CRITICAL ANALYSIS OF THE FOUNDATIONS OF THE THEORY OF NEGATIVE NUMBERS

Temur Z. Kalanov
Home of Physical Problems, Pisatelskaya 6a, 100200 Tashkent, Uzbekistan.
tzk_uz@yahoo.com, t.z.kalanov@mail.ru, t.z.kalanov@rambler.ru

Abstract. Critical analysis of the foundations of the theory of negative numbers is proposed. The unity of formal logic and of rational dialectics is methodological basis of the analysis. It is shown that the foundations of the theory of negative numbers contradict practice and contain formal-logical errors. The main results are as follows: a) the concept "number sign" is inadmissible one because it represents a formal-logical error; b) all the numbers are neutral ones because the number "zero" is a neutral one; c) signs "plus" and "minus" are only symbols of mathematical operations. The obtained results are the sufficient reason for the following statement. The existence of logical errors in the theory of negative numbers determines the essence of the theory: the theory is a false one.

Keywords: mathematics, number theory, mathematical physics, physics, geometry, engineering, formal logic, philosophy of science

MSC: 00A05, 00A30, 00A30g, 00A35, 00A69, 00A79, 03A05, 03A10, 03B42, 03B44, 03B80, 33B10, 03F50, 97E20, 97E30, 97G60, 97G70, 97F60, 97M50, 51M15, 51N35, 51P05

INTRODUCTION

Recently, the progress of science, engineering, and technology has led to appearance of a new problem – the problem of rationalization of the fundamental sciences. Rationalization of sciences is impossible without rationalization of thinking and critical analysis of the foundations of sciences within the framework of the correct methodological basis: the unity of formal logic and of rational dialectics. Critical analysis of the sciences within the framework of the correct methodological basis shows [1-21] that the foundations of theoretical physics and of mathematics (for example, classical geometry, the Pythagorean theorem, differential and integral calculus, vector calculus, trigonometry) contain logical errors.

As is well known, the theory of negative numbers is an important part of mathematics [22-26] and of mathematical formalism of physics [27]. This theory is widely and successfully used in the natural sciences. The main result of this theory is the following statement: negative numbers and the concept "negative sign of number" have scientific and practical meaning (for example, $1 - 2 = 0 - 1 = -1$). However, it does not mean that the problem of validity of the theory is now completely solved, or that the foundations of the theory are not in need of formal-logical and dialectical analysis. In my view, the theory of negative numbers cannot be considered as scientific truth if there is no formal-logical and dialectical substantiation of it in science.

Understanding of the essence of the theory of negative numbers is impossible without critical analysis of the foundations of this theory. And a complete understanding of the foundations of this theory is possible only within the framework of the correct methodological basis: the unity of formal logic and of rational dialectics. However, the formal-logical analysis of this theory is absent in science. The purpose of the present work is to propose critical analysis of the foundations of the theory of negative numbers within the framework of the correct methodological basis.

1. GEOMETRICAL ANALYSIS OF THE CONCEPT "NEGATIVE NUMBER"

1. As is known, if the Cartesian coordinate system XOY on a plane is given, then the coordinate lines (scales) X and Y divide the plane into four quarters (I, II, III, IV), and the point of intersection of coordinate lines – point O – determines the origin of coordinates (i.e., the number "zero"). The origin of coordinates – the number "zero" – is on the coordinate scales and divides each scale into two parts: the scale of positive numbers and the scale of negative numbers. In this case, the number "zero" belongs to both the scale of positive numbers and the scale of negative numbers. The following formal-logical contradiction arises: the number "zero" is both the positive number and the negative number.

Standard mathematics asserts that: (a) zero belongs to the positive and negative scales; (b) zero is neither a positive number nor a negative number; (c) zero has no sign; zero is not characterized by a sign: zero is a "neutral number". In this case, the formal-logical contradiction is conserved.

