The Relativistic Mechanic Theory of the String

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Abstract A new relativistic mechanic that governs the motion of the vibrating string within the compactified-dimensions space-time is speculated. This mechanic suggests that the relativistic mechanic of the special relativity is only valid for the motion within the familiar four-dimensional space-time. However, the new mechanic is valid for the motion within the compactified-dimensions space-time suggested by the string theory. The equations of this new mechanic show that the vibrating string can move in a speed that is faster than light. It is also shown that this new relativistic mechanic goes to the classical Newtonian mechanic whenever the speed of the vibrating string is much less than the speed of light. Since this mechanic does not prohibit the existence of faster than light particles, then it might uncover some of the mysteries regarding to the string theory, such as, the existence of tachyons, time travel, ...etc.

Keywords String Theory; Tachyon; Superluminal Motion; Special Relativity; Elementary Particles

1. Introduction

Many attempts have been made to prove whether matter can travel faster than light or not. In 1905, Albert Einstein proposed the Special Relativity which showed that a particle needs infinite energy to accelerate to the speed of light. In spite of that, the Special and General Relativity did not forbid totally the existence of superluminal particles. In 1962, Bilaniuk et al speculated on the possible existence of faster-than-light particles which were later named 'tachyons' by Feinberg (1967). Experimentally, there is no strong evidence of the existence of superluminal motion, which makes this puzzle unsolved yet. In my point of view, the relativistic mechanic of the special relativity prohibits the superluminal motion with the four-dimensional space-time. However, the motion within the compactified-dimensions space-time predicted by the string theory requires another relativistic mechanic that needs to be discovered. Therefore, a new speculative method is used to predict this relativistic mechanic.

2. The Relativistic Mechanic of the Special Relativity

The two famous postulates of the special theory of relativity lead to the following relativistic relationships,

\[ m = m_o \sqrt{1 - \left(\frac{u}{c}\right)^2} \]  
\[ E = mc^2 = m_e c^2 \sqrt{1 - \left(\frac{u}{c}\right)^2} \]  
\[ K = mc^2 - m_o c^2 = m_e c^2 \left(\sqrt{1 - \left(\frac{u}{c}\right)^2} - 1\right) \]  
\[ p = mu = m_o u \sqrt{1 - \left(\frac{u}{c}\right)^2} \]

where \( m \) is the relativistic mass, \( m_o \) is the rest mass, \( u \) is the velocity, \( c \) is the speed of light, \( E \) is the relativistic energy, \( K \) is the relativistic kinetic energy, and \( p \) is the relativistic momentum. As it can be easily noticed, these equations give imaginary values whenever \( u \) is greater than \( c \).

The special relativity equations can be used to any object moving in space-time with four dimensions without considering the motion of its inner structure (i.e. the motion of its string(s)) that is happening in space-time with compactified dimensions. For this inner motion of the object, we will, speculatively, derive different relativistic mechanic that give no imaginary mass for the superluminal motion which might give a new view for the concept of the tachyon.
3. The Relativistic Mechanic of the String

3.1. The Relativistic Mass

Experiments have proven the correctness of the special theory of relativity. The equations of this theory can be applied to any accelerated particle that is composed of one vibrating string (e.g. the electron) or more than one vibrating string (e.g. the proton). **This means that the motion within the four-dimensional space-time has an individual effect on every single vibrating string of the particle.**

Now, to know the relativistic mass represented by the motion of the vibrating string, we will consider both the electron and its anti-particle, the positron. According to the American physicist Richard Feynman, the positron is an electron that is traveling backward in time. Since they travel in different directions of time, then one of the vibrating strings should have a positive relativistic mass and the other should have a negative relativistic mass. Having relativistic masses with different signs for both strings of the electron and positron means having different velocities within the compactified-dimensions space-time. Since they are the same particle, then, **for the vibrating string moving within the compactified-dimensions space-time we need a mass-velocity equation that shows a similar dependency in different ranges of speed for both the positive and negative relativistic mass (i.e. symmetric function).** The only logic function that satisfies this condition is the following cosine function

