Review article

Critical review on cosmologically strengthening hydrogen atom

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Abstract: By considering the Stoney scale and current cosmic back ground temperatures the authors proposed a unified model mechanism for understanding the cosmological light emission mechanism in cosmologically 'strengthening hydrogen atom'. In this proposed model 'gravitational potential energy of proton' and '$\left(2n^2\right)$ states of electron' both seem to play a major role. Throughout the cosmic evolution, Planck's constant seems to be a constant whereas the currently believed 'reduced Planck's constant' seems to be a cosmological decreasing variable. With this new proposal - Hubble's redshift interpretation, Super novae dimming and currently believed cosmic acceleration can be reviewed at fundamental level and a correct model of cosmology can be developed.

Key words: Cosmic red shift, Stoney scale, Cosmic back ground temperature, Gravitational potential energy of proton, $\left(2n^2\right)$ states of electron, Black hole cosmology.

1. Introduction

The fundamental question to be answered is: During cosmic evolution, right from its birth, is hydrogen atom experiences any structural or physical changes? This question directly and indirectly seems to be linked with the currently believed cosmic redshift observations [1,2]. In this letter the authors reviewed their proposed new cosmic redshift interpretation [3] with reference to Black hole cosmology [4,5].

2. Motivating concepts and points

The authors request the science community to kindly look into the following points in a true scientific spirit.

1) As suggested by S.W. Hawking [6], there is no scientific evidence to Friedmann's second assumption [7].
2) If it is true that galaxy constitutes so many stars, each star constitutes so many hydrogen atoms and light is coming from any excited electron of any galactic star's any hydrogen atom, then considering redshift as an index of 'whole galaxy' receding may not be reasonable.
3) Merely by estimating 'galaxy distance' and without measuring any 'galaxy's actual receding speed', one cannot verify the cosmic acceleration. Note that, in 1947 Hubble himself thought for a new mechanism for understanding the observed red shift [2]. In his words: "We may predict with confidence that the 200 inch will tell us whether the red shifts must be accepted as evidence of a rapidly expanding universe, or attributed to some new principle in nature. Whatever may be the answer, the result may be welcomed as another major contribution to the exploration of the universe".
4) Even though it is very attractive, Einstein could not implement the Mach's principle [8,9] in Hubble-Friedmann-cosmology [10-13].
5) Until 1964, cosmologists could not believe in 'cosmic back ground temperature' [14].
6) In the past, 'quantum gravity' was in its beginning stage and now it is in an advanced theoretical phase.
7) Based on the Hubble's law and Super novae dimming, currently it is believed that, universe is accelerating [12,13]. In the authors' opinion, if magnitude of past Hubble's constant was higher than the current magnitude then magnitude of past $\left(c/H_t\right)$ will be smaller than the current Hubble length $\left(c/H_0\right)$. so the rate of decrease of Hubble constant can be considered as a true index of rate of increase in Hubble length and thus with reference to Hubble length, the rate of decrease of Hubble constant can be considered as a true index of cosmic rate of expansion.
8) In future, certainly with reference to current Hubble's constant, $d\left(c/H_0\right)/dt$ gives the true cosmic rate of expansion. Same logic can be applied to cosmic back ground temperature also. Clearly speaking $d\left(T_0\right)/dt$ gives the true cosmic rate of expansion. To understand the ground reality, accuracy of current methods of es-

timating the magnitudes of $(H_0 \text{ and } T_0)$ must be improved.

3. Reinterpreting cosmic red shift

During cosmic evolution, right from the beginning of formation of hydrogen atoms, as any baby hydrogen atom starts growing, cosmologically, bonding strength increases in between proton and electron causing increasing electron excitation energy to emit increased quantum of energy. With reference to the current strengthened or reinforced hydrogen atom, difference in 'emitted quantum of energy' may appear to be the observed cosmological redshift associated with galactic hydrogen atom. Observed Super novae dimming can be understood in this way [12]. Based on this new proposal, 'galaxy receding' concept suggested by Hubble can be reviewed and possibly can be relinquished. If cosmic time is running fast or if cosmic size/boundary is increasing fast or if cosmic temperature is decreasing fast then redshift seems to increase fast with reference to the current hydrogen atom. For a while guess that cosmological binding strength of proton and electron in the cosmologically evolving hydrogen atom is inversely proportional to the cosmic temperature, then with usual notation, observed cosmic red shift can be expressed as follows.

$$\left(E_{photon}\right)_t \cong \left(\frac{T_0}{T_t}\right)\left\{\left(\frac{e^4 m_e}{32\pi^2\varepsilon_0^2\hbar^2}\right)\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]\right\} \cong \frac{hc}{\lambda_t} \tag{1}$$

where, T_0 represents the current CMBR temperature, T_t represents past cosmic temperature and λ_t is the wavelength of photon 'emitted as well as received' from the galactic hydrogen atom.

