The Mandelbrot set and the fractal nature of light, the Universe, and everything.

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ABSTRACT

“There is always another way to say the same thing that doesn’t look at all like the way it was said before.” Richard Feynman. In this essay, a novel approach to cosmology is presented that mathematically models the Universe as an iterated function system (**IFS**) analogous to the famous Mandelbrot Set IFS (**M**): $\mathbf{z=z^2+c}$, where z and c are complex numbers. In theoretical physics, wavefunctions are functions of a complex space that are commonly used to model the dynamics of particles and waves. In the IFS framework presented herein, complex dynamical systems are generated via the iteration process, where the act of iteration corresponds to 1) a change in the state of the system and 2) a change to the wavefunction itself. In this manner, **M** can be considered a wavefunction generator. In this framework, all observables, including gravity and time, are thought to be generated by the iteration process. Feynman understood that there are many ways of looking at the Universe that are equivalent in nature but different psychologically. Understanding cosmology in terms of fractals and iterated function systems requires a paradigm shift in the way we approach cosmology. This is an evidence based dissertation and does not contradict the standard model; rather, it attempts to reconstruct it using the principles of the fractal paradigm as outlined in this essay. It is the contention of the author that in order to understand the true nature of light, the universe and everything, we must first understand the important role that fractal cosmology plays in the study of our complex dynamical universe.

Keywords: fractal, iteration, Mandelbrot set, wavefunction, black hole, event horizon, white hole, time, gravity, photon.

PRELUDE

This essay presents a scientific framework; that can be applied to the whole of the universe; that avoids some of the historical paradoxes; that answers some of the previously unanswered questions; and offers a new understanding of the nature of light, the universe and everything. This approach to cosmology has philosophical implications which are addressed throughout this essay.

1. INTRODUCTION

The concept of fractals is relatively new in the field of mathematics, developed and popularized by the late Benoit Mandelbrot (1924 - 2010). Mandelbrot coined the term **fractal** to mean any fragmented structure with (theoretically) infinite complexity that has the property of self-similarity[1]**.** A more complete definition that captures essence the term fractal is as follows:

> *Fractals are the emergent properties of iterative feedback systems, that exhibit both unpredictable and deterministic behaviours, forming patterns that manifest as complex coherent structures, with the property of scale invariance and self-similarity, displaying very specific boundary conditions, with complex morphologies that have a fractal dimension that uniquely quantifies the level of complexity of the emergent patterns within the system.*

This term **fractal**, however, was not in the minds of the founders of modern when they were developing the concepts that form the foundation of modern scientific thinking. Even Stephen Hawking admits that he does not know how to formulate physical laws on a fractal[2] since calculus, the main tool used by modern physics, does not work well on fractal manifolds. Understanding the universe in terms of fractal geometry requires a paradigm shift in the way we view the universe, which may require new laws and new tools. Although Hawking may concede that he does not know how to formulate physical

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laws on a fractal, what he does not consider is the possibility that these laws might emerge and evolve naturally and directly from fractal geometry itself.

In “Time Reborn”[3], Dr. Lee Smolin (Perimeter Institute for Theoretical Physics) argues that the standard approach to cosmology, which sees time as a dimension and the laws of physics as absolute and timeless, is incomplete, and that in order to move forward in the field of theoretical physics, “we need to make a clean break and embark on a search for a new kind of theory that can be applied to the whole universe”. Smolin proposes that this new theory must avoid the confusing paradoxes, must answer the unanswerable questions, as well as make physical predictions for cosmological observations. He refers to this as the “cosmological challenge”. Although Smolin admits that he does not have such a theory, his book outlines a set of principles to guide such a **new hypothesis**. “The central principle is that time must be real, and physical laws must evolve in that real time”.

The IFS framework presented in this essay addresses this cosmological challenge by proposing a new hypothesis that 1) can be applied to the whole of the universe; 2) avoids many of the confusing paradoxes associated with the standard model; and 3) is able to make physical predictions about the universe we observe. Using this new hypothesis, we begin to answer some of the unanswerable questions proposed by Smolin such as “what is time?” and “why these laws?”.

We begin by outlining the IFS framework from first principles in Section 2. Iteration is the key to understanding this framework and the fractal paradigm in general, since all observables, including time, and the laws of physics, are thought to be emergent properties of an iterative feedback system. In this framework, the act of iteration is analogous to the concept of action in the standard model and is considered an action constant. In Section 3, the Mandelbrot set, **M,** is presented as an example of a wavefunction generator where the process of iteration generates new, higher order wavefunctions. This manifests as continuous change throughout the system leading to the perception of time or the arrow of time. The laws of physics, including gravity, electromagnetism, the strong force and the weak force, are also seen as emergent properties of the iteration process. In the IFS framework, the concept of **black holes** is central to understanding how laws can emerge from fractal geometry. In Section 4, it is shown how black holes appear in the Mandelbrot set wavefunction generator. Although these black holes do not look anything like the black holes of the standard model, the IFS framework proposes that these are the black holes that nature makes. We follow this with a discussion of dimensionality in Section 5 which replaces the 4-dimensional curved space-time of the standard model with a three-dimensional complex curve bounded in four dimensions. Finally, in Section 6, we discuss the nature of light within the context of the IFS framework in an attempt to answer the question, “what is a photon?”. Although this new hypothesis is both philosophically and psychologically different than the standard model, and is highly speculative, the following argument is made: if the universe can be modeled as an iterative wavefunction system, as suggested by the IFS framework, then such a system would generate fractal patterns at all knowable (calculable) scales with no crossover to homogeneity. In other words, we might be able to answer the question, “why fractals?”.

2. THE IFS FRAMEWORK

2.1 Claims

The IFS framework begins with the following claims:

F1: The universe is a fractal and/or is fractal in nature at all knowable scales.
F2: There is a limit to the number of knowable scales in the universe.
F3: The universe can be mathematically modeled as an iterated wavefunction system that generates fractal patterns at all knowable (calculable) scales.

Although fractals do appear in our universe at many scales, including the quantum scale[4], in the standard model of cosmology (dominated by the big bang model), a crossover to homogeneity is assumed at some very large scale, although this is still a matter of considerable debate. The IFS framework, however, claims that there is no crossover to homogeneity at any knowable scale. In the fractal paradigm, its fractals all the way up, and fractals all the way down. Since complexity (fractals) can come from simplicity in iterated function systems, it is argued that the universe itself might be mathematically modeled as a kind-of IFS.

2.2 Framework

This section outlines the IFS framework from first principles. We start with the concept of **action**. In the standard model, action is associated with Planck's constant **h**. In the IFS framework, action is associated with the iteration process where the act of iteration (iterating the wavefunction) generates one unit of change. In this framework, iteration corresponds to change in the system and thus, iteration is considered an action constant. With the principle of action defined, the IFS framework is outlined as follows:

F4: Iteration is an action constant.

