A Critical Note on the Relativistic Doppler Shift

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Abstract

Analysis of the Einstein's general formula for the relativistic Doppler shift, derived in his 1905 paper *On the Electrodynamics of Moving Bodies*, revealed for the case of relative circular motions a critical contradiction with the relativistic time dilation prediction obtained in the same paper for reference frames in a relative rectilinear, uniform motion (i.e., relatively moving inertial frames). The time dilation under consideration was obtained for the traveling frame motion over an extremely small fraction (to the order of one light wavelength) of the circular path, so as the respective relative motion could be approximated with a rectilinear one.

Keywords: *Special Relativity, Einstein 1905, Relativistic Doppler shift.*

1. Introduction

In his 1905 paper,[1] Einstein derived the Lorentz transformation equations for the space and time coordinates on the basis of the relativity principle and the constancy of the speed of light. Transformation equations for the electric and magnetic forces were then deduced from the Maxwell-Hertz and the former equations. The obtained electrodynamics transformations applied on the wave equations for light led to the general relativistic Doppler shift formula.

The classical Doppler shift differs from relativistic one by the gamma factor; that is the Lorentz time dilation—or length contraction—factor. For circular motions where the relative motion is such that the observer perceives the light source as rotating in a circular path, the center of which is occupied by the observer, no Doppler effect would be observed when using the classical approach. However, it will be shown that the corresponding relativistic Doppler shift is in contradiction with the time dilation prediction of the Special Relativity.

2. Relativistic Doppler Shift Contradiction

In §3 of the cited paper,[1] the time transformation equation converting event time between two inertial frames in relative motion of velocity v, having the coordinate systems $K(x, y, z, t)$ and $k(\xi, \eta, \zeta, \tau)$ associated with what's considered as "stationary" and "moving" frame, respectively, is obtained as

$$\tau = \beta\left(t - \frac{vx}{c^2}\right), \qquad (1)$$

where

$$\beta = \left(\sqrt{1 - v^2 / c^2}\right)^{-1}, \text{ and } c = \text{ speed of light in empty space.}$$

In §7 of the same paper, a light (electrodynamics waves) source, with given wave characteristics, is considered in the stationary system at a sufficiently far distance from the origin. The characteristics of these waves were to be determined when observed from the moving frame. We quote the following passage:

From the equation for ω' it follows that if an observer is moving with velocity v relatively to an infinitely distant source of light of frequency ν, in such a way that the connecting line "source-observer" makes the angle ϕ with the velocity of the observer referred to a system of co-ordinates which is at rest relatively to the source of light, the frequency ν' of the light perceived by the observer is given by the equation

$$\nu' = \nu \frac{1 - \cos\phi \ . \ v / c}{\sqrt{1 - v^2 / c^2}}. \qquad (2)$$

This is Doppler's principle for any velocities whatever. When $\phi = 0$ the equation assumes the perspicuous form

$$\nu' = \frac{\sqrt{1 - v / c}}{\sqrt{1 + v / c}}. \qquad (3)$$

The angle ϕ is the instantaneous angle between the source-observer line and the velocity of the observer relative to the source rest frame. Since, as stated by Einstein, Eq. (2) is applicable for any velocities, the particular case when $\phi = \pi / 2$, not discussed in the cited paper, shall be considered. It corresponds to the observer moving in a circular motion around the source; or from the perspective of the observer, the source is rotating around the observer occupying the center of the source circular path. Using the above general Doppler shift equation for $\phi = \pi / 2$, we get

$$\nu' = \nu \frac{1}{\sqrt{1 - v^2 / c^2}}. \qquad (4)$$

Or, in terms of the wave period

$$T' = T\sqrt{1 - v^2 / c^2}, \qquad (5)$$

or

$$T' = \frac{1}{\beta} T. \qquad (6)$$

Since for the special case of $\phi = \pi \,/\, 2,$ there would be no change in the relative distance between the source and the observer (circular motion), the only justification of the Doppler shift would be the relativistic time dilation.—Obviously, there would be no Doppler effect for this case in the classical approach where $\nu' = \nu(1 - \cos\phi \ . \ v \,/\, c)$, yielding $\nu' = \nu$.

However, the events under consideration here, separated by the wave period, are the emission of two consecutive light impulses, defining a wave cycle, from the source that is at rest in its reference system K; i.e. co-local events in the light source system K. Thus, this corresponds to the event coordinates $x = 0$. Hence, the wave period time observed from the moving system k can be determined from Eq. (1) to be

$$T' = \beta(T - 0); \quad T' = \beta T, \qquad (7)$$

which is in contradiction with Eq. (6) resulting from the respective Doppler shift.

It is to be noted that for the considered duration of a light wave period, sufficiently small in terms of the relative motion effect, it can be considered with high degree of confidence that the respective relative distance traveled by the moving frame is infinitesimally small, so it can be approximated with a straight segment. Hence, the time transformation derived for inertial frames in relative motion (i.e., relatively moving in a straight line at a uniform speed) could be applied.

3. Conclusion

The relativistic Doppler shift equation for any arbitrary relative velocities, as derived in Einstein 1905 paper on the Special Relativity, revealed for the particular case of the observer-source circular relative motion, a critical contradiction with the time dilation prediction obtained in the same paper.

References

1 A. Einstein, "Zur elektrodynamik bewegter Körper," Annalen der Physik **322** (10), 891–921 (1905).