Einstein's 1905 Derivation of the Equations of Special Relativity Leads to its Refutation

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Abstract

Analysis of the Einstein's Special Relativity equations derivation, outlined from his 1905 paper *On the Electrodynamics of Moving Bodies*, revealed several contradictions. It imposed, through the speed of light principle, particular values on the space and time coordinates that, when used explicitly in Einstein's own equation substitutions, led to fundamental contradictions. Furthermore, the space and time coordinates used in the derived transformation equations to obtain the time dilation and length contraction predictions were found to be incompatible with the method used in the derivation to perform the time calculations; no such predictions were deemed feasible. The Special Relativity was hence found to be self-refuted.

Keywords: *Special Relativity, Einstein, 1905 Paper, SR refutation.*

1. Introduction

Originally, in an attempt to save the ether theory, Lorentz developed his well-known transformation to explain, with other physicists (Larmor, Fitzgerald, and Pointcaré), how the speed of light seemed to be independent of its propagation direction with respect to that of the earth motion around the sun, following the puzzling results of the famous Michelson-Morley experiment.[1] In Einstein's Special Relativity,[2] the ether conjecture was abandoned, and replaced by the relativity principle and the constancy of the speed of light principle, arriving at the same Lorentz transformation, yet under a different controversial context.

Einstein theory of special relativity has received much criticism.[3] [4] [5] [6] [7] [8] [9] [10] [11] [12] [13] Doubts on the bases of scientific, mathematical, and philosophical conflicts have been expressed. Criticism, on both academic and non-academic levels, has been mainly motivated by the unordinary physical phenomena of the time dilation ("slowing" of time in moving inertial frames) and length contraction of moving rigid bodies, emerging from the purely mathematical formulation of the theory, in addition to numerous paradoxes combined with the inconsistency and ambiguity in their resolutions.

In this paper, the mathematical derivation of the Special Relativity equations, given in Einstein's 1905 paper,[2] is exploited to carry out equation maneuverings leading to fundamental contradictions in the latter equations. In addition, the time dilation and length contraction predictions are shown to be erroneously deduced from the Special Relativity equations.

2. Contradictions in Einstein's 1905 Derivation

2.1. Derivation Outline

Firstly, for the sake of convenient referencing, the derivation section of Einstein's 1905 paper[2] shall be summarized. The summary will be followed by critical analyses.

In §3, entitled "Theory of the Transformation of Co-ordinates and Times from a Stationary System to another System in Uniform Motion of Translation Relatively to the Former", of the above mentioned paper, the transformation equations relating the coordinates of the stationary frame having the coordinates system $K(x,y,z,t)$ and the traveling frame having the system $k(\xi,\eta,\zeta,\tau)$ are derived. The first derivation step is set to determine a basic equation for τ as a function of the K coordinates. To accomplish this, the travel time for a light ray to go back and forth a certain distance in k is considered. This distance is set as

$$x' = x - vt, \qquad (1)$$

which is independent of time when it is fixed in k. In other words, a stationary point in k will have a set of values x', y, z independent of time. So, τ will be first determined as a linear function of $x', y, z,$ and t, i.e. $\tau(x', y, z, t)$. With respect to an observer in k,

$$\frac{1}{2}(\tau_o + \tau_2) = \tau_1, \qquad (2)$$

where $\tau_o, \tau_1,$ and τ_2 are the times in k at which the ray is emitted, reflected, and returned to its start point, respectively. Given the constant speed of light c, the stationary system time arguments of τ in $\tau_o, \tau_1,$ and τ_2 are $t, t+\frac{x'}{c-v}$, and $t+\frac{x'}{c-v}+\frac{x'}{c+v}$, respectively, corresponding to the space coordinates $(0,0,0),(x',0,0),$ and $(0,0,0),$ respectively. Therefore, Eq. (2) can be written as

$$\frac{1}{2}\left[\tau(0,0,0,t)+\tau\left(0,0,0,t+\frac{x'}{c-v}+\frac{x'}{c+v}\right)\right]=\tau\left(x',0,0,t+\frac{x'}{c-v}\right),$$

which can lead to

$$\frac{1}{2}\left(\frac{1}{c-v}+\frac{1}{c+v}\right)\frac{\partial\tau}{\partial t}=\frac{\partial\tau}{\partial x'}+\frac{1}{c-v}\frac{\partial\tau}{\partial t};$$

$$\frac{\partial\tau}{\partial x'}+\frac{v}{c^2-v^2}\frac{\partial\tau}{\partial t}=0. \qquad (3)$$