At any time in the past, at any galaxy, emitted photon energy can be expressed as follows.

$$\left.\begin{array}{l} E_t \cong \dfrac{hc}{\lambda_t} \cong \left(\dfrac{T_0}{T_t}\right)\left(\dfrac{hc}{\lambda_0}\right) \cong \left(\dfrac{T_0}{T_t}\right)E_0 \\ \rightarrow z_0 \cong \dfrac{\lambda_t-\lambda_0}{\lambda_0} \cong \dfrac{E_0-E_t}{E_t} \cong \dfrac{T_t-T_0}{T_0} \\ \text{and } \dfrac{E_0}{E_t} \cong \dfrac{\lambda_t}{\lambda_0} \cong \dfrac{T_t}{T_0} \cong (z_0+1) \end{array}\right\} \tag{2}$$

Here, z_0 is the current redshift, E_t is the energy of emitted photon from the galactic hydrogen atom and E_0 is the corresponding energy in the laboratory. λ_0 is the λ_t's corresponding wave length in the laboratory.

From laboratory point of view, above concept can be understood in the following way. After some time in future,

$$z_f \cong \frac{E_f - E_0}{E_0} \cong \frac{E_f}{E_0} - 1 \tag{3}$$

Here, E_f is the energy of photon emitted from laboratory hydrogen atom after some time in future. E_0 is the energy of current photon emitted from laboratory hydrogen atom. z_f is the redshift of laboratory hydrogen atom after some time in future. From now onwards, as time passes, in future - $\left[d(z_f)/dt\right]$ can be considered as an index of the absolute rate of cosmic expansion. Within the scope of experimental accuracy of laboratory hydrogen atom's redshift, it can be suggested that,

$$\left.\begin{array}{l} \text{Increasing } \left[d(z_f)/dt\right] \rightarrow \text{Cosmic Acceleration} \\ \text{Constant } \left[d(z_f)/dt\right] \rightarrow \text{Cosmic Uniform expansion} \\ \text{Decreasing } \left[d(z_f)/dt\right] \rightarrow \text{Cosmic Deceleration} \\ \left[d(z_f)/dt\right] = 0 \rightarrow \text{Cosmic halt} \end{array}\right\}$$

4. Reasons for considering the universe as a primordial evolving black hole

Even though the subject of black hole physics is very interesting, its back ground mathematics is very complicated and theoretical predictions are beyond the scope of current engineering and technology. So far no single prediction of black hole physics has been evidenced. In 1974 S.W. Hawking suggested that, black holes can have temperature [15]. In 2014, he suggested that black hole event horizons can be assumed to be 'apparent' and needs further investigation at fundamental level [16]. At this juncture, if one starts doubting the 'existence' of black hole event horizons, then whole black hole physics will certainly fall in a 'mathematical' singularity. Until a highly sophisticated satellite reaches a black hole event horizon, strange theoretical concepts like black hole thermal radiation, mass-inflation, black hole's gravitational radiation etc. cannot be addressed clearly and cannot be confirmed.

Now a days most of the cosmologists as well as astrophysicists strongly believe that each and every galaxy of the universe constitutes a fast spinning massive central black hole. Here the authors would like to stress the fact that, if primordial universe is able to produce so many galaxies with so many galactic central black holes that are having almost all closed curvatures, then cosmologists should not ignore the possibility of 'considering the whole universe as a primordial spinning black hole'. In reality - one may reach or may not reach a black hole, if one is willing to consider the whole 'observable universe' as a huge 'primordial evolving and light speed rotating black hole', quantum gravity can certainly acquire a clear physical identity [17-19] and many interesting things will come into visualization as proposed in the authors published papers[4,5].

5. Stoney scale Hubble constant and temperature

In the earlier published papers [4,5] the authors suggested that,

1) Universe can be considered as an evolving primordial black hole.
2) Stoney scale [20] can be considered the characteristic beginning scale of the baby primordial black hole universe.
3) Current back ground temperature can be considered as the current temperature of the current primordial black hole universe.

