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A note on the harmonic series and the logarithm

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Abstract

A relationship between the harmonic series and the logarithm is presented. The formula H(n)-log(n) for the Euler-Mascheroni constant is adopted accordingly.

$$\gamma \quad = \quad (\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\ldots\frac{1}{n}) \quad - \quad (\frac{1}{n+1}+\frac{1}{n+2}+\ldots\frac{1}{n+n^2})$$

Figure 1: Illustration of H(n) and log(n) as part of H(n+n^2)

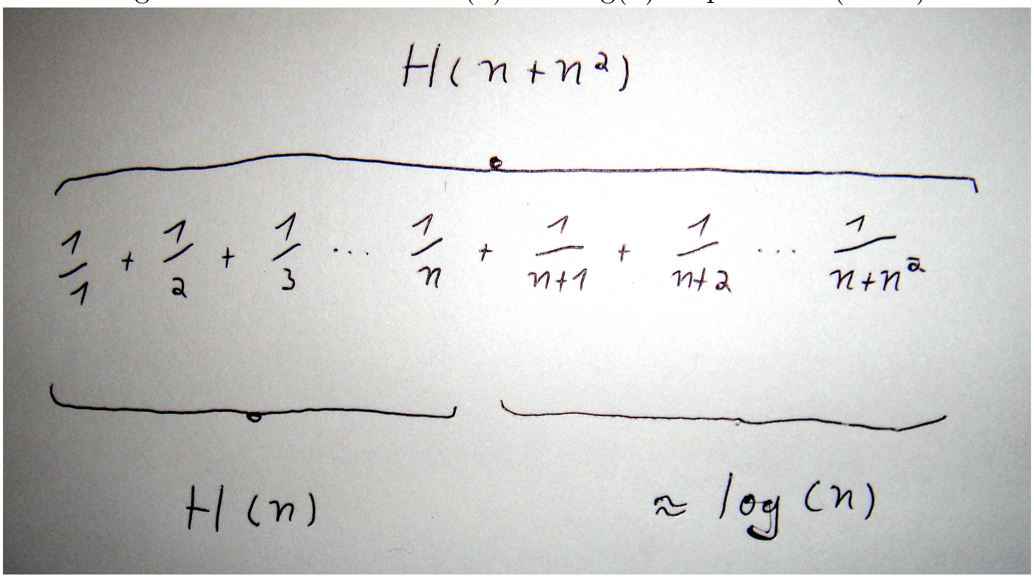
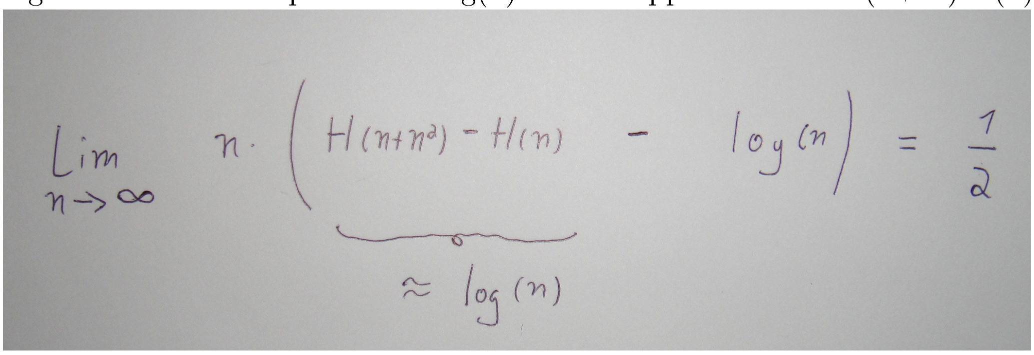


Figure 2: Relationship between log(n) and its approximation H(n+n^2)-H(n)



Mathematica Codes:

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Limit[2*HarmonicNumber[n] - HarmonicNumber[n + n*n], n -> Infinity]
Limit[n*(HarmonicNumber[n + n*n] - HarmonicNumber[n] - Log[n]), n -> Infinity]
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Generalization (with substitution of $n + n^x$ by n^x for simplification)

Figure 3: Illustration of H(n) and log(n) as part(s) of the Harmonic series