**The Abstract**

*An Approximation to the Mass Ratio of the Proton to the Electron.*
The mass ratio of the proton to the electron is a dimensionless physical constant. The following expression provides a good estimate.

$$(4\pi)\left(4\pi - \frac{1}{\pi}\right)\left(4\pi - \frac{2}{\pi}\right) = 1836.15 \tag{1}$$

The above expression is also the greatest lower bound (GLB) for a more general expression. A geometric persuasion is given and the more general expression is determined via the inversion of the spheres.

**The Background**

My name is Harry Watson. Today I am presenting a model for an approximation. I discovered the approximation on 28 June 1989. The idea begins with a ball of radius one and a line segment of length $4\pi$. The line segment is attached to the ball at one end point and is tangent to the ball. This is referred to as *the system.* Assume that the system is isolated in 3-space.

Figure 1: The "Stick and Ball" System

Figure 2 shows the region of space traced out by the line segment. Define the point of attachment of the line segment to the ball as the origin $(0, 0, 0)$ and the center of the ball to be $(0, 0, 1)$. We rotate the system about the three axes. Rotation about the $z$-axis defines a disk of radius $4\pi$ in the $xy$-plane.

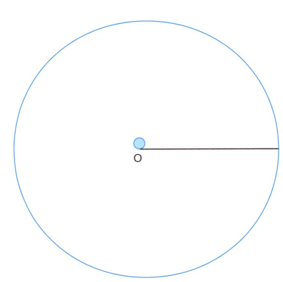

Figure 2: The Disk in the $yz$-plane

In Figure 3 we see the trace of the ball. The radius of 2 is used to do the Inversion of the Spheres, or more correctly: The Inversion of the Circles in two dimensions. This inversion creates a disk of radius $4\pi$ and an inner deleted disk of radius $1/\pi$.

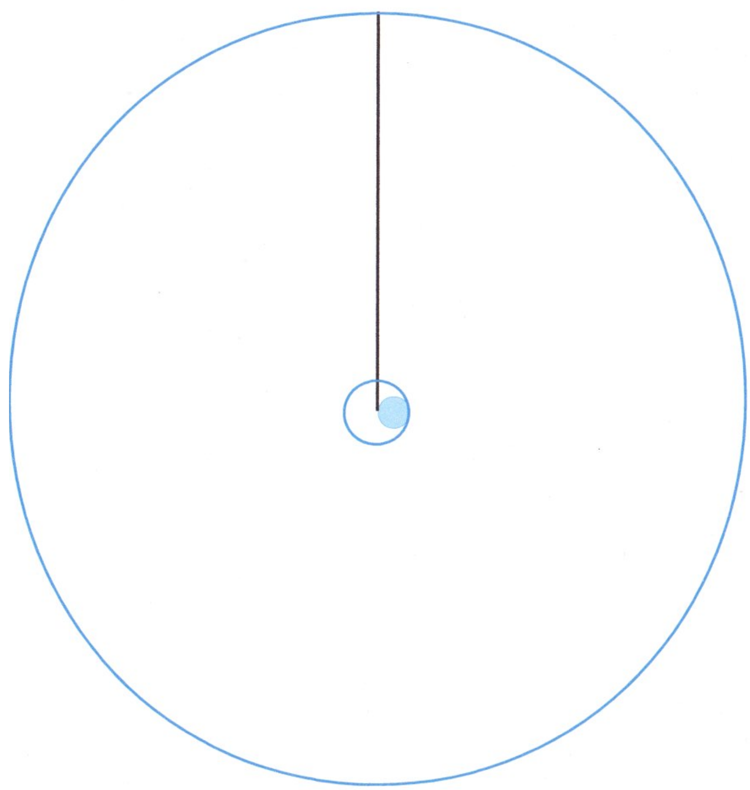

Figure 3: Rotation about the $y$-axis ($zx$-plane)

Rotating the system about the $y$-axis followed by rotation about the $x$-axis doubly defines the interior disk of radius 2. To accommate the area, doubling the area requires the interior disk in the $yz$-plane to have the effecctive radius of $2\sqrt{2}$.

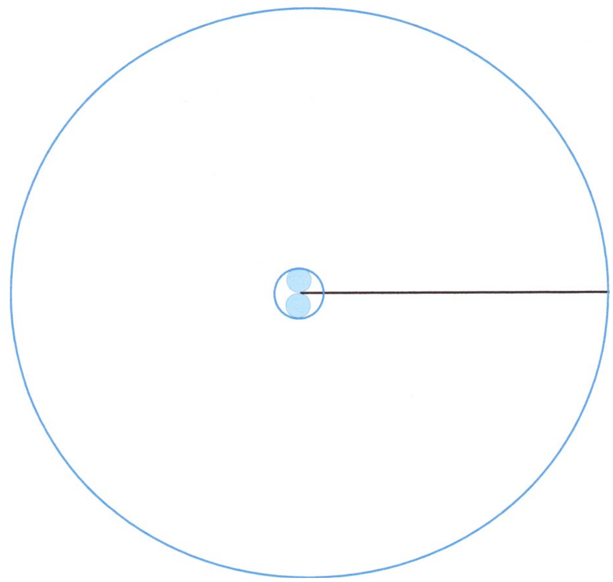

Figure 4: Rotation about the $x$-axis ($yz$-plane)

Multiply the three radii measures: $4\pi$, $4\pi - 1/\pi$, and $4\pi - 2/\pi$; this product nearly equals the mass ratio of the proton to the electron, a dimensionless physical constant. These operations underwrite an interesting result:

$$4\pi(4\pi - 1/\pi)(4\pi - 2/\pi) = 1836.15 \tag{2}$$

## The Model

*Preface:* This short article defines a function which models some characteristics of the proton. A general function is created which gives a close approximation to the proton-electron mass ratio. The *greatest lower bound* (GLB) is derived as a limiting case for the ratio.

