The Abstract

An Approximation to the Mass Ratio of the Proton to the Electron.
The mass ratio of the proton to the electron is a dimensionless physical constant. The following expression provides a good estimate.

$$(4\pi)\left(4\pi - \frac{1}{\pi}\right)\left(4\pi - \frac{2}{\pi}\right) = 1836.15 \tag{1}$$

The above expression is also the greatest lower bound (GLB) for a more general expression. A geometric persuasion is given and the more general expression is determined via the inversion of the spheres.

The Background

My name is Harry Watson. Today I am presenting a model for an approximation. The idea begins with a ball of radius one and a line segment of length 4π. The line segment is attached to the ball at one end point and is tangent to the ball. This is referred to as *the system*. Assume that the system is isolated in 3-space.

Figure 1: The "Stick and Ball" System

Figure 2 shows the region of space traced out by the line segment. Define the point of attachment of the line segment to the ball as the origin $(0, 0, 0)$ and the center of the ball to be $(0, 0, 1)$. We rotate the system about the three axes. Rotation about the z-axis defines a disk of radius 4π in the xy-plane.

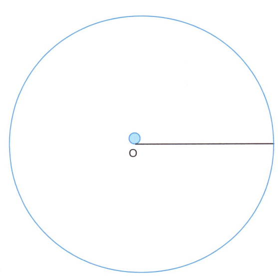

Figure 2: The Disk in the yz-plane

In Figure 3 we see the trace of the ball. The radius of 2 is used to do the Inversion of the Spheres, or more correctly: The Inversion of the Circles in two dimensions. This inversion creates a disk of radius 4π and an inner deleted disk of radius $1/\pi$.

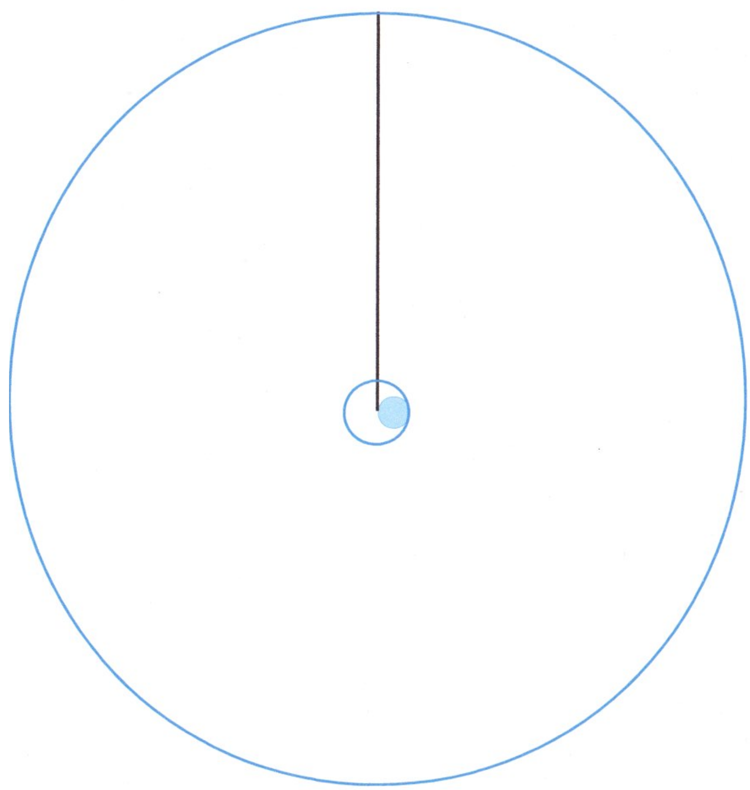

Figure 3: Rotation about the y-axis (zx-plane)

Rotating the system about the y-axis followed by rotation about the x-axis doubly defines the interior disk of radius 2. To accommatate the area, doubling the area requires the interior disk in the yz-plane to have the effecctive radius of $2\sqrt{2}$.