Similarly, considering light rays reflected back and forth along the Y and Z axes, it can be shown that

$$\frac{\partial \tau}{\partial y} = 0, \quad \frac{\partial \tau}{\partial z} = 0. \tag{4}$$

Equations (3) and (4) lead to the general form of the τ function:

$$\tau = a\left(t - \frac{v}{c^2 - v^2}x'\right), \tag{5}$$

where a is yet an unknown function of v, which shall be determined.

Next, the space coordinates transformation equations are determined. Using the constancy of the speed of light principle, the propagation speed of light in the traveling system k is also c, and for a light ray emitted at $\tau = 0$ in the positive ξ direction, we have $\xi = c\tau$. Therefore,

$$\xi = ac\left(t - \frac{v}{c^2 - v^2}x'\right). \tag{6}$$

But, as Einstein puts it, the ray moves relatively to the initial point of k, when measured in the stationary system, with the velocity $c - v$, so that

$$\frac{x'}{c - v} = t, \tag{7}$$

which, when inserted in Eq. (6), yields

$$\xi = a\frac{c^2}{c^2 - v^2}x'. \tag{8}$$

Similarly, in the η and ζ directions, $\eta = c\tau$, and $\zeta = c\tau$, with $t = \eta \,/\, \sqrt{c^2 - v^2}$, and $t = \zeta \,/\, \sqrt{c^2 - v^2}$, respectively, along with $x' = 0$ in both cases, Eq. (6) leads to

$$\eta = a\frac{c}{\sqrt{c^2 - v^2}}y, \tag{9}$$

$$\zeta = a\frac{c}{\sqrt{c^2 - v^2}}z. \tag{10}$$

Substituting the value of x' given by Eq. (1) into Eqs. (5), (8), (9), and (10) yields

$$\tau = a\frac{c^2}{c^2 - v^2}\left(t - \frac{vx}{c^2}\right), \tag{11}$$

$$\xi = a\frac{c^2}{c^2 - v^2}(x - vt). \tag{12}$$

—The above steps were skipped in Einstein's derivation.

Let

$$\beta = \frac{c}{\sqrt{c^2 - v^2}}, \text{ and } a\beta = \phi(v) \tag{13}$$

—not explicitly shown in Einstein's derivation— then

$$\tau = \phi(v)\beta\left(t - \frac{vx}{c^2}\right), \tag{14}$$

$$\xi = \phi(v)\beta(x - vt), \tag{15}$$

$$\eta = \phi(v)y, \tag{16}$$

$$\zeta = \phi(v)z. \tag{17}$$

At this stage of the derivation, the compatibility of the relativity principle with that of the constancy of the speed of light is demonstrated by considering the equation of a spherical light wave in the stationary system, and showing that it remains spherical in the moving system after applying the obtained transformation equations.

The remaining derivation is focused on determining the function $\phi(v)$, which is done by using: 1- symmetry, in applying the transformation equations to a third coordinate system K' in parallel translational motion with respect to k, such that its origin moves with the uniform velocity $-v$ along the X' axis coinciding with the X axes of the systems K and k, resulting in $\phi(v)\phi(-v) = 1$, and 2- the invariance of the length of a rod extended along the Y axis of the moving system, when viewed from the stationary system, whether its velocity be v or $-v$, leading to $\phi(v) = \phi(-v)$. Hence, $\phi(v) = 1$, reducing the obtained transformation equations to

$$\tau = \beta\left(t - \frac{vx}{c^2}\right), \tag{18}$$

$$\xi = \beta(x - vt), \tag{19}$$

$$\eta = y, \tag{20}$$

$$\zeta = z, \tag{21}$$

$$\text{where } \beta = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{22}$$