It is the act of iteration that generates change in the system. The term **iteration** is analogous to and plays the same role as the term **cycle** in the standard model.

F5: Each iteration corresponds to one unit of energy (oe):

$$\text{oe} = 6.26 \times 10^{-34} \text{ Joules.} \tag{1}$$

(Not to be confused with **h** which has units Joules × second.)

F6: Total energy of a particle or photon (E) is equal to:

$$E = \text{oe} \times \mathbf{n} \text{ Joules.} \tag{2}$$

Here, **n** is the number of iterations associated with a particle's internal structure. A. Worsley refers to this energy equation as harmonic quintessence[5]. Here, the total energy equivalence of a system depends solely on the number (**n**) of harmonic quintessence energy quanta contained within the system. In the IFS framework, the iteration process is responsible for the generation of each harmonic quanta. In "The Fundamental Physics of Electromagnetic Waves", Dr. J. Mortenson describes **n** as the much sought after hidden quantum variable described by Einstein and others[6]. Mortenson argues that this **frequency variable** was inadvertently removed from the language of theoretical physics by replacing the cycle/second with the 1/second in the International System of Units (SI), thus removing "an essential mathematical element of reality in quantum mechanics". The IFS framework replaces the term cycle with the term iteration. Iteration and cycle have different meaning in that **cycle** implies continuous repetition over time and thus can be counted (as one counts money), however, **iteration** implies continuous change over time (see Section 3) and thus, each iteration is not considered equal. In this manner, iteration is not a countable entity in the traditional sense (ie. iteration/iteration $\neq 1$). Also note that the above energy equation can be applied to both particles of light (ie. photons) and particles of matter (ie. electrons and protons) and therefore (2) can be considered more fundamental than Planck's energy equation, $\mathbf{E = hv}$ and Einstein's energy equation, $\mathbf{E = mc^2}$.

F7: The mass associated with one iteration (om) is:

$$\text{om} = \text{oe} / c^2 \text{ kg.} \tag{3}$$

(ie. the mass of an "om particle" or soliton is in the order of 10^{-51} kg.)

F8: Total mass of a particle (M) is then equal to:

$$M = \text{om} \times \mathbf{n}. \tag{4}$$

Next, the IFS framework adopts the principle that the universe is a kind-of computing device, since computers are generally used to calculate wavefunctions and iterated function systems. Here, the following analogy to the modern day computer is proposed:

F9: Planck time is the time for one iteration, and is analogous to one clock cycle of a computer.

F10: Planck frequency is analogous to the clock frequency of a computer.

F11: Planck limit is analogous to the limit to the digits of precision of a computer.

This last statement is a very important piece of the IFS framework and is discussed in detail in Section 4.4.

2.3 Summary of IFS framework

This outlines the basic foundation for the IFS framework. Here, we replace the term **cycle** with the term **iteration**. Cycle and iteration have intrinsically different meaning in that the term cycle implies continuous repetition over time, and iteration implies continuous change over time. This important difference is the key to understanding the IFS framework and the fractal cosmology outlined in this essay. In this framework, all particles, including photons, are modeled as complex fluctuations generated by the iteration process (ie. iterating the wavefunction).

3. THE MANDELBROT SET AS A WAVEFUNCTION GENERATOR

In computer science, iterated function systems (IFS) are commonly used to model the rough geometric shapes that we find in nature [7]. IFS are systems that change and evolve over **time** via the **iteration process**. It is this iteration process (feeding the output of a system back into the system) that is responsible for all the complexity associated with fractal patterns, and, it is argued, may even be responsible for the complex structures and behaviours of the universe itself.

3.1 Why wavefunctions?

In the IFS framework, a novel approach to cosmology is presented that mathematically models the universe as an **iterated wavefunction system**. In such a system, the act of iteration changes 1) the state of the system and 2) the wavefunction itself. In quantum mechanics, wavefunctions are commonly used to describe the wave dynamics of a particle or group of particles. Although the predictive powers of wavefunctions are very well known, no one can really answer the question "why wavefunctions?". In the IFS framework, wavefunctions are generated via the iteration process. In this framework, the Mandelbrot set iterated function system (**M**) is presented as an iterated wavefunction system (or complex wavefunction generator) and is defined by the following expression:

$$Z_{n+1} = Z_n^2 + C \quad \text{(where Z and C are complex variables).} \tag{5}$$

Although this expression appears very simple, the dynamics generated by the iteration process reveals complicated behavior at **all length scales**. How can such a simple mathematical expression create such vast complexity? This is the same question that we have been asking about our universe for thousands of years. How can such a complex universe come into being from (virtually) nothing? In the IFS framework, the answer to this question centers on the iteration process. For example, the first three iterations of (5) generate the following three wavefunctions:

$$Z_1 = (Z_0^2 + C)^2 + C. \tag{6}$$

$$Z_2 = ((Z_0^2 + C)^2 + C)^2 + C. \tag{7}$$

$$Z_3 = (((Z_0^2 + C)^2 + C)^2 + C)^2 + C. \tag{8}$$

As you can see, each iteration generates a new, higher order wavefunction. This translates to an increase in the degrees of freedom of the system and indirectly to an increase in the potential for complexity. This increase in complexity (via the iteration process) is depicted in Figure 1, where each image (A, B, C, D, E and F) corresponds to a different wavefunction.

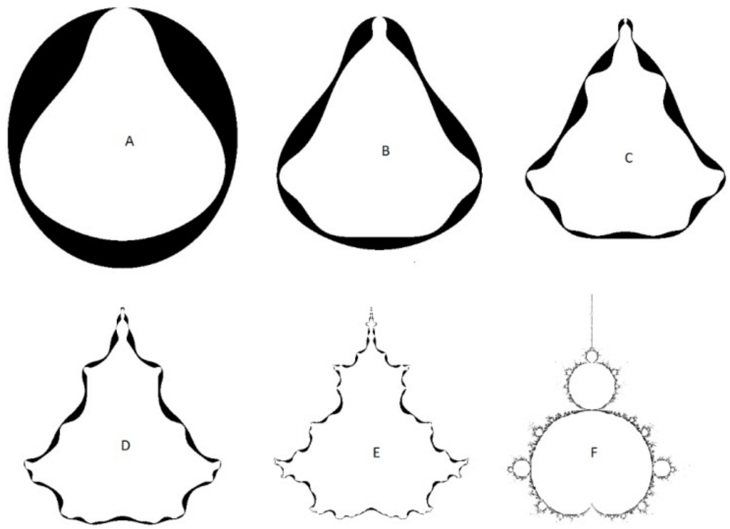

Figure1: Six regions of the complex plane, each corresponding to a different, higher order wavefunction.