It has been said that all models are wrong, but some are useful.[1] The proton is no longer considered a fundamental particle; it admits the possibility of being modeled by fundamental particles. In particular, we consider the "ball and stick" model. Let $M_e$ denote the mass of the electron and $M_p$ denote the mass of the proton. In a gravitational field, $M_p/M_e \approx 1836.15$. Let $\chi(x)$

[1] Box, George E. P., (1919-2013) British mathematician

and $\xi(y)$ be defined as follows.

$$\chi(x) = \begin{cases} 1 & \forall \quad 2/\pi \leq x \leq 4\pi \\ 2\varepsilon(\frac{x}{2}) & \forall \quad 0 \leq x < 2/\pi \end{cases}$$

$$\xi(y) = \begin{cases} 1 & \forall \quad 1/\pi \leq y \leq 4\pi \\ \varepsilon(y) & \forall \quad 0 \leq y < 1/\pi \end{cases}$$

The function $\varepsilon$ maps the interval $[0, 1/\pi]$ into the interval $[0, 1]$ such that $\varepsilon(0) = 0$, $\varepsilon(1/\pi) = 1$, and $\forall\, y \in (0, 1/\pi)$, $\varepsilon(y)$ is the fraction of surface area that particles of positive mass intersect with a sphere of radius $4/y$ and center $(0, 0, 0)$. Recall that a sphere of radius $r$ is the point set $\{(x, y, z) | \sqrt{x^2 + y^2 + z^2} = r\}$. The Inversion of the Spheres sends $y$ to $4/y$. Thus we have an elementary equation.

$$M_p/M_e = \int_0^{4\pi}\int_0^{4\pi}\int_0^{4\pi} \chi(x)\,\xi(y)\,dx\,dy\,dz \tag{3}$$

**The Environment** The environment external to the closed ball of radius $4\pi$ affects the determination of the mass ratio. This is modeled in Equation (3) by the functions $\chi$ and $\xi$, which are, in turn, determined by the function $\varepsilon$, described above.

**An Integral** The function $\varepsilon$ is dependent on the test environment. A related variable $\tau$ is defined as follows.

$$\int_0^{4\pi}\int_0^{4\pi}\int_0^{4\pi} \chi(x)\,\xi(y)\,dx\,dy\,dz = 4\pi\left(\int_0^{4\pi}\int_0^{4\pi} \chi(x)\,\xi(y)\,dx\,dy\right) \tag{4}$$

$$= 4\pi\left(\left(4\pi - \frac{1}{\pi}\right) + \int_0^{1/\pi} \xi(y)\ dy\right)\left(\left(4\pi - \frac{2}{\pi}\right) + \int_0^{2/\pi} \chi(x)\,dx\right) \tag{5}$$

$$\approx 4\pi\left((4\pi - 1/\pi) + \tau\right)\left((4\pi - 2/\pi) + 2\tau\right) \tag{6}$$

Let $M_p/M_e = 1836.15267245$. (This the current recommended CODATA value.) A small change in $\tau$ results in a significant change in the result. The relative difference between Equation (1) and the CODATA value is $(1836.15267245 - 4\pi(4\pi - 1/\pi)(4\pi - 2/\pi))/1836.15267245 = 0.000000508$.

Setting $\chi$ and $\xi$ equal to zero in the regions $[0, 1/\pi]$, and $[0, 2/\pi]$, respectively, an interesting integral can be obtained. This is the *Greatest Lower Bound* (GLB) that can be determined by experimentation.

$$M_p/M_e \approx \int_0^{4\pi} \int_{\frac{1}{\pi}}^{4\pi} \int_{\frac{2}{\pi}}^{4\pi} dx\, dy\, dz = 1836.15\ldots \tag{7}$$

$$\int_0^{4\pi} \int_{\frac{1}{\pi}}^{4\pi} \int_{\frac{2}{\pi}}^{4\pi} dx\, dy\, dz = (4\pi)\left(4\pi - \frac{1}{\pi}\right)\left(4\pi - \frac{2}{\pi}\right) \tag{8}$$

Setting $\chi$ and $\xi$ equal to one in the regions $[0, 1/\pi]$, and $[0, 2/\pi]$, respectively, an another interesting integral can be obtained:

$$\int_0^{4\pi} \int_0^{4\pi} \int_0^{4\pi} dx\, dy\, dz = (4\pi)^3 = 1984.40\ldots \tag{9}$$

This is what would model the event where there was no inversion of the spheres. This model gives the appearance of inherent instability.

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