2.2. Critical Analysis

2.2.1. Contradictory Findings

First let's specify the value of a first introduced in Eq. (5). From Eq. (13), we get

$$a = \frac{1}{\beta} = \sqrt{1 - \frac{v^2}{c^2}}.$$

Now, going back to the derivation of Eq. (8) for the value of ξ, it is obtained from the replacement of the time t of the stationary system in Eq. (6) with the time of travel of a light ray to go over the length x', when observed from K, given by Eq. (7). This must be the time in the stationary system K corresponding to the time in the moving system k considered in Eq. (6) by the relation $\xi = c\tau$. i.e., this time t must be, according to the light speed principle, given by $x = ct$, which is indeed the case, since Eq. (7) is actually equivalent to $x = ct$, obtained by replacing x' by its value $x - vt$ in Eq. (7). This point should have been emphasized in the derivation. It follows that Eq. (8)—and therefore Eq. (19)— as well as Eq. (18), are only valid for events satisfying $x = ct$. This shall be demonstrated further as follows.

Considering Eq. (6) for $\xi = c\tau$, it can be written as

$$\xi = ac\left(t - \frac{x'}{(c-v)}\frac{v}{(c+v)}\right). \qquad (23)$$

Replacing Eq. (7) $\left(t = x' / (c-v)\right.$, equivalent to $\left.x = ct\right)$ in Eq. (23), we get

$$\xi = \beta x\left(1 - \frac{v}{c}\right), \qquad (24)$$

and

$$\tau = \beta t\left(1 - \frac{v}{c}\right), \qquad (25)$$

Equations (24) and (25) shall yield the transformation Eqs. (19) and (18) if and only if $x = ct$. By symmetry, it is ascertained that the inverse transformation equations

$$t = \beta\left(\tau + \frac{v\xi}{c^2}\right), \qquad (26)$$

$$x = \beta(\xi + v\tau), \qquad (27)$$

shall be valid if and only if $\xi = c\tau$.

It is to be noted that Eqs. (24) and (25), obtained using Einstein's own derivation, are in line with the findings obtained in a critical paper refuting the Special Relativity.[11]

Now, substituting Eq. (18) into Eq.(26), returns

$$t = \beta\left(\beta\left(t - \frac{vx}{c^2}\right) + \frac{v\xi}{c^2}\right),$$

which can be simplified to

$$t\left(\beta^2 - 1\right) = \frac{vx}{c^2}\left(\beta^2 - \frac{\beta\xi}{x}\right). \qquad (28)$$

Since, as shown earlier, Eqs. (18) and (26) are only valid for $x = ct$ and $\xi = c\tau$, respectively, then Eq. (28) can be written as

$$t\left(\beta^2 - 1\right) = \frac{vx}{c^2}\left(\beta^2 - \frac{\beta\tau}{t}\right). \qquad (29)$$

For $\tau = 0$, according to Eq.(18), the transformed t- coordinate with respect to K would be $t = vx\,/\,c^2$. Consequently, for $t \neq 0$, Eq. (29) reduces to

$$t\left(\beta^2 - 1\right) = t\beta^2\ , \qquad (30)$$

yielding the contradiction $\beta^2 - 1 = \beta^2$, or $0 = 1$.

If instead, Eq.(26) was substituted into Eq. (18), a contradiction will follow for the transformation $t = 0,\ \tau = -v\xi\,/\,c^2$.

It can be shown that an analogous contradiction would be obtained from the space coordinate transformation equations by substituting Eq. (19) into Eq. (27) and applying Eq. (19) for the transformation $\xi = 0,\ x = vt$ on the resulting equation, along with $x = ct$ and $\xi = c\tau$, required for Eqs. (19) and (27), respectively, as shown earlier. Reversing the substitution results in a contradiction for $x = 0,\ \xi = -v\tau$.

It follows that using the transformations $[\tau = 0;\ t = vx\,/\,c^2]$, or $[\xi = 0; x = vt]$, in the obtained transformation equations; or $[t = 0;\ \tau = -v\xi\,/\,c^2]$, or $[x = 0;\ \xi = -v\tau]$, in the inverse transformation equations, will lead to contradictions, in agreement with earlier works. [10 11 12]

The obtained contradictions shall be confirmed in the next section via the length contraction and time dilation analysis through the outlined transformation equations derivation.