The black region in Figure 1A corresponds to all the complex variables (from the complex plane) that escape a certain boundary condition (a radius of 2) after only one iteration. This region corresponds to wavefunction (6). In a similar manner, Figure 1B and 1C represent all the complex variables that escape after two and three iterations and correspond to wavefunctions (7) and (8) respectively. It is found that each higher order wavefunction corresponds to a unique set of complex variables from the set of complex numbers. Figure 1F represents all the complex variables that escape after 100 iterations. As can be seen from this sequence of images, the curve or boundary generated by iterating the wavefunction (5) becomes more and more complex with each iteration. One can imagine that after 1000 iterations, the associated wavefunction can become quite complex. This corresponds to extreme curvature of the complex curve generated by the iterative wavefunction system **M**. In the standard model of cosmology, extreme curvature is associated with black holes. The IFS framework has a similar concept of black holes as will be discussed in Section 4.

The current probabilistic (statistical) approach to quantum mechanics and wavefunctions does not give any physical meaning to, nor can it make any specific predictions about any real event within the system it is said to be describing. Instead, it can only make predictions about the possibility (probability) of an event happening (or not happening) at any given place or time. It is the mechanism of norming (averaging) the wavefunction that essentially blurs the underlying real dynamics that are going on within the system (likely involving chaotic dynamics that cannot be predicted with any certainty). In short, the wavefunction normalization procedure obscures important details regarding the actual underlying dynamics of the waves or particles in question. At this point, there is another important question we should be asking: "What is the actual mechanism that drives the particle-wave dynamics of quantum mechanics to the solutions we observe statistically?". The IFS framework argues that it is the feedback process of iterating the wavefunction that drives the wave dynamics toward those solutions.

Louis de Broglie was one of the founders of quantum mechanics and the originator of the concept of wave-particle duality[8]. De Broglie was also very much a realist and was not a big fan of the statistical (subjective) interpretation of wavefunctions and quantum mechanics (the Copenhagen interpretation) that had nudged its way into favor[9]. What he was looking for was a more concrete physical (objective) representation for particle and wave dynamics that also matched reality. After many years of supporting the standard interpretation, De Broglie brushed off his old double solution theory which proposed an alternate interpretation of wavefunctions that did not involve statistical representations but instead, saw the wave-particle as a realistic entity. His new approach does not in any way contradict the standard model and may in fact explain why it works.

De Broglie's double solution theory proposes a real wave interpretation (v-wave) of wavefunctions, combined with the probabilistic interpretation (w-wave) of the standard model. The v-wave interpretation has physical meaning, cannot be arbitrarily normed (averaged) and is distinct from the Copenhagen interpretation in that it represents the pre-normalized wave. The v-wave encapsulates the chaotic, objective yet unpredictable characteristics of wavefunctions, and the standard statistical w-wave encapsulates the subjective yet deterministic characteristics. Both approaches are seen as equally valid ways of looking at the same data. The IFS framework supports De Broglie's v-wave interpretation of wavefunctions as will be discussed in more detail in Section 4. First, we need to address the concept of time within the IFS framework.

3.2 Why time?

In order to have a model of the universe that matches reality, we must first have a working model of time. The currently accepted and dominant view of cosmology has **time** embedded into a four dimensional space-time manifold with three dimensions of space and one dimension of time. Time is often thought of as the fourth dimension in this model. In the IFS framework, time, or the perception of time, is the result of an ever changing chaotic dynamical system. It is the unending unknowable uniqueness of each moment that gives us the sensation of time. "With clocks, we measure only numerical order of physical events and the numerical order of change runs only in space and not in time." [10]. Einstein also states that "time has no independent existence apart from the order of events by which we measure it." [11]. Smolin's book "Time Reborn" [3] centers on this very important question, "what is time?". Before we answer this question, we need to make sure we are asking the right question. Maybe the question should be "why is time?" not "what is time?" Why does time exist? This question is much more intrinsic and, in the IFS framework, has a very simple and obvious answer:

F12: Time is an emergent property of change brought about by the iteration process.

As demonstrated in Section 3.1, iterating the wavefunction not only changes the state of the system; it changes the wavefunction itself. Thus, the iteration process guarantees continuous change after each iteration (cycle) since each iteration generates a new, higher order wavefunction. This increases the degrees of freedom of the underlying system, allowing ever increasing complex behaviours to emerge. It is the continuous emergence of these ever changing wavefunctions that gives us the perception of time or the arrow of time.

Now we can answer the question, "Why does time exist?" Time exists because **things** change. Change is built into the equation (by iterating the wavefunction). Time is an emergent property of change. Change is both real and physical (measurable); in fact, all that is ever measured is change; (ie. change in position, change in speed, change over time). In the IFS framework, time both emerges and evolves, since the degrees of freedom are continuously changing (increasing) with each iteration. The same can be said for the laws of physics which also emerge and evolve within this framework, as will be demonstrated in the following sections. In the IFS framework, the question, "why is change?" can be answered: because **change is built into the 'equation'.**

4. THE MANDELBROT SET AND THE LAWS OF PHYSICS

In the IFS framework, the emergence of the laws of physics centers on the concept of black holes. Section 3 demonstrates how iterating the wavefunction generates higher order wavefunctions resulting in the extreme curvature associated with the Mandelbrot set boundary. The IFS framework presents a new description of black holes which are fractal structures (strange attractors) that can appear at many scales including the atomic and quantum scales. The standard model of black holes are also scalable in that they appear at both the stellar scale and the galactic scale, however, in this model, the atom is not considered a black hole. In the IFS framework, the laws of physics are emergent properties of scalable IFS black holes (which in turn are emergent properties of an iterative wavefunction system) and in this manner, the laws of physics are also scalable. In this framework, dark energy corresponds to electromagnetism at the small scale. Dark matter corresponds to the strong force at the small scale and gravity corresponds to the weak force at the small scale. These concepts will be discussed in great detail throughout this section.

4.1 Schwarzschild black holes

According to standard cosmology, black holes are cosmic objects with massive gravitational fields that curve space-time so drastically that nothing, including light, can escape its event horizon. When Einstein's theory of relativity first predicted the existence of black holes, it was considered as a mathematical curiosity, and thus, black holes were not thought to exist in reality. It is now commonly understood that black holes do exist, and are found in many places in the universe including at the center of all galaxies. Recent observations suggest that black holes may even play a role in the formation and evolution of the galaxies themselves[12]. Although we are quite certain of the reality of black holes, the reason for their existence is still somewhat of a mystery.