\[ m = \frac{m'_o}{\cos(v/c)} \]  

where \( m \) is the relativistic mass, \( m'_o \) is the rest mass, \( v \) is the velocity of the vibrating string, and \( c \) is the speed of light. From this equation, which is plotted in Fig. 1, we can notice that \( m=m'_o \) when \( v << c \). We can also notice that,

\[
\begin{align*}
  m &= m'_o & \text{at} & \quad v = 0, 2\pi c, 4\pi c, \ldots \\
  m &= 1.82m'_o & \text{at} & \quad v = c \\
  m &= \pm \infty & \text{at} & \quad v = (\pi/2)c, (3\pi/2)c, (5\pi/2)c, \ldots \\
  m &= -m'_o & \text{at} & \quad v = \pi c, 3\pi c, 5\pi c, \ldots
\end{align*}
\]

![Fig 1. The change of the relativistic mass with the velocity of vibrating string](image-url)
Equation (5) gives the relativistic mass in terms of the rest mass and velocity within the compactified-dimensions space-time. However, equation (1) gives the relativistic mass in terms of the rest mass and velocity within the four-dimensional space-time. Equation (5) also suggests that the speed of the vibrating string can travel faster than light within the compactified-dimensions space-time. However, this superluminal motion cannot be noticed in the four dimensional space-time (according to the special relativity). Fig. 1 shows the similarity between the curve that is above (which represents the positive relativistic mass) and the curve that is below (which represents the negative relativistic mass) the abscissa. It also shows an infinite number of speed ranges for both the positive and negative relativistic mass.

If we assume that in the rest case of the electron (i.e. \( u = 0 \) and \( m = m_o \)), its vibrating string has a velocity of \( v_j \), then from equation (5) we can write,

\[
m_o' = \frac{m_o}{\cos(v_j / c)}
\]  

(6)

by substituting from equations (5) and (6) in (1) we will find the relationship between \( u \) and \( v \) as following,

\[
\frac{m_o'}{\cos(v / c)} = \frac{m_o' / \cos(v_j / c)}{\sqrt{1-(u/c)^2}}
\]

\[
u = c \sqrt{1- \left[ \frac{\cos(v / c)}{\cos(v_j / c)} \right]^2}
\]

(7)

Interestingly, if we rewrite equation (5) as following,

\[
m = \frac{m_o'}{\sqrt{\cos^2(v / c)}}
\]

then we expand the term under the square root into a Taylor’s series, it becomes

\[
m = \frac{m_o'}{\sqrt{1-(v/c)^2 + (2/3)(v/c)^4 + \ldots}}
\]

which is very similar to equation (1).

3.2. The Relativistic Energy

From equation (2) and (5), we can find that the relativistic energy represented by a vibrating string is,

\[
E = mc^2 = \frac{m_o'c^2}{\cos(v / c)}
\]

(8)

We still can see that the energy equals mass times the speed of light squared. This relationship gives the total relativistic energy in terms of the rest mass and velocity within the compactified-dimensions space-time. However, equation (2) gives the total relativistic energy in terms of the rest mass and velocity within the four-dimensional space-time.

3.3. The Relativistic Kinetic Energy

Consequently, the relativistic kinetic energy will be,

\[
K = mc^2 - m_o'c^2
\]
or

$$K = \frac{m^r v^2}{\cos(v/c)} - m^r c^2$$  \hspace{1cm} (9)$$

If we expand equation (9) into a Taylor's series, we will obtain,

$$K = \frac{1}{2} m^r v^2 + \frac{5}{24} m^r v^4 + \ldots$$

which goes to the classical Newtonian expression for \( v << c \).