The contradiction between the qualitative determinacy of the positive number, the qualitative determinacy of the negative number, and the qualitative determinacy of the neutral number has the form of the law of identity:

(positive number) = (negative number);
(positive number) = (neutral number);
(negative number) = (neutral number).

Then the following questions arise: How does one can eliminate this contradiction? Are negative numbers admissible ones in science and practice? Are there negative and positive numbers in science and practice? The answer to these questions is as follows. The contradiction is eliminated if and only if the law of absence of contradictions,

(positive number) $\neq$ (negative number),
(positive number) $\neq$ (neutral number),
(negative number) $\neq$ (neutral number),

is not violated. The only correct assertion follows from the law of absence of contradictions: if there exists a neutral number (i.e. the number "zero") on the numerical scale, then all the numbers on the numeric scale are neutral ones. Thus, neither positive numbers nor negative numbers do not exist on the numerical scale (i.e., they are not admissible numbers).

2. Let the material geometrical figure "square with identical sides $a\,meter$ " be in the quarter I of the coordinate system XOY (in which all the numbers on scales have the dimension "$meter$") (Figure 1).

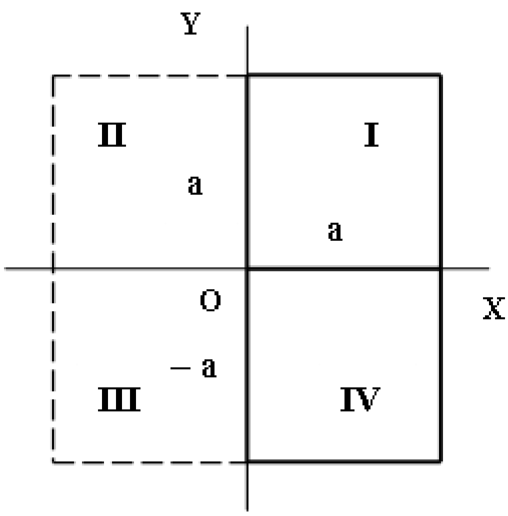

Figure 1. The position of the material geometrical figure "square with identical sides $a\ meter$ " in the quarters I and IV of the coordinate system XOY .

If the figure "square with identical sides $a\ meter$ " is situated in the quarters II and IV, then the sides "$a\ meter$ " and "$-a\ meter$ " are not identical ones: $a \neq -a$. In other words, the geometrical figure "square with identical sides $a\ meter$ " turns into the geometrical figure "square with non-identical sides $a\ meter$ and $-a\ meter$ " in the quarters II and IV of the coordinate system XOY . In this case, the correct mathematical relationship $a \neq -a$ is expressed by the formal-logical law of absence of contradictions:

(square with identical sides $a\ meter$) $\neq$
(square with non-identical sides $a\ meter$ and $-a\ meter$).

And the incorrect mathematical relationship $a = -a$ is expressed by the formal-logical law of identity:

(square with identical sides $a\ meter$) =
(square with non-identical sides $a\ meter$ and $-a\ meter$).

3. The material geometrical figure "square with identical sides $a\ meter$ " has the area $S = a \times a = a^2$. The calculation of the area of this geometrical figure in the quarters I and IV of the coordinate system XOY leads to appearance of the concept "imaginary unit". Really, if $S_I \neq S_{IV}$, then

$$S_I = a \times a = a^2; \ \sqrt{S_I} = a;$$
$$S_{IV} = -a \times a = -a^2;$$
$$\sqrt{S_{IV}} = a\sqrt{-1} = ai; \quad i \equiv \sqrt{-1},$$

where S_I, S_{IV} and $\sqrt{S_I}$, $\sqrt{S_{IV}}$ are areas and sides of the figure in the quarters I and IV, respectively; i is imaginary unit. In this case, the following logical error appears:

$$\sqrt{S_{IV}} = a\sqrt{-1} = ai,$$

because

$$\sqrt{S_{IV}} \neq a, \ \sqrt{S_{IV}} \neq -a.$$

In other words, these relationships signify that S_{IV} represents the area of the square whose sides are equal to $\sqrt{S_{IV}} = a\sqrt{-1} = ai$. But $\sqrt{S_{IV}} = ai$ contradicts the condition that the sides of this square are equal to a and $-a$ in the expression $S_{IV} = -a \times a = -a^2$. Consequently, the concepts “negative number” and “imaginary unit” represent a formal-logical error in the case of $S_I \neq S_{IV}$.