The Schwarzschild solution to Einstein's GR equations (depicted in Figure 2) represents one of the simplest and most general black hole models. According to this model, nothing can escape from inside the event horizon of a black hole, not even light. Just outside the event horizon, escape is possible, but only if one is travelling at the speed of light, thus, only photons can escape this region. Inside the photon sphere, photons are forced to travel in orbits due to extreme gravity. According to GR, photons cannot stay in the photon sphere forever due to the unstable orbits in this region, thus, they will eventually escape, even if after a very long time. In the standard model, extreme gravity is thought to be the cause of black holes. Also, in this model, the internal structure of the black hole cannot be known.

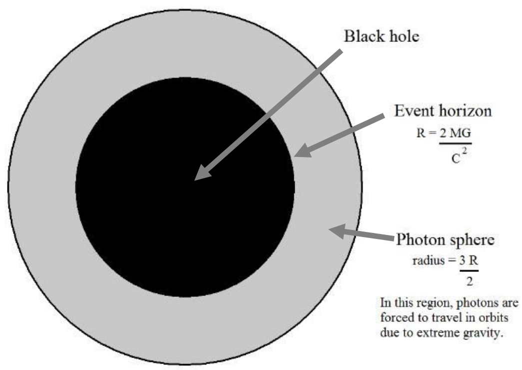

Figure 2: Model of a Schwarzschild black hole depicting the black hole, the event horizon and the photon sphere regions. In this model, gravity alone is responsible for the extent and behaviours of these regions.

4.2 IFS black holes

In the IFS framework, black holes exist and, like time, are the emergent properties of the iteration process (iterating the wavefunction). In this section, an analogy between the Schwarzschild black hole (Figure 2) and the Mandelbrot set (Figure 3) is proposed.

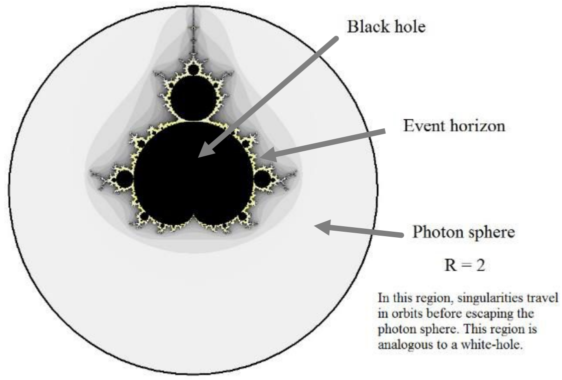

Figure 3: The Mandelbrot set as a black hole. Here, the black region of **M** is analogous to a black hole. The gradient region inside the boundary (R=2) is analogous to the photon sphere. In the IFS framework, this region also corresponds to a white hole. The event horizon is the fuzzy boundary that exactly separates a white hole from a black hole. This is where all the complex fractal patterns associated with **M** are found.

To generate this image, a set of complex variables is iterated through the wavefunction (5). These variables are considered as the initial conditions of the wavefunction generator. Each iteration generates a new state (new complex variable) as well as a new, higher order, wavefunction. After each iteration, a test is done to see if the new state has reached the escape condition, R=2. If it escapes, we give it a color depending on the number of iterations (energy) it took to escape. *Mandelbrot referred to this escape condition as* ***escape*** **velocity**[1] *and, for the sake of the analogy presented in this section, this term will be adopted.* The complex variables that can and do reach escape velocity correspond to the gradient regions in Figure 3. This region is analogous to the photon sphere in Figure 2. In the IFS framework, this region is also associated with white holes. More on this in Section 4.5. The inner black region corresponds to the trajectories that never reach escape velocity even after an infinite number of iterations. This region is analogous to the black hole region in Figure 2. The fuzzy boundary that exactly separates the black hole from the photon sphere is analogous to the event horizon in Figure 2. The trajectories associated with this region will be discussed further in Section 4.6. For now, we will be focusing on the black hole regions of **M**. Unlike standard black holes, in the IFS framework, the internal structure of the inside of a black hole can be known. The trajectories (singularities) associated with the IFS black hole will be discussed in the next section.

4.3 Singularities

The standard notion of a singularity, as an object with infinite density in an infinitely small volume is one of the greatest paradoxes facing modern physics. Infinities in an equation are usually a sign that something is wrong. The IFS framework solves this problem by interpreting the Planck limit as a **limit to the digits of precision** of the universal computer. In section 4.4, we demonstrate how this limit is able to prevent the infinite collapse of singularities (at the very smallest scale) thus avoiding the infinite mass condition. (*This model is in close alignment with loop quantum gravity which assumes a minimum unit of space and a minimum unit of time. In the IFS framework, a complex space governed by a wavefunction generator is analogous to a spin foam and the complex curve or event horizon generated by a wavefunction generator is analogous to a spin network. In the IFS framework, spin foams generate spin networks.)*

In his double solution theory, De Broglie sees particles as high energy concentrations of small amplitude chaotic fluctuations that are likened to a singularity[8]. The particle's internal vibration is likened to a small clock. The dynamic (chaotic) fluctuations are associated with the particles internal structure where an increase in fluctuations corresponds directly to an increase in energy (and indirectly to an increase in mass). De Broglie thought it premature to try to describe the internal structure of his real v-waves since, he argues, "it would probably involve complicated non-linear equations". His reluctance to describe these structures was likely due to the fact that calculus, the main tool used in theoretical physics, does not work well on fractal manifolds.

In the IFS framework, it is argued that the non-linear dynamics associated with De Broglie's v-waves can be generated by iterating the wavefunction. Each iteration is associated with one unit of energy. Particles (singularities) that require more iterations (per unit time) are considered more energetic in the case of **light** particles such as photons, and more massive in the case of **matter** particles such as electrons and protons. In this manner, the IFS framework gives a mechanism for De Broglie's **sub-quantum medium** where the act of iterating the wavefunction generates real and measurable phenomenon. In short, it is argued that iterating the wavefunction generates exactly the v-wave **particles** that De Broglie imagined. Next, we demonstrate how iterating a wavefunction can reproduce the very real, complex dynamical fluctuations that could be considered as a physical manifestation of De Broglie's v-wave. For example, Figure 4 depicts the trajectories of three distinct complex variables selected from the black hole region of **M**. In the top row, the order of points is shown by connecting consecutive points with lines. This demonstrates the actual path of the **solitons** during the iteration process.

