

# Association Performance Enhancement Through Classification

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Association of spatial information about targets is conventionally based on measures such as the Euclidean or the Mahalanobis distance. These approaches produce satisfactory results when targets are more distant than the resolution of the employed sensing principle, but is limited if they lie closer. This paper describes an association method combined with classification enhancing performance. The method not only considers spatial distance, but also information about class membership during a post-processing step. Association of measurements that cannot be uniquely associated to only one estimate, but to multiple estimates, is achieved under the constraint of conflict minimization of the combination of mutual class memberships.

With Monte Carlo simulations the performance of this new method is compared with a Kalman filter. This evaluation is performed in a multi-target environment with unknown correspondence between measurements and targets. The evaluation can not be only based on the root mean square error (RMSE) of the position estimate, but requires a performance assessment of the underlying target number estimation and the association. Therefore, two new measures are introduced.

The new method outperforms the Kalman filter approach with respect to association performance and RMSE.

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## 1. INTRODUCTION

This paper describes the improvement of localization of targets through information fusion. In particular, information about static targets is fused, that are closely lying below surface, for so named *Ground Penetrating Localization* (GPL).

GPL is used in various fields such as in geology, mine sweeping, and *Urban Search and Rescue* (USAR). This paper focuses on USAR. The employed methods for GPL can be classified into three categories illustrated in Fig. 1: detection, localization, and verification methods.

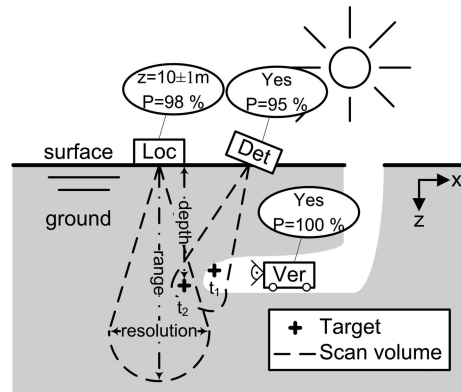


Fig. 1. Ground-penetrating localization (GPL) of targets  $t_i$  by different types of methods in  $xz$ -plane: **D**etection, **L**ocalisation, and **V**erification. In the scan volume of the detection method, two targets can not be discerned because they lie within the resolution of the sensing principle.

Detection methods produce a binary result with a given detection probability of whether there is a target within their scan volume. The detection probability density is uniform within the scan volume as detection methods are unable to localize targets. A detection is determined by the signal to noise ratio and the detection threshold. An example detection method is a search dog sniffing out a victim.

In contrast, a localization method not only detects (with a given detection probability) a target within its scan volume, but can also localize its position. Localization can consist of a one-dimensional range measurement, a direction, or three-dimensional coordinates. Examples for localization methods are: *Ground Penetrating Radar* (GPR) systems [29], cellular phone localization [13].

Whereas both of the previous methods can only produce uncertain results with respect to the existence of a target, verification methods can provide evidencethrough visual or physical inspection. The problem with verification methods consists of knowing where evidence about a target has been collected. This is often not trivial during ground-penetrating exploration that is often operated in an unknown environment without a map. Endoscopes and rescue robots can for instance be classified under verification methods since they can penetrate into a rubble pile and provide evidence about the whereabouts of victims [15].

GPL with detection and localization methods corresponds to remote sensing because the perceptive organs or devices remain at the surface while the targets are in the ground (see Fig. 1). In such remote sensing situation, a single sensor often only provides an incomplete, imprecise, and uncertain (for definitions refer to [5]) picture about the target location. Therefore, it is common practice to employ multiple sensors of different types—i.e., heterogeneous sensors—and to fuse the readings. However, if multiple targets lie close to each other, the resolution capabilities are limited to that of each single sensor.

The idea behind the proposed performance enhancement solution is that the sensing by heterogeneous sensors is based on target attributes that are not necessarily the same among all targets and can be sensed by search methods. These attributes are for instance the health condition, respiration frequency, and size of a buried victim. For instance, the GPR is capable of measuring the respiration frequency which might be different for every individual. Association is compromised if a measurement can be associated to multiple closely lying position estimates of a target. In this case, if the association method not only considers spatial aspects but previously detected unique attributes of the targets as well, measurements can be unambiguously attributed to the appropriate targets. The hypothesis is that implementing this idea enhances localization performance.

Unknown material between the surface and the target impedes not only the target position estimation, but also the fundamental detection accuracy. The scan volume is commonly unknown as well. The sensing range corresponds to the maximal distance between target and surface for which a true-positive detection occurs with sufficient probability. Resolution expresses the minimal mutual distance at which two targets can be detected individually. For instance, the distribution of steel reinforced walls may be unknown, but will influence the range and resolution of GPR. If the localization method, despite some expectable position measurement error, is capable of identifying the targets within its scan volume, the distinction of targets is easy, but this is often not possible. In order to circumvent these limitations, sensing can be carried out multiple times at different locations and orientations, as presented in Fig. 1.

When errors are random, redundant fusion may be used to increase the reliability, accuracy of and confidence in the information [21]. Systematic errors can be minimized using heterogeneous GPL methods. For instance, a systematic error may only affect one sensor type, but not others. Applying multiple sensing principles may allow decreasing, or even detecting and eliminating systematic errors.