Also, a logical error appears if $S_I \equiv S_{IV}$. Really, if $S_I \equiv S_{IV}$, then $1 \equiv -1$, $1 \equiv \sqrt{-1}$. In order to eliminate the logical error $1 \equiv -1$, one should introduce the concept of modulus of number: $|1| = |-1| \equiv 1$. (According to the standard mathematics, modulus of number is a unsigned number. The algebraic quantity of the number always has a sign: plus or minus). The use of the modulus sign signifies the movement of the geometric figure from the quarters II, III, and IV into the quarter I of the coordinate system XOY. In this case, the geometrical figure represents the “square with identical sides *a meter* ” in the quarter I.

Thus, the geometrical analysis leads to the conclusion that the concepts “negative number” and “imaginary unit” represent a formal-logical error in all cases.

2. LOGICAL ANALYSIS OF THE CONCEPT “NUMBER SIGN”

1. As is well known, practice is a criterion of truth. From practical point of view, operations such as

$$\$1 - \$2 = \$0 - \$1 = -\$1,$$
$$1\,\text{kilogram} - 2\,\text{kilograms} =$$
$$0\,\text{kilogram} - 1\,\text{kilogram} = -1\,\text{kilogram},$$
$$1\,\text{meter} - 2\,\text{meters} =$$
$$0\,\text{meter} - 1\,\text{meter} = -1\,\text{meter},$$
$$1\,\text{second} - 2\,\text{seconds} =$$
$$0\,\text{second} - 1\,\text{second} = -1\,\text{second},$$

and the results of these operations are meaningless ones, wrong in essence. Interpretation of these operations does not represent a mathematical explanation, has no mathematical meaning. Really, the standard interpretation of negative numbers is that the quantity $-a$ is interpreted as modulus $|-a|$, and then one add an explanation which is not related to mathematics. In other words, the interpretation of negative numbers signifies a change of qualitative determinacy (meaning) of these numbers.

Since $a \neq -a$, $|a| = |-a|$ (where a is some number), positive and negative numbers have identical quantitative determinacy (i.e., $|a| = |-a|$) but non-identical qualitative determinacy (i.e., $a \neq -a$). Non-identity of qualitative determinacy is expressed by the formal-logical law of absence of contradiction:

(positive number) $\neq$ *(negative number);*
(positive number) $\neq$ *(unsigned number);*
(negative number) $\neq$ *(unsigned number).*

The following logical statements are true:

(a) positive numbers have identical quality (quantitative determinacy), and therefore they satisfy the formal-logical law of identity:

(positive number) = (positive numbers).

(If the number "zero" was a positive number, then the number "zero" would have to obey this law);

(b) negative numbers have identical quality (quantitative determinacy), and therefore they satisfy the formal-logical law of identity:

(negative numbers) = (negative numbers).

(If the number "zero" was a negative number, then the number "zero" would have to obey this law);

(c) the number "zero" is the unique (special, particular) number, and it satisfies the formal-logical law of identity:

(number "zero" not having a sign) =
(number "zero not having a sign).

(d) the number "zero" satisfies the formal-logical law of absence of contradiction:

(unsigned number) $\neq$ *(signed number).*

But the equations of a type such as

$$1\,\text{kilogram} - 2\,\text{kilograms} =$$
$$0\,\text{kilogram} - 1\,\text{kilogram} = -1\,\text{kilogram}$$

represent violation of the formal-logic law of absence of contradiction. Really, violation of the formal-logic law of absence contradiction is that the left-hand side and the right-hand side of such mathematical equations belong to different qualitative determinacy. In other words, the left-hand side contains positive numbers and neutral number "zero", and the right-hand side contains negative numbers:

(positive numbers and unsigned number "zero") =
(negative numbers).