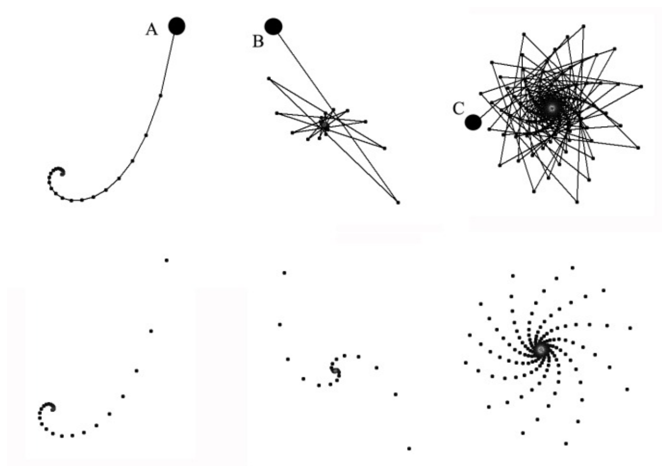

Figure 4: The above image depicts the trajectories of three complex variables that originate from the black hole region of **M**. The large dots labeled **A**, **B** and **C** correspond to the starting point or initial condition for each **particle** or singularity as specified in Table 1. In this image, the smaller dots correspond to the path or trajectory generated by the iteration process.

Table 1: Initial conditions of the three trajectories from Figure 4.

Singularity	Real Component	Imaginary Component
A	0.25671428571428567	-0.02857142857142847
B	-0.53757142857142881	-0.17142857142857149
C	-0.27471428571428591	-0.61142857142857143

In the IFS framework, these collapsing trajectories are analogous to black hole singularities and (it is argued) De Broglie's v-wave particles. Mandelbrot argued that if a singularity is not a geometric shape such as a point, a line or a surface, then it must be a fractal[1]. In this framework, black holes are associated with the strong force (quarks) at the small scale and dark matter, at the large scale. In short, the IFS framework presents a mechanism for the continuous collapse of singularities that avoids the inherent infinities associated with the standard model (due to the Planck limit or limit to the digits of precision of the universal computer. See next section.), as well as reproducing many of the more complex patterns that we observe in nature.

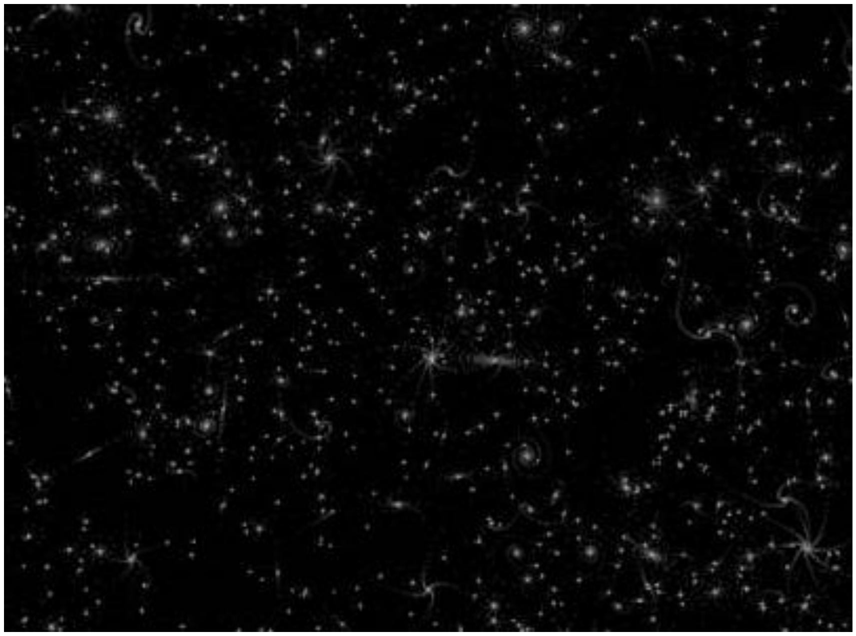

Figure 5: In this figure, a random set of IFS black hole singularities were generated using the iterative wavefunction generator **M**. Notice the morphological similarity between this image and a Hubble deep field image.

4.4 Why Planck limit?

While studying the collapsing singularities such as the ones in Figure 4 and 5, it was found that the infinite collapse of these singularities was prevented by the **limit to the digits of precision** of the computer. This is due to the breakdown of the computer's ability to compute wavefunctions at that scale. In the IFS framework, it is argued that the Planck limit is analogous to the limit to the digits of precision of a computer and, for all intents and purposes, serves the same purpose as the computer quantum limit in that it stops the infinite collapse of black hole singularities. Coincidentally (or not), the Planck limit, at ~34 digits of precision, has exactly the same quantum limit as a 64-bit computer according to the IEEE standard of floating point numbers (IEEE 754-1985)**.** In short, the IFS framework gives a reason for the existence of a Planck limit. Next, we demonstrate exactly what is happening to the IFS singularities at the Planck limit.

It is found that the continuous collapse (contraction) of these singularities is suddenly stopped when it reaches this limit, and instead, it slips into an endlessly repeating sequence or loop. It is also found that the number of elements in this **loop** exactly matches the number of **spiral arms** seen within the structure being generated. In the IFS framework, this loop condition is referred to as **loop singularity**. For example, image **A** in Figure 4 represents a loop-1 singularity, **B** is a loop-2 singularity and **C** is a loop-11 singularity. It is also found that the collapse of these singularities can continue unabated past the Planck limit when given more digits of precision (by utilizing a special mathematical tool to extend the digits of precision) however, in the case of the universe, there is a limit, the Planck limit. In short, the IFS framework gives a real and understandable explanation for the Planck limit and even suggests a reason for its existence.

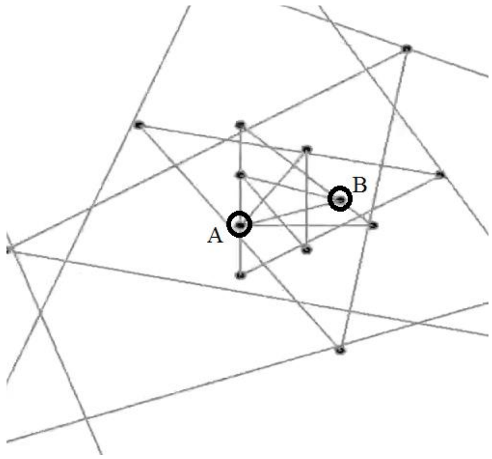

Figure 6: Video 1: The movie linked to this figure follows the continuous collapse of an IFS black hole singularity down to the quantum limit (the limit to the digits of precision of the computer). The initial conditions for this singularity are specified in Table 2. The above image represents the final break down of the singularity at the quantum limit. The two circles labelled **A** and **B** correspond to the final repeating pattern that the singularity falls into. This condition is referred to as loop-singularity in the IFS framework. The above figure is thus considered a loop-2 singularity.

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Table 2: Initial conditions of the singularity in Figure 6.

