Three functions based on the digital sum of a number and ten conjectures

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Abstract. In this paper I present three functions based on the digital sum of a number which might be interesting to study and ten conjectures. These functions are: (I) F(x) defined as the digital sum of the number 2^x – x^2; (II) G(x) equal to F(x) – x and (III) H(x) defined as the digital sum of the number 2^x + x^2.

(I)

Let F(x) be the sum of the digits of the number 2^x – x^2, where x is an odd positive number. Then:

Conjecture 1:

There exist an infinity of primes p such that F(p) = p. Such primes p are 13, 61 (...). Note that, up to x = 241, there is no other odd number x for which F(x) = x.

Conjecture 2:

There exist an infinity of pairs of twin primes (p, q) such that F(p) = F(q). Such pairs are (59, 61), (239, 241) (...) with corresponding F(p) = F(q) equal to 61, 331 (...).

Conjecture 3:

There exist an infinity of pairs of primes (p, q) such that F(p) = q. Such pairs are (5, 7), (11, 19), (23, 43), (29, 37), (43, 61), (59, 61), (101, 109), (157, 229), (167, 241), (239, 331), (241, 331) (...).

Conjecture 4:

There exist an infinity of pairs of primes (p, q) such that F(p) = q^2. Such pairs are (31, 7), (83, 11), (103, 11), (...).

Conjecture 5:

There exist an infinity of pairs of primes (p, q) such that F(p^2) = q. Such pairs are (13, 223), (19, 541), (29, 1129), (...).

Conjecture 6:

There exist an infinity of pairs of primes (p, F(p)) such that F(p) - p = 2 (in other words, p and F(p) are twin primes). Such pairs of twin primes are (5, 7), (59, 61) (...).

(II)

Let G(x) = F(x) - x, where x and F(x) are those defined above. Then:

Conjecture 7:

There exist an infinity of pairs of primes (p, F(p)) such that G(p) is a multiple of 9. Such pairs of primes are (43, 61) (...) with corresponding G(p) equal to 18 (...).

Conjecture 8:

There exist an infinity of pairs of primes (p, F(p)) such that G(p) is a power of the number 2. Such pairs of primes are (5, 7), (11, 19), (29, 37), (101, 109) (...) with corresponding exponents (powers of 2): 1, 3, 3, 3 (...).

Conjecture 9:

There exist an infinity of primes p such that G(p) is also prime. Such pairs of primes (p, G(p)) are (17, 5), (41, 11), (47, 17), (53, 23), (71, 5), (113, 47), (173, 53) (...).

Problem 1:

Which is the longest possible sequence of ordered odd numbers n such that F(n) has the same value for all of them? The longest sequence I met is: 75, 81, 87, 93, 99, for all of them F(n) having the value 116.

Problem 2:

Which have in common the odd numbers n for that F(n) is equal to a power of two (such number is the prime 179 for which F(p) = 256)?

(III)

Let H(x) be the sum of the digits of the number 2^x + x^2, where x is an odd positive number. Then:

Conjecture 10:

There exist an infinity of pairs of twin primes (x = 11 + 18*k, y = 13 + 18*k) such that H(x) = H(y). Such pairs of twin primes are: (11, 13), (29, 31), (101, 103), (191, 193), (227, 229), (569, 571) with corresponding H(x) = H(y) equal to: 18, 45, 117, 243, 315, 810 (...).