# Symmetric Mass Hierarchies of the Standard Model 

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The fundamental particles of the Standard Model are arranged in symmetric mass hierarchy within a geometric framework that is connected to Planck scale. Atomic scales are incorporated within the scheme.

All values of subatomic particle mass used in this letter are the evaluations of the Particle Data Group, 2014 [1].

The scales of atomic and subatomic physics lie on levels and sublevels within each of three geometric sequences that descend from Planck scale (1.220932(73) x $\left.10^{19} \mathrm{GeV}[2]\right)^{1}:$ Sequence 1, of common ratio $1 / \pi$; Sequence 2 , of common ratio $2 / \pi$; and Sequence 3, of common ratio $1 / \mathrm{e}$ [3]. Levels in Sequences 1,2 and 3 are assigned the numbers $n_{1}, n_{2}$ and $n_{3}$, respectively. The Planck framework may originate from the geometry of spacetime [4]. Five principal scales, including the Rydberg constant $R_{\infty}=13.6 \mathrm{eV}$, the Bohr radius $a_{0}=0.529 \times 10^{-10} \mathrm{~m}$ and the pion charge radius $r_{\pi}=0.672(8)$ $\times 10^{-15} \mathrm{~m}$ [1], are shown on the levels of Sequences 1 and 2 in Figure 1 .


Figure 1: Scales of atomic and subatomic physics on the levels of Sequences 1 and 2. Multiplets e-u and $\mathrm{W}^{ \pm}-\mathrm{Z}^{0}$ are represented by the geometric mean of the two masses. All scales lie on a straight line since $n_{1}$ and $n_{2}$ are in constant ratio.

[^0]The weak gauge bosons, $\mathrm{W}^{ \pm}$and $\mathrm{Z}^{0}$, the up quark (2.3 MeV [1]) and the electron are found in a symmetric arrangement centred on $(100,45)$ in Sequences 2 and 3, at the approximate three-sequence type 2 superlevel $^{2}$ coincidence ( $40,100,45$ ). The arrangement is shown on the mass levels of Sequences 2 and 3 in Figure 2.


Figure 2: The weak gauge bosons in symmetric arrangement with the lightest quark (u) and lightest charged lepton (e) about $(100,45)$ in Sequences 2 and 3. Multiplets e-u and $W^{ \pm}-Z^{0}$ are each represented by the geometric mean of the two masses. All mass scales lie on a straight line since $\mathrm{n}_{2}$ and $\mathrm{n}_{3}$ are in constant ratio.

[^1]The quarks of each generation are arranged symmetrically about type 2 mass superlevels within the Planck framework, perhaps as the result of symmetry breaking upon spacetime boundaries. The quark doublets lie in symmetric hierarchy within the Planck framework, as shown in Figure 3. The arrangement is centred on $(100,45)$ in Sequences 2 and 3, at the approximate three-sequence type 2 superlevel coincidence $(40,100,45)$, as is the $(e-u)-\left(W^{ \pm}-Z^{0}\right)$ arrangement of Figure 2.


Figure 3: The quark doublets upon type 2 superlevels in Sequences 2 and 3. Each doublet is represented by the geometric mean of the two masses.

The charged leptons must be considered in conjunction with their apparent pseudoscalar partners [4]. An arrangement of charged leptons and pseudoscalar mesons on type 1 superlevels within Sequences 1 and 2 is shown in Figure 4. The muon and the $\pi^{ \pm}$mesons are arranged symmetrically about Level 102 in Sequence 2 , while the tau lepton and the $\mathrm{D}^{ \pm}$mesons are arranged symmetrically about Level 96. The $\mathrm{K}^{ \pm}$mesons occupy Level 39 in Sequence 1. The arrangement is centred on (39, 99) in Sequences 1 and 2, at the approximate three-sequence type 1 superlevel coincidence ( $39,99,45$ ).


Figure 4: An arrangement of charged leptons and pseudoscalar mesons on type 1 superlevels within Sequences 1 and 2. Partnerships are represented by the geometric mean of the two masses.

The electron occupies Level 45, a superlevel of type 1 and type 2, in Sequence 1. The electron is arranged symmetrically with the $\mathrm{K}^{ \pm}$mesons about $(42,48)$ in Sequences 1 and 3 , at the type 1 superlevel/sublevel/superlevel coincidence ( $42,106.5,48$ ), which lies at the centre of a symmetric arrangement of three-sequence coincident type 1 superlevels that spans many orders of magnitude [5]. The e-K ${ }^{ \pm}$arrangement is shown on type 1 superlevels within Sequences 1 and 3 in Figure 5.


Figure 5: The electron and the $\mathrm{K}^{ \pm}$mesons in arrangement about $(42,48)$ in Sequences 1 and 3.

The proton and the ${ }^{56} \mathrm{Fe}$ nucleus, a scalar boson of mass 52.09 GeV , are shown in symmetric arrangement on the mass levels and sublevels of Sequences 2 and 3 in Figure 6. The proton is the only stable hadron, while the ${ }^{56} \mathrm{Fe}$ nucleus has the highest binding energy per nucleon of any atomic nucleus. The arrangement is centred on the close type 1 superlevel coincidence $(93,42)$ in Sequences 2 and 3 , at the approximate three-sequence type 1 superlevel coincidence ( $36,93,42$ ).


Figure 6: The proton and the ${ }^{56} \mathrm{Fe}$ nucleus in symmetric arrangement about (93, 42) in Sequences 2 and 3.

The framework of Figure 6 includes, on sublevels, the $\mathrm{W}^{ \pm}-\mathrm{Z}^{0}$ multiplet and the Higgs field vacuum expectation value ( 246 GeV ), as shown in Figure 7. The top quark and the Higgs boson are arranged symmetrically about Level 86.25 in Sequence 2.


Figure 7: The Higgs field vacuum expectation value and the high mass particles, in multiplets, on the levels and sublevels of Sequences 2 and 3. The uncertainties in location are smaller than the marker diameter.

The $\mathrm{W}^{ \pm}-\mathrm{Z}^{0}$ multiplet occupies an exceptional location within the Planck framework, lying on closely coincident half-levels in Sequence 1, Sequence 2 (see Figures 1 and 2) and Sequence 3 (see Figure 7). The bosons are shown in symmetric arrangement within Sequences 1 and 3 in Figure 8.


Figure 8: The $W^{ \pm}$and $Z^{0}$ bosons in symmetric arrangement about half-levels in Sequences 1 and 3 . The $W^{ \pm}-Z^{0}$ multiplet is represented by the geometric mean of the two masses.

## References

1. K. A. Olive et al. (Particle Data Group), Chin. Phys. C38, 090001 (2014) (URL: http://pdg.lbl.gov)
2. CODATA 2010
3. B. F. Riley, The Planck Model, vixra:1311.0053
4. B. F. Riley, Standard model scales from warped extra dimensions, arxiv:0809.0111
5. B. F. Riley, Supersymmetric light and dark matter, vixra:1405.0253

[^0]:    ${ }^{1}$ Length scales ascend from the Planck length (1.616199(97) $\left.\times 10^{-35} \mathrm{~m}[2]\right)$.

[^1]:    ${ }^{2}$ Type 1 superlevels are levels whose level-numbers are multiples of 3 . Type 2 superlevels are levels whose level-numbers are multiples of 5 .