Real Component	**Imaginary Component**	**Iterations to Loop Singularity**
-0.53399654140623998	-0.23168956718750006	197

4.5 White holes

Along with black holes, general relativity (GR) also predicts something referred to as a white hole. These are regions of space-time from which matter (and light) may escape. They are for all intents and purposes, the reverse of black holes. According to GR, a white hole event horizon in the past becomes a black hole event horizon in the future. However, also according to GR, there is no mechanism with which a white hole can form and therefore, they are not thought to exist in reality. In the IFS framework, it is argued that white holes can and do exist and correspond to the escaping regions of **M**, the gradient region in Figure 3 and the photon sphere in Figure 2.

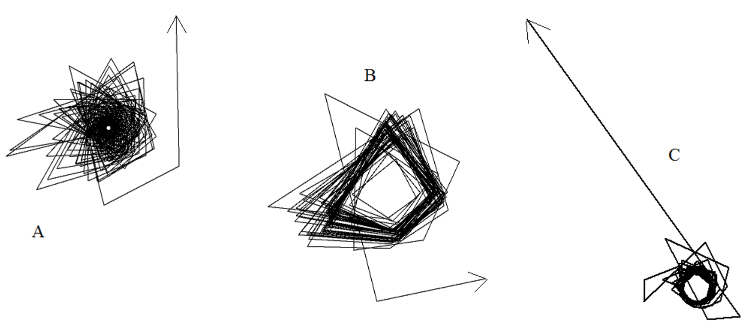

Figure 7: This image depicts three trajectories that are able to escape the boundary condition of **M**. These correspond to white hole singularities in the IFS framework. Unlike the black hole trajectories, these ones are spiraling (orbiting) away from the center until they reach escape velocity.

Table 3: Initial conditions of the three trajectories from Figure7.

Singularity	Real Component	Imaginary Component
A	0.08141887555369647	0.62448274333267884
B	0.23968372492222095	0.54659111668112559
C	0.36109748007296816	0.11689364183160467

Figure 7 depicts three singularities from the escaping region of **M** with initial conditions specified in Table 3. Although these look very similar to the collapsing black hole singularities from Figure 4, they are in fact expanding out to the boundary condition. Since these singularities originate from the photon sphere region in Figure 3, in the IFS framework, these singularities are associated with particles of light or photons. Like the photons in the Schwarzschild model, the photons in Figure 7 are travelling in orbits up until the point that they reach escape velocity, at which point their singularities become more like the straight line trajectories of photons far away from extreme gravity. In the IFS framework, speed of light is seen as the escape velocity of a photon from a black hole. (*In this framework, the escape velocity of more energetic photons is thought to be slightly lower than that for the less energetic photons. This is in close alignment with loop quantum gravity which also predicts a slightly different speed of light for photons of different energies. Further investigation required.)*

The reason white holes can exist in the IFS framework is because time is an emergent property of change and not a dimension attached to a space-time manifold. The GR description of time has created many paradoxes associated with time which cannot be reconciled within the context of the standard model. Since general relativity predicts the existence of white holes, then, like black holes, white holes should also exist in reality. The IFS framework gives a mechanism for white holes to exist without creating any unresolvable paradoxes associated with the standard interpretation. In this framework, white holes are associated with photons (the photon sphere) at the small scale and dark energy at the large scale. White holes and black holes are the reverse of each other in the IFS framework. Here, the escaping singularities (white holes) correspond to the expansion dynamic and the non-escaping singularities (black holes) correspond to the contraction dynamic. The iteration process is responsible for the generation of these two opposing forces, which are indirectly responsible for the emergence and evolution of time, the laws of physics and, as we will see in the next section, the self-organization of all organized matter.

4.6 Event horizons

In general relativity, an event horizon is a boundary condition where the gravitational pull of a black hole becomes so extreme that nothing, not even light can escape this boundary. It is often referred to as the **boundary of no return** and in its simplest form, is modeled as the surface of a spherical black hole as depicted in Figure 2. In the IFS framework, the term **event horizon** has a different meaning. In the **M** model (Figure 3), the event horizon is the boundary that exactly separates the black hole region from the white hole region. Mandelbrot referred to this boundary as **S** for separator[1]. This is an event horizon in the truest sense in that is separates the expanding events from the contracting events. These two opposing forces (expansion and contraction) conspire to the generation of an infinitely complex fractal curve. This is where all the complex fractal patterns associated with **M** are found. Many self-similar (not exactly the same) copies of this black-hole-white-hole pattern are embedded throughout this complex fractal structure at many scales including the quantum scale (as depicted in Figure 8).

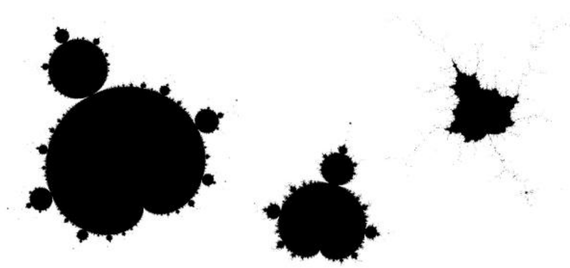

Figure 8: Self-similar copies of the Mandelbrot set imbedded into the event horizon of the wavefunction generator **M**.

In the IFS framework, this scalable structure is thought to be responsible for the emergence of the laws of physics, and thus, the laws of physics are also thought to be scalable. Like the black holes in the standard model, in the IFS framework, the closer one gets to the event horizon of a black hole, the more energy (iterations) it takes to reach escape velocity. The singularities with these initial conditions turn out to be the most interesting as seen in Figure 9, 10 and 11. Notice the morphological similarities between these singularities and the Sting Ray nebula in Figure 9, the Cartwheel galaxy in Figure 10 and the galaxy cluster as seen in Figure 11. The standard model (governed by gravity) has difficulty explaining these kinds of morphologies, yet, in the IFS framework, they seem to come for free.