Stoney scale mass-energy scale can be expressed as follows [3,4].

$$\left.\begin{array}{l} \left(M_S\right)^{\pm} \cong \sqrt{\dfrac{e^2}{4\pi\varepsilon_0 G}} \cong 1.859272\times10^{-9}\text{ kg} \\ \text{(OR)} \\ M_S \cong \left(\dfrac{\pi}{xy\sqrt{45}}\right)\sqrt{\dfrac{hc}{G}} \cong 1.824\times10^{-9}\text{ kg} \end{array}\right\} \quad (4)$$

Stoney scale characteristic Hubble radius and Hubble constant can be expressed as follows.

$$\left.\begin{array}{l} R_S \cong \dfrac{2GM_S}{c^2} \cong 2.7613\times10^{-36}\text{ m and} \\ H_S \cong \dfrac{c}{R_S} \cong \dfrac{c^3}{2GM_S} \cong 1.0857\times10^{44}\text{ rad/sec} \end{array}\right\} \quad (5)$$

Stoney scale characteristic thermal energy density and temperature can be expressed as follows.

$$\left.\begin{array}{l} aT_S^4 \cong 8.47\times10^{81}\text{ J/m}^{-3}\text{ and} \\ T_S \cong \left(\dfrac{3H_S^2c^2}{8\pi G}\right)^{\frac{1}{4}} \cong 2.2371\times10^{32}\text{ K} \end{array}\right\} \quad (6)$$

At any time in the past,

$$\left.\begin{array}{l} aT_t^4 \cong \left[1+\ln\left(\dfrac{H_S}{H_t}\right)\right]^{-2}\left(\dfrac{3H_t^2}{8\pi G}\right)\text{ and} \\ T_t \cong \left(\dfrac{3H_t^2c^2}{8\pi Ga}\right)^{\frac{1}{4}} \Big/ \sqrt{1+\ln\left(\dfrac{H_S}{H_t}\right)} \end{array}\right\} \quad (7)$$

If $H_0 \cong 71$ km/sec/Mpc,
$aT_0^4 \cong 4.16\times10^{-14}\,\text{J.m}^{-3}$ and $T_0 \cong 2.723$ K.

At any time, matter density can be expressed as follows.

$$\left(\rho_m\right)_t \cong \sqrt{\left(\frac{3H_t^2}{8\pi G}\right)\left(\frac{aT_t^4}{c^2}\right)} \quad (8)$$

$$\cong \left\{1+\ln\left(\frac{H_U}{H_t}\right)\right\}\left(\frac{aT_t^4}{c^2}\right) \cong \left(\frac{3H_t^2}{8\pi G}\right)\Big/\left\{1+\ln\left(\frac{H_U}{H_t}\right)\right\}$$

If $H_0 \cong 71$ km/sec/Mpc, $\left(\rho_m\right)_0 \cong 6.62\times10^{-32}$ gram.cm^{-3}

6. Stoney scale model mechanism for understanding the cosmic red shift in hydrogen atom

In a cosmological approach, starting from the Planck scale, in this section the authors proposed a simple and ad-hoc model mechanism for understanding the binding energy of electron and proton in the hydrogen atom. It is for further study and development. In hydrogen atom, in a cosmological approach, potential energy of electron be:

$$\left(E_{\text{pot}}\right)_t \cong -\frac{e^2}{4\pi\varepsilon_0 r_t} \quad (9)$$

where r_t is the cosmologically changing distance between proton and electron. From Bohr's theory of Hydrogen atom, maximum number of electrons that can be accommodated in any principal quantum shell are $\left(2n^2\right)$ where n=1,2,3,.. This proposal can be reinterpreted as follows: **In Hydrogen atom, in n^{th} principal quantum shell, electron can exist in $\left(2n^2\right)$ different states.**

With reference to standard notation of gravitational potential energy, in nuclear physics, quantitatively and qualitatively it is possible to guess that [21],

$$-\frac{3}{5}\frac{Gm_p^2}{R_s} \cong -\frac{Gm_p^2}{2R_p} \quad (10)$$

where, m_p is the rest mass of proton , R_p is the 'rms' radius of proton [22,23] and R_s is the strong interaction dominating range [24]. Here $\left(-\frac{3}{5}\frac{Gm_p^2}{R_s}\right)$ can be considered as the gravitational potential energy of proton.