This signifies that the mathematical equations containing positive and negative numbers and zero are inadmissible ones in science and practice. It follows that all the numbers are neutral numbers: the numbers have no signs because the number "zero" have no sign. If the number "zero" had a sign, then there would be both the positive and negative numbers.

2. From practical point of view, the number (figure) is a symbol designating some amount or absence of amount. Numbers can have dimensions (i.e., qualitative determinacy), but they can have no dimensions. The number "zero" is a symbol designating absence of amount. Mathematically, the essence of number "zero" is manifested in the following statements.

(a) The definition of zero is as follows:

$$a \equiv a\,,\; a - a \equiv 0\,,\; a = a + 0\,,$$

where a is a dimensional or dimensionless number. The definition of zero satisfies the formal-logical law of identity:

(number not having a sign) = (number not having a sign).

(b) The admissible operations on zero are as follows:

$$\frac{a-a}{a} = \frac{0}{a}, \quad \frac{0}{a} = 0; \qquad \frac{a\left(a-a\right)}{a} = a\,0, \quad a\,0 = 0.$$

(c) The inadmissible operation on zero is as follows:

$$\frac{a-a}{0} = \frac{a}{0} - \frac{a}{0} = \frac{0}{0},$$

because the number "zero" does not designate some amount, i.e. the number "zero" designates absence of amount.

(d) Zero is a special (particular) number. Zero is not a part of any number a, zero is not divided into parts, zero is not composed of parts: $a = a + 0$, $0/a = 0$; $a/0$ is not a part of a; $0/0$ is not a part of 0. Zero is neither integer number nor fractional number; zero has no sign.

Therefore, firstly, the addition operation on zero and the subtraction operation on zero do not change amount: $0 \pm 0 = 0$; secondly, the multiplication operation on zero (i.e., $a \times 0 = 0$) and the division operation on zero by some number (i.e., $0/a = 0$) do not lead to change of zero; thirdly, the operations $0 - a$ and $a/0$ are inadmissible ones.

This signifies that zero is the beginning of amount counting out (i.e., the beginning of amount measuring). By definition, the concept "beginning of amount counting out" has the single sense: it is the designation of absence of amount. Therefore, the subtraction of numbers from zero (i.e., $0 - a$) and division of numbers by zero (i.e., $a/0$) are inadmissible operations. The appearance of negative numbers in standard mathematics is stipulated by the following logical error: the assumption that the number "zero" is composed of two parts (i.e., zero is divided into two parts): a and $-a$, i.e.

$$0 - a = -a, \qquad a + \left(-a\right) = 0.$$

This assumption contradicts the definition of zero and the formal-logical law of absence of contradiction.

Thus, the formal-logical analysis of the concept "number sign" leads to the following conclusion: all the numbers are neutral ones; numbers have no signs; the concepts "positive number" and "negative number" represent a formal logical error.

3. DIALECTICAL ANALYSIS OF THE CONCEPT "SYMBOLS OF MATHEMATICAL OPERATIONS"

1. Movement is change in general. Movement is a change of the qualitative and quantitative determinacy of the object. If the qualitative determinacy of the object is not changed, then the movement of the object represents the process of transition of some states of the object into the other states of the object. The process of change is characterized by a direction. If one of the directions can be called a positive direction, then the opposite direction can be called a negative direction.

2. From practical point of view, mathematics is a science of calculations. In mathematics, the quantitative determinacy of the object (i.e., the state of the object) is characterized by a number, and a change in the quantitative determinacy of the object (i.e., the process of transition of some states into other states on condition that the qualitative determinacy of the object is not changed) is described by means of symbols of operations on quantities (numbers). The concepts "quantitative determinacy of the object (i.e., the state)" and "change of the quantitative determinacy of the object (i.e., process)" are not identical ones. Therefore, the identification of

the concepts "state; number" and "change of state; mathematical operation" represents a formal-logical error (i.e., violation of the law of absence of contradiction). Mathematical operations are carried out by people. Therefore, mathematical formalism contains only quantities (numbers) and symbols of operations on quantities (numbers), but mathematical formalism do not contain movement (action).