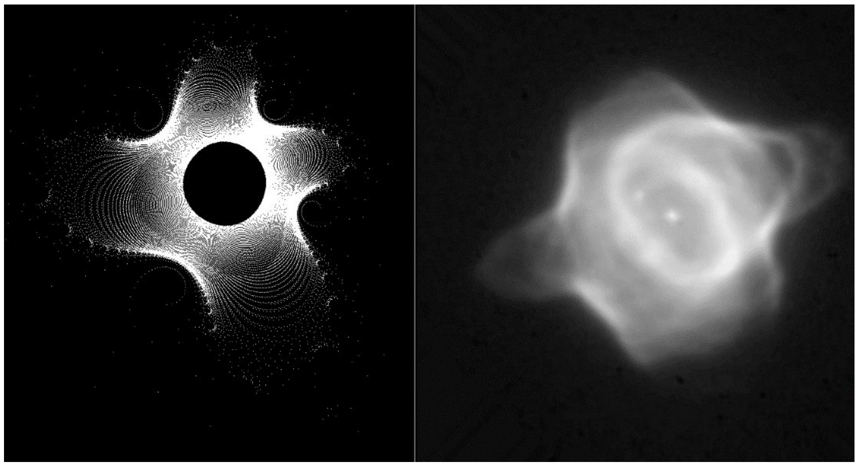

Figure 9: The image on the left depicts a white-hole singularity (from the escaping region of **M**) with initial conditions specified in Table 4. The image on the right is the famous Stingray Nebula (*Image credit: Nasa*.). In the standard model, patterns such as these are very difficult to explain and reproduce using gravity alone. In the IFS framework, patterns such as these are emergent properties of an iterative wavefunction system.

Table 4: Initial conditions and number of iterations associated with Figure 9.

Real Component	Imaginary Component	Iterations (n)
0.25520149659878483	0.49473324077959746	73,953

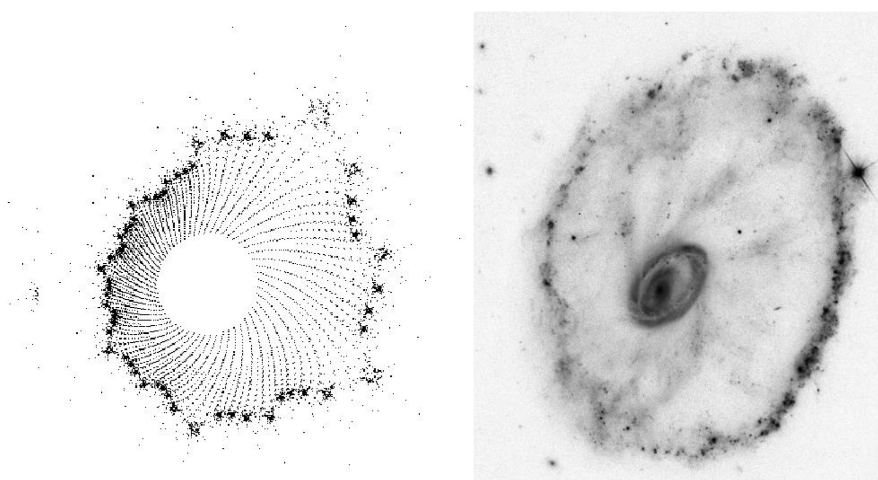

Figure 10: Image on the left depicts a complex white-hole singularity (from the escaping region of **M**) with initial conditions specified in Table 5. The image on the right is the famous Cartwheel Galaxy (*Image credit: NASA)*. Notice the morphological similarity between these two forms. Many examples of these are found close to the edge of the event horizon of an IFS black hole.

Table 5: Initial conditions and number of iterations (n) associated with singularity in Figure 10.

Real Component	Imaginary Component	Iterations (n)
0.31324370654012845	0.03877109020960720	9309

Figure 11: Image on the left depicts a complex white hole singularity (from the escaping region of **M**) with initial conditions specified in Table 6. Notice the morphological similarity between this object and the Virgo Cluster on the right (*Image: Credit Nasa).* Many examples of these are found close to the edge of the event horizon of an IFS black hole.

Table 6: Initial conditions and number of iterations associated with the singularity in Figure 11.

Real Component	Imaginary Component	Iterations (n)
0.05103771361715907	0.64098319549560490	48710

4.7 Particles, fields and information

John Archibald Wheeler is one of the leading black hole experts and is credited with coining the term **black hole**. Wheeler went through many phases in his physics career. Earlier on, everything was particles, then everything was fields, and finally, everything was information[13]. In the IFS framework, everything is singularity. In this framework, singularities correspond to particles, fields and information. Each iteration generates higher order wavefunctions which in turn generates new information. The singularities presented in this section represent information. It is found that each unique complex variable generates an equally unique complex trajectory and, since the complex numbers represents a dense set, there are an infinite number of possible paths these trajectories can take. The Schwarzschild black hole contains very little information, but the IFS black hole is in fact information.

4.8 Why these laws?

In the standard model of cosmology, gravity alone is thought to be responsible for the formation of black holes. The IFS framework provides a mechanism for the existence of black holes, event horizons and white holes which avoids the historical paradoxes and misinterpretations, and gives a simple mathematical formalism for their emergence. This framework describes a scalable mathematical structure that sees this white-black hole as a scalable primordial structure (Figure 3) which can be applied to the whole of the universe, at all scales. Although they don't look anything like the black holes, event horizons and white holes of the standard model, the IFS framework argues that these are the black holes, event horizons and white holes that nature makes.

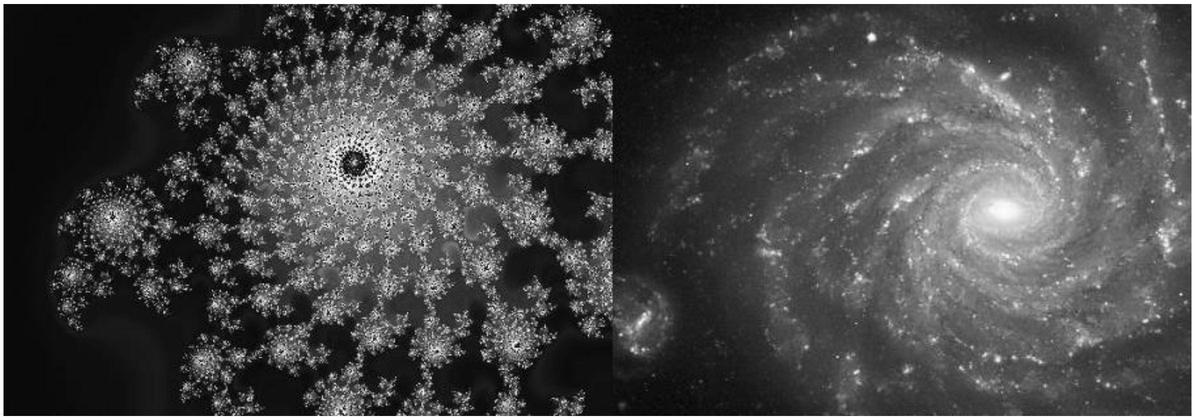

Figure 12: Image on the left depicts a sub-region of the Mandelbrot. The center and extent of this region is specified in Table 7. These kinds of patterns are found close to the edge of the event horizon in **M**. The image on the right is the famous Grand Spiral Galaxy (NGC 1232) (*Credit: FORS1, 8.2-meter VLT Antu, ESO*). Notice the morphological similarities between these two images. In the IFS framework, these patterns are emergent properties of an iterative wavefunction system.