2.2.2. Inconsistency of Einstein's Derivation

Going back to the derivation section, what if we considered the light ray traveling in the negative ξ direction? In this case, we would have $\xi = -c\tau$, and a simple calculation could show that the corresponding time in the stationary system would be

$$\frac{x'}{c + v} = t,$$

which, when inserted in the foregoing Einstein's Eq. (5) for $\tau(x', t)$, using $\xi = -c\tau$, yields

$$\xi = a\frac{c^2}{c^2 - v^2}x'\left(\frac{2v}{c} - 1\right),$$

or

$$\xi = \beta(x - vt)(2v/c - 1),$$

undermining the whole derivation of the Special Relativity transformation equations!

2.2.3. Unviability of the Length Contraction and Time Dilation Predictions

In §4 of the considered paper, the physical meaning of the obtained transformation equations in terms of moving rigid bodies and moving clocks is addressed. The length contraction and time dilation are the main aspects that appear to be anticipated by these equations.

The length contraction is predicted in the Special Relativity by setting the time t of the stationary system to zero in the space coordinate transformation Eq. (19), leading to

$$x = \xi / \beta. \qquad (31)$$

However, setting the stationary frame time to zero has actually the predetermined result of $x = 0$, in conformity with the transformation derivation assumptions. In fact, the space coordinate transformation Eq. (19) is obtained from Eq. (8) by substituting the time t into Eq. (6). Yet, the only stationary system time t that can be substituted into Eq. (6) must be the time that corresponds to the time τ given by $\xi = c\tau$ (the basis of Eq. (6) where τ is substituted by its value as a function of t and x'), i.e. the time given by $x = ct$ (equivalent to $t = x' / (c - v)$). If the time $t = 0$ was instead substituted for the time in Eq. (6), we get

$$\xi = \frac{1}{\beta}c\left(-\frac{v}{c^2 - v^2}x'\right);$$

$$\xi = -\frac{v}{c}\beta x, \text{ or } x = -\frac{c}{v}\frac{\xi}{\beta},$$

implying $v = -c$, when compared to Eq. (31). Thus, carrying the derivation method for the space transformation equation with $t = 0$, leads to a contradiction. Or alternatively, when $t = 0$, Eq. (7) returns $x' = x = 0$ (in addition Eq. (5) yields $\tau = 0$). In other words, no length contraction would be produced.

On the other hand, the time dilation is envisaged by evaluating the rate of a clock located at the origin of the moving system, as observed in the stationary system. With respect to the stationary system, the position of such a clock is given by $x = vt$. Substituting this value of x in the time transformation Eq. (18), we get

$$\tau = \beta t\left(1 - \frac{v^2}{c^2}\right);$$

$$t = \beta\tau, \qquad (32)$$

which is interpreted such as the stationary system time is dilated by a factor of β with respect to the moving system. i.e., the clock in the moving system runs slower than the stationary frame's by a factor of β.

However, Eqs. (1) and (19) imply that for $x = vt,\ \ x' = 0$ and $\xi = 0$. Hence, under such conditions, Eq. (6) results in $t = 0$. Alternatively, the derivation of Eq. (5) wouldn't be feasible for $x' = 0$, since it is used as the basis for the time calculation. The Einstein's outlined derivation of the transformation equations is based on providing and calculating the time coordinates by means of light rays traveling certain distances $(x', \xi, etc.)$. Therefore, setting one of these distance to zero means the corresponding time would vanish. Hence, following the derivation of Eq. (5), when $x = vt$, $x' = 0$ and $\tau = 0$. Thus, Eq. (5), the basic time transformation equation, leads to $t = 0$.

An equivalent way to arriving at the above result would be by replacing x with $x' + vt$ in the final time transformation Eq. (18), leading to

$$\tau = \beta\left(t - \frac{v(x' + vt)}{c^2}\right).$$

In the context of the latter equation outlined derivation, for $x = vt,\ \ x' = 0,$ which evidently leads to $\tau = 0,$ resulting in $t = 0$. It follows that the obtained time dilation Eq. (32) is deemed invalid.

3. Conclusion

Following Einstein's own 1905 derivation of the Special Relativity transformation equations, it was shown that these equations were limited to events with space and time coordinates satisfying the constancy of the speed of flight principle. They resulted in contradictions when applied to certain events occurring at the coordinate origins. Furthermore, it was revealed that the Special Relativity approach used to obtain the time dilation and length contraction predictions was incompatible with the derivation employed method for generating time intervals and calculating their variance, using traveling light rays.

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