3. The basic mathematical (quantitative) operations on quantities and numbers are as follows: addition operation (designated by the symbol " + "), subtraction operation (designated by the symbol " – "), multiplication operation (designated by the symbol " × "), division operation (designated by the symbol " : " or " / "). The quantitative relationship between quantities, symbols of operations on quantities, and result of operations is called mathematical equation. It is designated by the symbol " = ".

4. The addition operations and multiplication operations are actions which lead to an increase in the numerical value of the result of operations; subtraction operations and division operations are actions which lead to a decrease in the numerical value of the result of operations. Operations of increase of the numerical value (i.e., increase of amount) and operations of decrease of the numerical value (i.e., decrease of amount) are mutually opposite operations. If the direction of the operation of increase of amount may be called positive direction, then the direction of the operation of decrease of amount should be called negative direction. If some operation is called direct one, then the operation of inversion of direct operation is called inverse operation. For example, if the operations $a \times b$, $b \times a$, $a + b$, $b + a$ are called direct ones, then the operations $a : b$ (or a/b), $b : a$ (or b/a), $a - b$, $b - a$ are called inverse ones. Direct and inverse operations are called mutually opposite operations. In this connection, the following problem arises: How does one can express symbolically the inversion of the direction of operation?

5. The solution to this problem is as follows.

a) The symbols of mathematical operations have practical meaning and can be practically used only in combination with numbers and the designations of the numbers in letters: for example,

$$a + b = c,$$
$$a - b = d, \quad a > b,$$
$$b - a = h, \quad b > a,$$
$$a \times b = b \times a = k,$$
$$a/b = l, \; b/a = 1/l,$$

where the letters designate numbers. In other words, the symbol of the operation relates two quantities (numbers). Therefore, the symbol of the operation of inversion of direction should contain a letter (number) and the symbol of the mathematical operation.

b) The definition of operational form of operations and correspondence between the standard form of operations (left-hand side of relationships) and the operational form of operations (right-hand side of relationships) are as follows:

$$a + b \equiv \langle a+\rangle b, \quad b + a \equiv \langle b+\rangle a;$$
$$a - b \equiv \langle a-\rangle b, \quad b - a \equiv \langle b-\rangle a;$$
$$a \times b \equiv \langle a\times\rangle b, \quad b \times a \equiv \langle b\times\rangle a;$$
$$a/b \equiv \langle \;/b\rangle a, \quad b/a \equiv \langle \;/a\rangle b;$$
$$(a/b) \times (b/a) \equiv \langle \;/b\rangle a \times \langle \;/a\rangle b \equiv 1;$$
$$-1 \equiv \langle -1\times\rangle, \quad -a \equiv \langle -1\times\rangle a, \quad -b \equiv \langle -1\times\rangle b;$$

$$(-a)\times(-a)\equiv\langle -1\times\rangle\langle -1\times\rangle a^2 \equiv a^2;$$
$$(-b)\times(-b)\equiv\langle -1\times\rangle\langle -1\times\rangle b^2 \equiv b^2;$$
$$\langle -1\times\rangle\langle -1\times\rangle \equiv \langle 1\times\rangle, \quad \langle -1\times\rangle\langle 1\times\rangle \equiv \langle -1\times\rangle;$$
$$b-a\equiv\langle -1\times\rangle(a-b), \quad a-b\equiv\langle -1\times\rangle(b-a),$$

where expression in angle brackets $\langle\ \rangle$ designates an operator, $\langle -1\times\rangle$ is the operator of the inversion of direction of operation. Multiplication of operators represents successive fulfilment of operations: for example, the expression $\langle -1\times\rangle\langle -1\times\rangle \equiv \langle 1\times\rangle$ represents the inversion of the operation of inversion.

c) The establishing of correspondence between the standard form of the operations and the operational form of the operations is a necessary condition for understanding of the qualitative distinction between a number sign and a symbol of operation. If the understanding is achieved, it is possible to use standard mathematical designation. However, it is not allowed to ascribe sign "plus" or "minus" to quantities (numbers).

