C_c^{f-1}/2A_{\text{raf}}V_{\text{raf}}[\text{raf (A, } 2^YV, C)]^{1+\varepsilon} \).

Let \( C_\varepsilon = C_1^{j-1} \sim C_c^{f-1}/2A_{\text{raf}}V_{\text{raf}} \), we get \( C \leq C_\varepsilon [\text{raf (A, } 2^YV, C)]^{1+\varepsilon} \).

Manifestly when \( R = 0 \), it has \( \varepsilon = 0 \), and \( C = C_\varepsilon [\text{raf (A, } 2^YV, C)]^{1+\varepsilon} \).

We have concluded \( C_\varepsilon = 2^XS_1^{t-1}S_n^{m-1}/A_{\text{raf}}B_{\text{raf}} \) and \( C_\varepsilon = C_1^{j-1} \sim C_c^{f-1}/2A_{\text{raf}}V_{\text{raf}} \) in preceding proofs, evidently each and every \( C_\varepsilon \) is a constant because it consists of known numbers.

Besides, for a smaller non-negative real number \( R \geq 0 \), actually, it is merely comparatively speaking, if \( \text{raf (A, B, } 2^{X+1}S) \) or \( \text{raf (A, } 2^YV, C) \) is very great a positive even number such as \( 2 \times 11 \times 13 \times 99991 \times 99989 \times 99961 \times 99929 \times 99923 \times 87641 \times 72223 \times 8117 \times 12347 \), then even if \( R = 2015.11223 \sqrt{2} \), it is also a
smaller non-negative real number. Since raf \((A, B, 2^{X+1}S)\) or raf \((A, 2^YV, C)\) may be infinity, so \(R\) may tend to infinity.

Taken one with another, we have proven that there are both infinitely more \(2^{X+1}S \leq C_ε [raf (A, B, 2^{X+1}S)]^{1+ε}\) when \(X\) is each and every natural number, and infinitely more \(C \leq C_ε [raf (A, 2^YV, C)]^{1+ε}\) when \(C\) is each and every positive odd number \(\geq 1\).

But then, when \(X\) is a concrete natural number, even if the concrete natural number tends to infinity, there also are merely finitely more \(2^{X+1}S \leq C_ε [raf (A, B, 2^{X+1}S)]^{1+ε}\) in which case \(A+B=2^{X+1}S\).

When \(C\) is a concrete positive odd number, even if the concrete positive odd number tends to infinity, there also are merely finitely more \(C \leq C_ε [raf (A, 2^YV, C)]^{1+ε}\) in which case \(A+2^YV=C\).

To sum up, the proof is completed by now. Consequently the ABC conjecture does hold water.