$$\left.\begin{array}{l} R_s \cong 1.05\times10^{-15} \text{ m and} \\ R_p \cong 0.8775\times10^{-15} \text{ m} \end{array}\right\}$$

Note that, $2R_p \cong 1.75\times10^{-15}$ m may be taken as the approximate ending range of strong interaction from the center of proton. Within the nucleus, at distances larger than 0.7 fm the force becomes attractive between spin-aligned nucleons, becoming maximal at a center-center distance of about 0.9 fm. Beyond this distance nuclear force drops essentially exponentially, until beyond about 2.0 fm separation, the force drops to negligibly small values. At short distances (less than 1.7 fm or so), the nuclear force is stronger than the Coulomb force between protons; it thus overcomes the repulsion of protons inside the nucleus.

In hydrogen atom, potential energy of possible $\left(2n^2\right)$ quantum states be:

$$\left.\begin{array}{l} \left(\in_{\text{pot}}\right)_t \cong -2n^2\left(E_{\text{pot}}\right)_t \\ \cong -2n^2\left(\frac{e^2}{4\pi\varepsilon_0 r_t}\right) \cong -\left(\frac{T_U}{T_t}\right)\left(\frac{Gm_p^2}{2R_p}\right) \end{array}\right\} \quad (11)$$

where, $T_S \cong \left(\frac{3H_S^2c^2}{8\pi Ga}\right)^{\frac{1}{4}} \cong 2.2371\times10^{32}$ K and T_t represents the past cosmic temperature. This expression is very simple and tightly connected with quantum nature, gravity and evolving cosmic back ground and needs further study.

Based on the Virial theorem [21], in a central force field, quantitatively potential energy is twice of kinetic energy or kinetic energy is half the potential energy. Following this idea,

$$\left(\in_{\text{kin}}\right)_t \cong \frac{1}{2}\left|2n^2\left(\frac{e^2}{4\pi\varepsilon_0 r_t}\right)\right| \cong \left(\frac{T_S}{T_t}\right)\left(\frac{Gm_p^2}{4R_p}\right) \quad (12)$$

Total energy of one electron in $\left(2n^2\right)$ possible quantum states be:

$$\left.\begin{array}{l} \left(\in_{\text{pot}}\right)_t + \left(\in_{\text{kin}}\right)_t \cong \left(\in_{\text{tot}}\right)_t \\ \to \left(\in_{\text{tot}}\right)_t \cong -\left(\frac{T_S}{T_t}\right)\left(\frac{Gm_p^2}{4R_p}\right) \end{array}\right\} \quad (13)$$

Total energy of one electron out of possible $\left(2n^2\right)$ quantum states can be :

$$\left.\begin{array}{l} \left(E_{\text{tot}}\right)_t \cong -\left(\frac{1}{2n^2}\right)\left(\frac{T_S}{T_t}\right)\left(\frac{Gm_p^2}{4R_p}\right) \\ \cong -\left(\frac{T_S}{T_t}\right)\left(\frac{Gm_p^2}{8n^2R_p}\right) \end{array}\right\} \quad (14)$$

$$\left.\begin{array}{l} \text{If } R_p \cong 0.8775\times10^{-15} \text{ m, } \mathrm{T}_0 \cong 2.725 \text{ K,} \\ T_S \cong 2.25868\times10^{32} \text{ K and n=1} \\ \left(\frac{T_S}{T_0}\right)\left(\frac{Gm_p^2}{8R_p}\right) \cong 13.76 \text{ eV.} \end{array}\right\}$$

Potential energy of one electron out of possible $\left(2n^2\right)$ quantum states can be:

$$\left.\begin{array}{l} \left(E_{\text{pot}}\right)_t \cong -\left(\frac{1}{2n^2}\right)\left(\frac{T_S}{T_t}\right)\left(\frac{Gm_p^2}{2R_p}\right) \\ \cong -\left(\frac{T_S}{T_t}\right)\left(\frac{Gm_p^2}{4n^2R_p}\right) \end{array}\right\} \quad (15)$$

Orbiting radius of one electron out of possible $2n^2$ quantum states can be :

$$r_t \cong \left(\frac{T_t}{T_S}\right)\left(2n^2\right)\left(\frac{e^2}{4\pi\varepsilon_0 Gm_p^2}\right)\left(2R_p\right) \quad (16)$$