Thus, the dialectical analysis of the concepts "mathematical operation" and "symbol of mathematical operations" leads to the conclusion that the symbol of the mathematical operation can not be ascribed to a number. Number is not characterized by a symbol of mathematical operation, and, therefore, it has no sign. The concept "number sign" or the identification of the concepts "number sign" and "symbol of mathematical operation" represents a formal logical error.

4. DISCUSSION

1. As is well known, the concept of negative numbers appeared in ancient mathematics in the 7th century, and finally formed in the 19th century. The great mathematicians of antiquity were wise men because they understood that practice is criterion of truth. Therefore, they called negative numbers by "false", "dummy", "absurd", and "imaginary" numbers. "By the beginning of the 19th century Caspar Wessel (1745-1818) and Jean Argand (1768-1822) had produced different mathematical representations of 'imaginary' numbers, and around the same time Augustus De Morgan (1806-1871), George Peacock (1791-1858), William Hamilton (1805-1865), and others began to work on the 'logic' of arithmetic and algebra and a clearer definition of negative numbers, imaginary quantities, and the nature of the operations on them began to emerge . Negative numbers and imaginaries are now built into the mathematical models of the physical world of science, engineering and the commercial world. There are many applications of negative numbers today in banking, commodity markets, electrical engineering, and anywhere we use a frame of reference as in coordinate geometry, or relativity theory" (Encyclopedia). However, the concept "methodological basis of science" is not contained in mathematics until now.

2. The standard theory of negative numbers, first worded in the article *"Theory of Conjugate Functions, or Algebraic Couples; with a Preliminary and Elementary Essay on Algebra as the Science of Pure Time"* by William Rowan Hamilton, legalized the existence of negative numbers in mathematics and put an end to criticism of negative numbers. The scientists called positive and negative numbers, and number 0 by rational numbers and were satisfied that need for serious thinking about the true sense of negative numbers and of zero fell off. The stage of interpretation of negative numbers in science was begun. For example, the interpretation of some well-known negative numbers is as follows:

a) Number $-273{,}15^{\circ}\,C$ is the absolute zero of temperature, i.e. zero degrees Kelvin. Interpretation of this negative number is as follows: number $-273{,}15$ represents the modulus

$\left|-273{,}15\right|$; sign “minus” signifies that number $\left|-273{,}15\right|$ is below zero; concepts “negative” and “below” are identical ones; the term “below” has no mathematical meaning.

b) Number $-1{,}602176565 \times 10^{-19}\, Cl$ is the electron charge. Interpretation of this negative number is as follows: number $-1{,}602176565 \times 10^{-19}$ represents the modulus $\left|-602\,176\,565 \times 10^{-19}\right|$; number $\left|-602\,176\,565 \times 10^{-19}\right|$ is quantity of charge (i.e., quantitative determinacy); sign “minus” signifies qualitative determinacy of the electron; concepts “minus sign” and “electron” are identical ones; the term “electron” has no mathematical meaning. Also, the term “proton” has no mathematical meaning if one identifies the concepts “plus sign” and “proton charge”.

c) Number $-13{,}7\ milliard\ years$ is the beginning of formation of the Universe. Interpretation of this negative number is as follows: number $-13{,}7$ represents the modulus $\left|-13{,}7\right|$; number $\left|-13{,}7\right|$ is quantitative determinacy; “minus sign” signifies qualitative determinacy of number $-13{,}7$; concepts “minus sign” and “beginning” are identical ones; the term “beginning” has no mathematical meaning.

Thus, qualitative determinacy of negative numbers is expressed by concepts which have no mathematical meaning:

(mathematical concept) = (non-mathematical concept).