3.4. The Relativistic Momentum

If we substitute from equations (5) and (7) into equation (4), it becomes

$$p(u) = mu = \frac{m^r}{\cos(v/c)} \sqrt{1 - \left(\frac{\cos(v/c)}{\cos(v_1/c)}\right)^2}$$

This equation represents the relativistic momentum in \( u \)-direction (i.e. the four-dimensional space-time momentum). However, the relativistic momentum for the vibrating string in \( v \)-direction (i.e. the compactified-dimensions space-time momentum) can be obtained by letting \( v_1 = 0 \). Therefore, by substituting in the last equation we obtain

$$p = m^r c \frac{\sin(v/c)}{\cos(v/c)}$$

which gives,

$$p = m^r c \frac{\sin(v/c)}{\cos(v_1/c)}$$

or

$$p = m^r c \tan(v/c)$$  \hspace{1cm} (10)$$

Similarly, if we expand equation (10) into Taylor’s series, we will obtain,

$$p = m^r v + \frac{1}{3} m^r \frac{v^3}{c^2} + \ldots$$

which also goes to the classical Newtonian expression for \( v << c \).

Equation (10) can also be obtained by the classical Newtonian mechanic with one modification. From the classical mechanic we have

$$dK = F \cdot dx = \frac{dp}{dt} \cdot dx$$

thus

$$dK = dp \cdot u$$  \hspace{1cm} (11)$$

Since we are dealing with different type of motion, we should replace the velocity \( u \) here with the velocity \( v \). How can we do this? In fact, we can just easily use equation (7) after letting \( v_1 = 0 \). This will make equation (7) looks like

$$u = c \sin(v/c)$$  \hspace{1cm} (12)$$

This equation relates the velocity of the vibrating string (\( v \)) within the compactified-dimensions space-time to the velocity of the vibrating string within the four-dimensional space-time. Now, by substituting from Equations (9) and (12) into ((11), we obtain

\[1\] In fact, if we expand equation (12) into a Taylor’s series, then we will see that \( u = v \) at \( v << c \).
\[
d \left( \frac{m'_c c^2}{\cos(v/c)} - m'_c c^2 \right) = dp \cdot c \sin(v/c)
\]

this gives
\[
\frac{m'_c c \cdot \sin(v/c)}{\cos^2(v/c)} \cdot dv = dp \cdot c \sin(v/c)
\]
or
\[
dp = \frac{m'_c}{\cos^2(v/c)} \cdot dv
\]

by performing the integration we obtain
\[
p = m'_c c \tan(\psi/c)
\]

which is equation (10).

Also, from equations (8) and (10) we can obtain the following relationship,
\[
E^2 - p^2 c^2 = m'_c c^4
\]

This relationship represents Lorentz invariant.

### 3.5. The Relativistic Force

By differentiating the momentum equation (10) with respect to time, we will obtain the force acting on the moving vibrating string
\[
F = \frac{m'_c}{\cos^2(v/c)} \frac{dv}{dt}
\]  
(13)

### 6. Discussions

The reason we have never observed a particle that is traveling faster than light is because this superluminal motion is happening inside the inner structure of the elementary particles and within the compactified-dimensions space-time. For example, it is shown experimentally that the electron (that is composed of one string) cannot move faster than light. However, the vibrating string of the electron can be traveling faster than light within the compactified-dimensions space-time.

If we look into Fig. 1 (or equation (5)) we can notice that \( m = m'_c \) at different velocities (in different ranges) such as \( 0, 2\pi c, 4\pi c, 6\pi c, \ldots \) etc. This indicates that the term "rest mass" in this relativistic mechanic is inaccurate. This figure also shows that we can have an infinite number of the same curve at different velocity ranges for both the positive and negative relativistic mass which might predict the existence of an infinite number of the same elementary particle with different vibrating string velocity.

### 7. Conclusion

This brief paper suggests that the motion within the compactified-dimensions space-time described by the string theory requires a different relativistic mechanic that is different than that presented by the special theory of relativity. A speculative method is used to predict this relativistic mechanic. It is, accordingly, shown that the vibrating strings of a particle (or any matter) can move faster than light within the compactified-dimensions space-time. In my point of view, this new mechanic could be the road into more understanding to the laws of nature.

### References