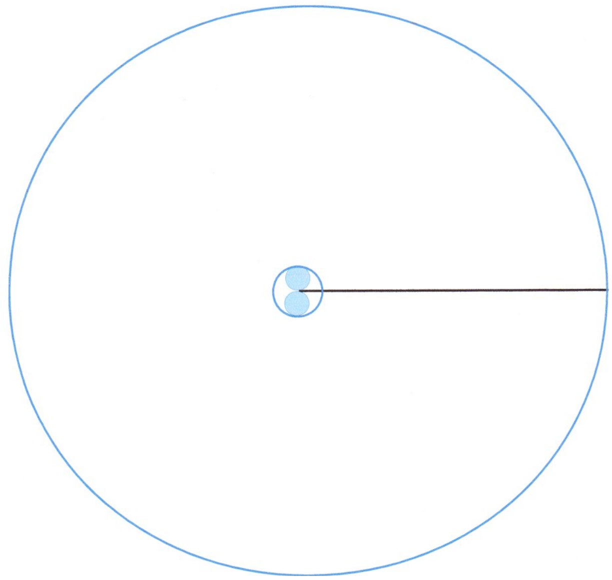

Figure 4: Rotation about the x-axis (yz-plane)

Multiply the three radii measures: 4π, $4\pi - 1/\pi$, and $4\pi - 2/\pi$; this product nearly equals the mass ratio of the proton to the electron, a dimensionless physical constant. These operations underwrite an interesting result:

$$4\pi(4\pi - 1/\pi)(4\pi - 2/\pi) = 1836.15 \tag{2}$$

The Model

Preface: This short article defines a function which models some characteristics of the proton. A general function is created which gives a close approximation to the proton-electron mass ratio. The *Greatest Lower Bound* (GLB) is derived as a limiting case for the ratio.

It has been said that all models are wrong, but some are useful.[1] The proton is no longer considered a fundamental particle; it admits the possibility of being modeled by fundamental particles. In particular, we consider the "stick and ball" model. Let M_e denote the mass of the electron and M_p denote the mass of the proton. In a gravitational field, $M_p/M_e \approx 1836.15$.

[1] Box, George E. P., (1919-2013) British mathematician

Let T denote a "Test Function" such that for each $\tau_y \in [0, \pi^{-1}]$ and for each $\tau_x \in [0, 2\pi^{-1}]$

$$T(\tau_y, \tau_x) = (4\pi)\left(4\pi - \frac{1}{\pi} + \tau_y\right)\left(4\pi - \frac{2}{\pi} + \tau_x\right) \tag{3}$$

When $\tau_y = \tau_x = 0$, we have $T(0, 0) \approx 1836.15$; when $\tau_y = \pi^{-1}$, $\tau_x = 2\pi^{-1}$ we have $T(\pi^{-1}, 2\pi^{-1}) = (4\pi)^3 \approx 1984.40$. For convenience, let $T_0 = T(0, 0)$ and let $T_\infty = T(\pi^{-1}, 2\pi^{-1})$

$$T : [0, \pi^{-1}] \times [0, 2\pi^{-1}] \to [T_0, (4\pi)^3]$$

For each $T_1 \in [T_0, T_\infty]$ there exists a point (τ_{y_1}, τ_{x_1}) such that

$$T_1 = T(\tau_{y_1}, \tau_{x_1})$$

T_1 is a measure of the relative mass inherent in the space inverted into the ball having radius 4π. It represents one test result from measuring the mass ratio of the proton to the electron. Claim that T_0 is the GLB of all such measurements.

Remark: Although $2\tau_y$ and τ_x may not be equal, we will assume their equality for ease of computation. Let $\tau = \tau_y = \tau_x/2$. Let $T_1 = M_p/M_e = 1836.15267245$. (This the current recommended CODATA value.) We can solve Equation (4) for τ.

$$T(\tau, 2\tau) = 1836.15267245 = (4\pi)\left(4\pi - \frac{1}{\pi} + \tau\right)\left(4\pi - \frac{2}{\pi} + 2\tau\right) \tag{4}$$

(It is *quadratic* in τ.) Using a spreadsheet, the positive root is given by $\tau =$ `2.03869333148532E-006`. We substitute it back into Equation (4) to check. $T(0.00000203869) = 1836.15267245$.

Again, using a spreadsheet, we can iterate and determine the values of τ_y and τ_x to high precision. We have:

```
tau_y       tau_x       CODATA value
0.000002036 0.000004080 1836.15267245
0.000002035 0.000004081 1836.15267245
```

These values for τ_y and τ_x satisfy Equation (3) to eight decimal places. Notice that $\tau_y \approx \tau_x/2$. (But we knew that already.)

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