Newton's Aether

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Abstract

This text demonstrates that how we think about Gravity can be influenced by the mathematical tools that are available to us. The author attempts to predict what Newton might have thought if he had known of the works of Euler and Hamilton and been familiar with Linear Algebra. A definition is presented for the natural logarithm of a vector. The author determines that Newton would have inferred that the force of gravity and the resulting acceleration are not collinear.

Preface

Knowledge of quaternions and Linear Algebra is required.

"It is inconceivable that inanimate brute matter should, without the mediation of something else which is not material, operate upon and affect other matter, without mutual contact, as it must do if gravitation in the sense of Epicurus be essential and inherent in it. And this is one reason why I desired you would not ascribe 'innate gravity' to me. That gravity should be innate, inherent, and essential to matter, so that one body may act upon another at a distance, through a vacuum, without the mediation of anything else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity, that I believe no man who has in philosophical matters a competent faculty of thinking can ever fall into it. Gravity must be caused by an agent acting constantly according to certain laws; but whether this agent be material or immaterial, I have left to the consideration of my readers." - Isaac Newton, third letter to Bentley in 1692

Discussion

Isaac Newton lived from 1642 - 1726. Leonhard Euler lived from 1707 - 1783. William Rowan Hamilton lived from 1805 - 1865. Therefore, Newton preceded both of the other men and hence did not have the benefit of their respective works. The author has recently wondered what might have been if Newton had known of their respective works. The author assumes that Newton was familiar with the works of Copernicus (1473 – 1543) and Kepler (1571 – 1630).

The author will begin by stating the concepts that Newton would have known.

Kepler's Laws are understood to be[1]:

1. The orbit of a planet is an ellipse with the Sun at one of the two foci.
2. A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.
3. The square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit.

Euler's Equation[2] is written as:

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Hamilton[3] defined a quaternion as being the ratio between two generic space vectors. He established the following:

$$\mathbf{Q} = q_0 + q_i\mathbf{i} + q_j\mathbf{j} + q_k\mathbf{k};\ where\ \mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = -1 = \mathbf{ijk}$$

Equation 0:

$$\mathbf{Q} = \frac{\mathbf{y}}{\mathbf{x}} = \frac{y_i\mathbf{i} + y_j\mathbf{j} + y_k\mathbf{k}}{x_i\mathbf{i} + x_j\mathbf{j} + x_k\mathbf{k}}$$

The convention used here is that a quaternion is represented by a **bold**-faced CAPITAL letter. A vector is represented by a **bold**-faced lower case letter. A scalar is represented by a lower case letter in regular font. By combining a scalar and a vector, Hamilton developed a Geometric Algebra. The vectors **i**, **j**, and **k** are unit vectors in the direction of the x, y, and z axes respectively.

Next, let us state Newton's two most famous equations. These are:

Newton's Second Law:

$$\mathbf{f} = m\mathbf{a}$$

and

Newton's Law of Gravity:

$$f = G\frac{m_1 m_2}{r^2}; therefore\ a_2 = G\frac{m_1}{r^2}; also\ f_1 = -f_2$$

If Newton had known of Hamilton's work, he would have realized immediately that there was something missing in this formulation of gravity. Force is a vector. Acceleration is a vector. He determined an equation that gave him the magnitude for the force of gravity but the equation gave him a scalar value rather than a vector. It was understood that the direction of the force was an attraction between the centers of mass. Somehow, he would have needed for mass divided by the square of a distance to produce acceleration as a vector.

The author considers the vectors **x** and **y** to be Kepler's line segments between the Sun and a planet at two successive times. Rearranging the definition of a quaternion from Equation 0 above allows the following:

$$\mathbf{Qx} = (q_0 + q_i\mathbf{i} + q_j\mathbf{j} + q_k\mathbf{k})(x_i\mathbf{i} + x_j\mathbf{j} + x_k\mathbf{k}) = \mathbf{y} = y_i\mathbf{i} + y_j\mathbf{j} + y_k\mathbf{k}$$

This multiplication results in a system of four simultaneous equations. These can be represented by the following matrix multiplication:

$$\begin{bmatrix} 0 & -x_i & -x_j & -x_k \\ +x_i & 0 & +x_k & -x_j \\ +x_j & -x_k & 0 & +x_i \\ +x_k & +x_j & -x_i & 0 \end{bmatrix} \begin{bmatrix} q_0 \\ q_i \\ q_j \\ q_k \end{bmatrix} = \begin{bmatrix} 0 \\ y_i \\ y_j \\ y_k \end{bmatrix}$$