Therefore, the interpretation of negative numbers represents a formal-logical error.

3. There is no logical definition of the concept “negative number” in science and practice. And definition such as “negative number represents the number which is not a positive number” is inadmissible one in formal logic because such definition represents “contradictious (negative) definition”. The correct definition should be “confirmatory (positive) definition”.

Positive and negative numbers and the number “zero” have different qualitative determinacy (even if these numbers have the same dimension). This signifies that the scale of positive numbers and the scale of negative numbers cannot have common point O (i.e., the number 0) in the Cartesian coordinate system XOY. Therefore, the existence of the coordinate system XOY represents a formal-logical error.

From a practical point of view, all the numbers (having dimension or not) are always a result of measurement (or comparison). Negative numbers do not represent a measuring result or a consequence of the existence of positive numbers. This signifies that the set of negative numbers is not a supplement (expansion, extension) of the set of positive integers because positive and negative numbers have different qualitative determinacy. (In other words, if the existence of negative numbers would be cause of the existence of positive numbers, then one could be built negative numbers on the basis of positive numbers). Consequently, the existence of negative numbers is not consistent with practice and is not confirmed by practice.

Negative numbers are inadmissible ones: they should exist neither in science nor in practice. All the numbers obtained in measurements and having the same dimension are characterized by identical qualitative determinacy. Number “zero” is a neutral number. Consequently, all the numbers represent neutral numbers (i.e., the numbers which have no sign “plus’ or “minus”), and the concept “number sign” is inadmissible one. Sign “plus” and “minus” are only symbols of mathematical operations.

4. The theory of negative numbers is not unique erroneous theory in mathematics. As shown in the works [6-28], differential and integral calculus, the Pythagorean theorem, vector calculus, and trigonometry are erroneous theories too. Therefore, today mathematics stands in front of the dilemma: either to recognize the existence of formal-logical errors or to continue movement on the wrong track.

CONCLUSION

Thus, the results of the critical analysis of the theory of negative numbers within the framework of correct methodological basis – the unity of formal logic and of rational dialectics – are as follows:

1) negative numbers are inadmissible ones in science because they represent a formal-logical error;
2) the concept "number sign" is inadmissible one because it represents a formal-logical error;
3) all the numbers are neutral ones because the number "zero" is a neutral one;
4) signs "plus" and "minus" are only symbols of mathematical operations;
5) the operational form of mathematical operations furnishes the clue to understanding of the operation of inversion of operation.

The obtained results are the sufficient reason for the following statement: the essence of the theory of negative numbers is that the theory is a false one.