Kinetic energy of one electron out of possible $\left(2n^2\right)$ quantum states can be:

$$\begin{array}{l} \left(E_{\text{kin}}\right)_t \cong \frac{1}{2}m_e v_t^2 \\ \cong -\left(\frac{1}{2n^2}\right)\left(\frac{T_S}{T_t}\right)\left(\frac{Gm_p^2}{4R_p}\right) \cong -\left(\frac{T_S}{T_t}\right)\left(\frac{Gm_p^2}{8n^2R_p}\right) \end{array} \quad (17)$$

Orbiting velocity of one electron out of possible $2n^2$ quantum states can be:

$$v_t \cong \left.\begin{array}{l}\sqrt{\left(\frac{1}{2n^2}\right)\left(\frac{T_S}{T_t}\right)\left(\frac{Gm_p^2}{2R_p m_e}\right)} \\ \cong \frac{1}{n}\sqrt{\left(\frac{T_S}{T_t}\right)\left(\frac{Gm_p^2}{4R_p m_e}\right)}\end{array}\right\} \quad (18)$$

Angular momentum of one electron out of possible $\left(2n^2\right)$ quantum states can be:

$$\left.\begin{array}{l} m_e r_t v_t \cong \sqrt{\left(2n^2\right)\left(\frac{T_t}{T_S}\right)\left(\frac{2R_p m_e}{Gm_p^2}\right)}\left(\frac{e^2}{4\pi\varepsilon_0}\right) \\ \cong n\sqrt{\left(\frac{T_t}{T_S}\right)\left(\frac{4R_p m_e}{Gm_p^2}\right)}\left(\frac{e^2}{4\pi\varepsilon_0}\right) \cong n\left(\hbar_t\right)\end{array}\right\} \quad (19)$$

Here the key point to be noted is that,

$$\hbar_t \cong \frac{e^2}{4\pi\varepsilon_0 v_t} \cong \left(\frac{e^2}{4\pi\varepsilon_0}\right)\sqrt{\left(\frac{T_t}{T_S}\right)\left(\frac{4R_p m_e}{Gm_p^2}\right)} \quad (20)$$

With reference to current cosmic back ground temperature,

$$\hbar_0 \cong \frac{e^2}{4\pi\varepsilon_0 v_0} \cong \left(\frac{e^2}{4\pi\varepsilon_0}\right)\sqrt{\left(\frac{T_0}{T_S}\right)\left(\frac{4R_p m_e}{Gm_p^2}\right)} \quad (21)$$

Here it should be noted that, throughout the cosmic evolution, Planck's constant is a constant where as the currently believed 'reduced Planck's constant' is a cosmological decreasing variable .

Considering the jumping nature of electrons, now emitted quantum of energy for one electron can be expressed as follows.

$$\left(E_{\text{photon}}\right)_t \cong \left(\frac{T_S}{T_t}\right)\left(\frac{Gm_p^2}{8R_p}\right)\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right) \quad (22)$$

In the current laboratory hydrogen atom,

$$\left(E_{\text{photon}}\right)_0 \cong \left(\frac{T_S}{T_0}\right)\left(\frac{Gm_p^2}{8R_p}\right)\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right) \quad (23)$$

Clearly speaking, total energy of one electron can be:

$$\left(E_{\text{tot}}\right) \propto \left(\frac{1}{2n^2}\right) \quad (24)$$

This idea is connected with quantum nature.

$$\left(E_{\text{tot}}\right) \propto \left(\frac{Gm_p^2}{4R_p}\right) \quad (25)$$

This idea is connected with final unification of gravity and atomic interactions.

$$\left(E_{\text{tot}}\right)_t \propto \left(\frac{T_S}{T_t}\right) \quad (26)$$

This idea is connected with cosmic evolution and changing cosmic back ground.

7. Conclusion

In this brief report, in a cosmological approach the authors proposed a new interpretation for the observed galactic redshift. By considering this new cosmic redshift interpretation a novel model of cosmology can be developed. It can be suggested that,

1) In Hydrogen atom, in n^{th} principal quantum shell, electron can exist in $\left(2n^2\right)$ different states.
2) Gravitational potential energy of proton play a crucial role in the past and current hydrogen atoms' light emission mechanism.
3) Stoney scale and current cosmic back ground temperatures play a vital role in laboratory hydrogen atom's light emission mechanism.
4) 'Galaxy receding' concept suggested by Hubble can be reviewed at fundamental level and possibly 'Hubble's law' and its dependent 'cosmic acceleration' concepts can be relinquished.

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