Table 7: Parameters for generating the above sub-region of the Mandelbrot set.

Real Center	Imaginary Center	Real Extent	Imaginary Extent
0.238291710520393	0.38038103197243328	0.00020176911157024794	0.00015132683367768596

In the IFS framework, gravity is an emergent property of black holes and not the other way around. It is at this point that this framework begins to challenge (contradict) the standard model. If gravity (large scale force) is not required to make a black hole, then there is nothing preventing the atom from being a black hole. In this model, the nucleus of the atom (strong force) corresponds to the black hole region of **M**, the electron shell (electromagnetism) corresponds to the white hole region, and the region that separates the nucleus from the electron shell region (weak force) corresponds to the event horizon. In this model, the laws of physics are not primordial and timeless, but are emergent properties of an iterative wavefunction system.

5. THE MANDELBROT SET AND DIMENSIONALITY

As demonstrated in Figure 1, the boundary generated by the Mandelbrot set iterated function system (**M**) corresponds to a complex 1-dimensional curve (event horizon) bounded in 2-dimensions (ie. the 2D complex plane). This curve is referred to as a space filling curve and has a maximal fractal dimension of two[14]. In fractal geometry, an object is defined as a fractal if its fractal dimension exceeds its topological dimension[1]. In the case of **M**, the fractal dimension of the boundary greatly exceeds the topological dimension, and therefore, this boundary or curve is by definition a fractal. However, our universe is not 1-dimensional but is spatially perceived to be 3-dimensional.

In the standard model of cosmology, space-time is presented as a 4D manifold with three dimensions of space and one dimension of time. Here, time is thought of as the 4th dimension. In the IFS framework, time is an emergent property of change brought about by the iteration process, and therefore is not considered a dimension in the traditional sense. Instead, we imagine the universe as a 3-dimensional fractal curve bounded in four dimensions. In other words, (in the IFS framework) the universe could be described as a 4-dimensional Mandelbrot set, which generates a 3-dimensional fractal curve (event horizon) that corresponds to the observable universe. This still leaves us with a 4-dimensional universe; however, the fourth dimension becomes a bounding dimension and not a spatial dimension in the traditional sense.

As the universe evolves in this fractal paradigm (with each iteration), so do the degrees of freedom driven by the never ending iteration process. This increase in the degrees of freedom manifests as an increase in the fractal dimension of the 3+ dimensional fractal curve that is the visible universe. In short, the fractal dimension of the universe ever increases over

time. The IFS framework can be extended to 4-dimensions by utilizing a mathematical construct known as the quaternion which consists of one real component and three imaginary components, Q(r,i,j,k). The quaternion numbers are similar to the complex numbers only they have two extra imaginary dimensions (j and k). It is found that the quaternions do generate similar behaviours (singularities) as the complex numbers. This will be the focus of future research.

6. THE MANDELBROT SET AND THE NATURE OF LIGHT

The main motivation for this essay was to answer the question "what is a photon?". Now that the foundation for the IFS framework has been laid, we can begin to answer this question. In Section 3.1, we discussed the iteration process in terms of iterating the wavefunction, and determined that such a system would generate higher order wavefunctions over time. In the standard model, wavefunctions are associated with the quantized energy levels of particles. In the atom, the energy levels within the electron shell region are quantized and each quantized region can be modeled as a wavefunction. The white hole region of **M** has similar properties in that each quantized region depicted in Figure 1 corresponds to one wavefunction and one energy level. Thus, the IFS framework provides a mechanism for the quantization of energy levels. In the standard model, wavefunctions can be applied to collections of bosons. Here, it is argued that the regions depicted in Figure 1 are analogous to collections of bosons and, since the complex numbers represents a dense set, there are an infinite number of complex variables that correspond to each energy state. This is the first clue that the white hole region of **M** is related to photons, since photons are bosons in the standard model.

In Section 4.5, we presented an analogy between the white hole region of **M** and the photon sphere of the Schwarzschild black hole. In both the standard black hole and the IFS black hole, photons (singularities) in this region are forced to travel in orbits until they reach escape velocity or, in the case of the standard model, the speed of light. In the IFS framework, black holes are self-similar structures that can appear at many scales including the atomic and quantum scales. Here, there is nothing preventing the atom from being a black hole. In the IFS framework, the atom is modeled as a mini white-black hole analogous to Figure 3. In this model, the strong force is associated with the black hole region of **M**. *The singularities within this region are analogous to quarks and, like quarks, are found to exhibit asymptotic behavior within the black hole region*. Electromagnetism is associated with the white hole region of **M.** In Figure 7 we saw how singularities from the white hole region of **M** behave like the photons inside the photons sphere of a Schwarzschild black hole. In this region, the photon's orbital behavior continues unabated until it reaches the escape condition or speed of light in the case of a Schwarzschild black hole. In other words, the speed of light is the escape velocity of a photon from a black hole. With this new interpretation of the speed of light, it becomes clear that the atom must be a black hole; not the gravitational black hole from the standard model, but the white-black hole from the IFS framework.

7. CONCLUSION

In this essay, a novel approach to cosmology is presented which argues that the universe is fractal in nature at all observable scales with no crossover to homogeneity. A framework is presented that connects the standard model to the fractal paradigm via the iteration process. In this framework, **iteration** is analogous to **cycle**, and the iteration process is responsible for the emergence of time, the laws of physics and all organized matter via an iterative wavefunction system. A new description of singularity is presented leading to a new understanding of black holes, event horizons and white holes which avoids the historical paradoxes and contradictions associated with the standard model.

In short, it is argued that the fractal paradigm offers new information that was not in the minds of the founders of modern physics. This approach to cosmology not only replicates many of the physical observables, it is also able to address some of the more perplexing problems associated with modern physics, as well as being able to answer some of the unanswerable questions such as: "What is time? What is light? Why Planck limit? Why wavefunctions? Why curved space? Why singularities? Why black holes? Why quantization", "Why these laws?" and, last but not least, "Why fractals?" We used to think the world was flat. We were wrong. We used to think that the sun went around the earth. We were wrong. We used to think that extreme gravity created black holes. The IFS framework says we were wrong. In this framework, gravity is an emergent property of black holes which in turn are emergent properties of an iterative wavefunction system. We used to think that the universe was homogeneous and isotropic at some very large scale. In the IFS framework, its fractals all the way up, and fractals all the way down.

Future work: The IFS framework will be studied to further understand the fractal nature of light, the universe, and everything. The concepts of dark energy, dark matter, antimatter, relativity, the equivalence principle, particle dynamics, and black hole entropy will be addressed within the context of this framework. The extension these concepts to the 4D quaternion space will also be explored.

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