This system is solved for the elements of **Q** by multiplying by the inverse of the coefficient matrix as follows:

$$\begin{bmatrix} q_0 \\ q_i \\ q_j \\ q_k \end{bmatrix} = \frac{1}{x_i^2 + x_j^2 + x_k^2} \begin{bmatrix} 0 & +x_i & +x_j & +x_k \\ -x_i & 0 & -x_k & +x_j \\ -x_j & +x_k & 0 & -x_i \\ -x_k & -x_j & +x_i & 0 \end{bmatrix} \begin{bmatrix} 0 \\ y_i \\ y_j \\ y_k \end{bmatrix}$$

This allows **Q** to be expressed as follows:

Equation 1:

$$\mathbf{Q} = \frac{\mathbf{y}}{\mathbf{x}} = \frac{1}{\|\mathbf{x}\|^2}(\mathbf{x}\cdot\mathbf{y} + \mathbf{x}\times\mathbf{y});\ q_0 = \frac{\mathbf{x}\cdot\mathbf{y}}{\|\mathbf{x}\|^2}\ and\ \mathbf{q} = \frac{\mathbf{x}\times\mathbf{y}}{\|\mathbf{x}\|^2};\mathbf{Q} = q_0 + \mathbf{q}$$

Please note that this form of **Q** is inversely proportional to the square of the length of vector **x**. This is one of the features of Newton's Law of Gravity. Please note also that the complex conjugate **Q*** satisfies the equation **xQ** = **y**.

Now let us add another relationship between vector **x** and vector **y**. Please refer to Figure 1 below. Let us make the following definition:

$$\mathbf{y} = \mathbf{x} + \Delta\mathbf{x}$$

When this is substituted into Equation 1, rearranged, and simplified the result is:

Equation 2:

$$\frac{(\mathbf{x}\cdot\Delta\mathbf{x}) + (\mathbf{x}\times\Delta\mathbf{x})}{\|\mathbf{x}\|^2} = \frac{\Delta\mathbf{x}}{\mathbf{x}}$$

The Δ**x** terms can then be reduced to d**x** by taking the limit as Δ**x** goes to zero.

Equation 3:

$$\frac{(\mathbf{x}\cdot d\mathbf{x}) + (\mathbf{x}\times d\mathbf{x})}{\|\mathbf{x}\|^2} = \frac{d\mathbf{x}}{\mathbf{x}}$$

The dot product term causes the length of vector **x** to change. The cross product term causes vector **x** to sweep out a surface in space. These are precisely the features that were required by Kepler.

Furthermore, Equation 3 forms the basis for the definition of the natural logarithm of a vector as follows:

Equation 4:

$$\int_{\mathbf{u}}^{\mathbf{x}}\frac{\mathbf{x}\cdot d\mathbf{x}}{\|\mathbf{x}\|^2} + \int_{\mathbf{u}}^{\mathbf{x}}\frac{\mathbf{x}\times d\mathbf{x}}{\|\mathbf{x}\|^2} = \int_{\mathbf{u}}^{\mathbf{x}}\frac{d\mathbf{x}}{\mathbf{x}} = \ln(\mathbf{x})\,;where\ \mathbf{u}\ is\ a\ unit\ vector$$

Given the scalar versus vector problem in the Law of Gravity and given Euler's Equation, he might have chosen to rewrite the Second Law as follows:

$$\mathbf{M} = \frac{\mathbf{f}}{\mathbf{a}} = m_0 + m_i\mathbf{i} + m_j\mathbf{j} + m_k\mathbf{k};\mathbf{M} = \|\mathbf{M}\|\big(\cos(\theta_0) + \sin(\theta_i)\mathbf{i} + \sin(\theta_j)\mathbf{j} + \sin(\theta_k)\mathbf{k}\big)$$

From Equation 1, he would then have the following:

Equation 5:

$$\mathbf{M} = \frac{\mathbf{f}}{\mathbf{a}} = \frac{1}{\|\mathbf{a}\|^2}(\mathbf{a}\cdot\mathbf{f} + \mathbf{a}\times\mathbf{f}) = \|\mathbf{M}\|\left(\cos(\theta_0) + \frac{\mathbf{m}}{\|\mathbf{M}\|}\right);\ \frac{\mathbf{a}\cdot\mathbf{f}}{\|\mathbf{a}\|^2} = \|\mathbf{M}\|\cos(\theta_0)\ and\ \frac{\mathbf{a}\times\mathbf{f}}{\|\mathbf{a}\|^2} = \mathbf{m}$$

He might also have chosen to rewrite the Law of Gravity as follows:

Equation 6:

$$\frac{r^2}{G}\mathbf{f} = \mathbf{M}_1\mathbf{M}_2$$

At this point, he would have to think long and to think hard. It is simple enough to use Equation 5 to substitute for $\mathbf{M}_1$ and $\mathbf{M}_2$ in Equation 6. The potential problem with this is that he does not know that inertial mass and gravitational mass are equivalent. The Eötvös Experiment would not be until 1885. Remember that $\mathbf{f}_2 = -\mathbf{f}_1$. Making these substitutions produces the following:

$$\frac{r^2}{G}\mathbf{f} = \left[\frac{1}{\|\mathbf{a}_1\|^2}(\mathbf{a}_1\cdot\mathbf{f}_1 + \mathbf{a}_1\times\mathbf{f}_1)\right]\left[\frac{-1}{\|\mathbf{a}_2\|^2}(\mathbf{a}_2\cdot\mathbf{f}_1 + \mathbf{a}_2\times\mathbf{f}_1)\right] = \|\mathbf{M}_1\|\left(\cos(\theta_1) + \frac{\mathbf{m}_1}{\|\mathbf{M}_1\|}\right)\|\mathbf{M}_2\|\left(\cos(\theta_2) + \frac{\mathbf{m}_2}{\|\mathbf{M}_2\|}\right)$$

Rearranging slightly gives:

$$\frac{r^2}{G}\mathbf{f} = \frac{-1}{\|\mathbf{a}_1\|^2\|\mathbf{a}_2\|^2}[(\mathbf{a}_1\cdot\mathbf{f}_1 + \mathbf{a}_1\times\mathbf{f}_1)][(\mathbf{a}_2\cdot\mathbf{f}_1 + \mathbf{a}_2\times\mathbf{f}_1)] = \|\mathbf{M}_1\|\|\mathbf{M}_2\|\left(\cos(\theta_1) + \frac{\mathbf{m}_1}{\|\mathbf{M}_1\|}\right)\left(\cos(\theta_2) + \frac{\mathbf{m}_2}{\|\mathbf{M}_2\|}\right)$$

Force is a vector. Therefore, the scalar terms must sum to zero and the vector terms must have a non-zero sum. Only the acceleration-force multiplication will be presented here due to length considerations.

$$(\mathbf{a}_1\cdot\mathbf{f}_1 + \mathbf{a}_1\times\mathbf{f}_1)(\mathbf{a}_2\cdot\mathbf{f}_1 + \mathbf{a}_2\times\mathbf{f}_1) = (\mathbf{a}_1\cdot\mathbf{f}_1)(\mathbf{a}_2\cdot\mathbf{f}_1) + (\mathbf{a}_1\times\mathbf{f}_1)(\mathbf{a}_2\cdot\mathbf{f}_1) + (\mathbf{a}_1\cdot\mathbf{f}_1)(\mathbf{a}_2\times\mathbf{f}_1) + (\mathbf{a}_1\times\mathbf{f}_1)(\mathbf{a}_2\times\mathbf{f}_1)$$

The second and third groups of terms on the right-hand side appear to be similar since they are each a dot product multiplied by a cross product. Therefore, the author will group them together. The remaining terms are the first and fourth groups of terms. They will also be grouped together.

Scalar:

$$(\mathbf{a}_1\cdot\mathbf{f}_1)(\mathbf{a}_2\cdot\mathbf{f}_1) + (\mathbf{a}_1\times\mathbf{f}_1)(\mathbf{a}_2\times\mathbf{f}_1) = 0$$

Vector:

$$(\mathbf{a}_1\times\mathbf{f}_1)(\mathbf{a}_2\cdot\mathbf{f}_1) + (\mathbf{a}_1\cdot\mathbf{f}_1)(\mathbf{a}_2\times\mathbf{f}_1) \neq \mathbf{0}$$

Newton would have been more interested in the vector equation since he was working on the force of gravity. Therefore, let us simplify the vector equation.

$$\left((\mathbf{a}_2\cdot\mathbf{f}_1)\mathbf{a}_1\times\mathbf{f}_1\right) + \left((\mathbf{a}_1\cdot\mathbf{f}_1)\mathbf{a}_2\times\mathbf{f}_1\right) \neq \mathbf{0}$$

$$[(\mathbf{a}_2 \cdot \mathbf{f}_1)\mathbf{a}_1 + (\mathbf{a}_1 \cdot \mathbf{f}_1)\mathbf{a}_2] \times \mathbf{f}_1 \neq \mathbf{0}$$

When two vectors are collinear, their cross product is zero. The physical implication of this vector requirement is that the force of gravity and the resulting acceleration are not collinear! Newton would have used this as a basis to describe the Aether.

Lastly, Equation 6 has two solutions, these are **f** and the complex conjugate **f***. These are equal in magnitude but opposite in direction.

Conclusions

The author speculates as to what Newton might have done if he had studied Euler and Hamilton. Newton would have been able to produce two equations for gravity with one being a vector equation and the other being a scalar equation. The author finds that Newton would have concluded that the force of gravity is not collinear with the resulting acceleration. The two separate equations might have pointed Newton towards General Relativity and towards Quantum Mechanics.

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References

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Figure 1