REFERENCES

[1] T.Z. Kalanov. "The critical analysis of the foundations of theoretical physics. Crisis in theoretical physics: The problem of scientific truth". Lambert Academic Publishing. ISBN 978-3-8433-6367-9. (2010).
[2] T.Z. Kalanov. "Analysis of the problem of relation between geometry and natural sciences". Prespacetime Journal. Vol. 1, No 5, (2010), pp. 75-87.
[3] T.Z. Kalanov. "Logical analysis of the foundations of differential and integral calculus". Bulletin of Pure and Applied Sciences. Vol. 30 E (Math.& Stat.), No. 2, (2011), pp. 327-334.
[4] T.Z. Kalanov. "Logical analysis of the foundations of differential and integral calculus". Bulletin of Pure and Applied Sciences. Vol. 30 E (Math.& Stat.), No. 2, (2011), pp. 327-334.
[5] T.Z. Kalanov. "Critical analysis of the foundations of differential and integral calculus". MCMS (Ada Lovelace Publications). (2011), pp. 34-40.
[6] T.Z. Kalanov. "Critical analysis of the foundations of differential and integral calculus". International Journal of Science and Technology, Vol. 1, No. 2, (2012), pp. 80-84.
[7] T.Z. Kalanov. "On rationalization of the foundations of differential calculus". Bulletin of Pure and Applied Sciences. Vol. 31E (Math.& Stat.), No. 1, (2012), pp. 1-7.
[8] T.Z. Kalanov. "The logical analysis of the Pythagorean theorem and of the problem of irrational numbers". Asian Journal of Mathematics and Physics. Vol. 2013, (2013), pp. 1-12.
[9] T.Z. Kalanov. "The critical analysis of the Pythagorean theorem and of the problem of irrational numbers". Global Journal of Advanced Research on Classical and Modern Geometries. Vol. 2, No 2, (2013), pp. 59-68.
[10] T.Z. Kalanov. "The critical analysis of the Pythagorean theorem and of the problem of irrational numbers". Basic Research Journal of Education Research and Review. Vol. 2, No. 4, (2013), pp. 59-65.
[11] T.Z. Kalanov. "The critical analysis of the Pythagorean theorem and of the problem of irrational numbers". Bulletin of Pure and Applied Sciences. Vol. 32 (Math & Stat), No. 1, (2013), p. 1-12.
[12] T.Z. Kalanov. "On the logical analysis of the foundations of vector calculus". Research Desk. Vol. 2, No. 3, (2013), pp. 249-259.
[13] T.Z. Kalanov. "On the logical analysis of the foundations of vector calculus". International Journal of Scientific Knowledge. Computing and Information Technology. Vol. 3, No. 2, (2013), pp. 25-30.
[14] T.Z. Kalanov. "On the logical analysis of the foundations of vector calculus". Journal of Computer and Mathematical Sciences Vol. 4, No. 4, (2013), pp. 202-321.
[15] T.Z. Kalanov. "On the logical analysis of the foundations of vector calculus". Journal of Research in Electrical and Electronics Engineering. Vol. 2, No. 5, (2013), pp. 1-5.
[16] T.Z. Kalanov. "The foundations of vector calculus: The logical error in

mathematics and theoretical physics". Unique Journal of Educational Research. Vol. 1, No. 4, (2013), pp. 54-59.
[17] T.Z. Kalanov. "On the logical analysis of the foundations of vector calculus".
Aryabhatta Journal of Mathematics & Informatics. Vol. 5, No. 2, (2013), pp. 227-234.
[18] T.Z. Kalanov. "Critical analysis of the mathematical formalism of theoretical physics. II. Foundations of vector calculus". Bulletin of Pure and Applied Sciences. Vol. 32 E (Math & Stat), No. 2, (2013), p.121-130.
[19] T.Z. Kalanov. "On the system analysis of the foundations of trigonometry". International Journal of Science Inventions Today. Vol. 3, No. 2, (2014), pp. 119-147.
[20] T.Z. Kalanov. "On the system analysis of the foundations of trigonometry". Pure and Applied Mathematics Journal. Vol. 3, No. 2, (2014), pp. 26-39.
[21] T.Z. Kalanov. "On the system analysis of the foundations of trigonometry". Bulletin of Pure and Applied Sciences. Vol. 33E (Math & Stat), No. 1, (2014), pp. 1-27.
[22] N. Bourbaki. Elements of the History of Mathematics. Berlin, Heidelberg, and New York: Springer-Verlag. ISBN 3-540-64767-8. (1998).
[23] D.J. Struik. A Concise History of Mathematics. New York: Dover Publications. (1987).
[24] G. Ifrah. The Universal History of Numbers. Harvill Press, London. (1998).
[25] C.B. Boyer. "A history of mathematics (Second ed.). John Wiley & Sons, Inc. ISBN 0-471-54397-7. (1991).
[26] W.R. Hamilton. "Theory of conjugate functions, or algebraic couples; with a preliminary and elementary essay on algebra as the science of pure time". (Transactions of the Royal Irish Academy, vol. 17, part 1 (1837), pp. 293-422).
Edited by David R. Wilkins, 2000.
[27] E. Madelung. Die mathematischen hilfsmittel des physikers. Berlin, Gottingen, Heidelberg, Springer-Verlag. (1957).