In terms of the *Joint Directors of Laboratories* (JDL) Data Fusion Working Group, this paper is limited to Level 1 Object Refinement [12]. In multi-target tracking applications using multiple sensors, several fusion architectures exist [11, 21]. The particularity of the

*Association with Classification* (AC) method presented in this paper, is that it combines centralized fusion of locational and feature information. However, we restrict the focus to features that do not allow for the identification of a target. AC only demonstrates its full potential, if the sensing by heterogeneous sensors is based on multiple attributes. It is applicable for GPL of multiple, unique targets, such as USAR of trapped victims or for geologic exploration.

In Section 2, the state of the art is presented. In Section 3, the problem of uncertain and imprecise information association is formulated. The AC method based on an initial probabilistic association and post-processing using a possibilistic approach, is presented in Section 4. The simulation environment and the performance measures used for benchmarking AC with a Kalman Filter with an association gate are described in Section 5. The results of a Monte Carlo simulation are presented in Section 6 and discussed in Section 7.

## 2. STATE OF THE ART

Combining data originating from different sources with the goal to improve the quality of information, is called data fusion [28]. In pattern recognition, the data originates mostly from a single source and is “complete” when processed [16]. This paper focuses on scenarios where information can be “incomplete” because it is collected on demand.

Data fusion techniques have been developed for tracking mobile targets. Hence, they are particularly suited for dynamic worlds. In early work, the filters were restricted to situations where a single target is being monitored. At every time step, only one measurement was selected to update the state. Filters such as *Nearest Neighbor* or *Strongest Neighbor* satisfied these requirements. However, multiple observations may be available to improve the accuracy of the estimate for the single target. The *Probabilistic Data Association* (PDA) filter is an “all-neighbor” approach that uses all neighboring observations within some gated region and improves the state estimation ([2], p. 299). However, the tracking performance of multiple targets using the PDA filter is poor, since “...the computation of the association probabilities separately for each target is not effective in the presence of a neighboring target.” ([3], p. 325) The stability of target tracks while crossing is compromised by the persistent interference of the measurements of neighboring targets. This applies also to static worlds.

The *Joint PDA* (JPDA) filter suggested by Bar-Shalom jointly calculates the probability of measurements belonging to targets, to overcome the mentioned limitations of PDA filters. However, the underlying assumption of a JPDA filter is that a measurement only originates from one target at every time step of a scan.

In static worlds, multiple measurements may support a single target. The bijective constraint between measurement and target has to be withdrawn to be able

to associate multiple measurements with a single target. Probabilistic association techniques for multi-target environments in data fusion systems rely on validation gates.<sup>1</sup> The gates around position estimates allow the association of measurements based on a probabilistic approach. An alternative is *Multiple Hypothesis Tracker* (MHT) that represents a measurement centric approach. Because it is difficult to consider all association vectors, this approach requires pruning [27]. Furthermore, it is designed to make a single inference on one target object [17]. Hence, it would need to be extended to handle multiple targets.

To overcome the resolution limitation in dense multi-target environments, the **JPDA** with *Merged measurements filter* (JPDAM) has been suggested. It accounts for situations where a measurement may have originated from the detection of multiple targets that are indistinguishable. The membership of a merged measurement to a track is calculated depending on its respective signal strength ([3], p. 366), i.e., an *unresolved or merged measurement* carries less information (relative to its signal strength) than a *resolved measurement*. In GPL this approach is often not possible since a measurement's relative signal strength can not be expressed. Hence, hard association is favored.

The possibilistic association of Ayoun, et al. [1] based on the *Transferable Belief Model* (TBM) of Smets [24] is limited on the one hand by a discretized resolution grid and on the other by the computational effort [14], but has the interesting distinction of allowing to search for the target location based on conflict minimization within a finite set of measurements. Possibilistic approaches of association based on class membership for target type estimation are presented in Chapter 13 of [23], but are limited to a single target and are designed for dynamic worlds.

Clustering methods such as *k-means* are inappropriate because they require the number of expected targets which is unknown. There are methods to iteratively evaluate the measurements with varying estimated target number and choose the one which minimizes or maximizes a given measure. However, this represents a bigger computational burden which decreases efficiency. Furthermore, the results of *k-means* clustering are not reproducible since they are dependent upon the initial conditions. Methods of statistical pattern recognition such as *Expectation Maximization* or *density estimation* have the advantage of considering class membership, but need a considerable amount of information

<sup>1</sup>The validation gate is depicted by an ellipse centered about the nominal position estimate that represents the contour of constant probability (in two dimensions) for a multivariate Gaussian distribution [26]. To filter measurements that are not associated with an estimate (i.e., the information source) is the purpose of a validation gate that corresponds to the volume around the position estimate ([20], p. 157). If the Mahalanobis distance between an estimated target and a measurement is smaller than a predefined threshold, the latter is associated with the former.

to work properly, their computational effort is greedy [16]. Furthermore, their design is particularly adapted for static fusion and not for dynamic fusion.

### 3. FORMULATION OF THE ASSOCIATION PROBLEM

The association aims to link *measurements to a position estimate* (M2E). A measurement  $\vec{r}$  and a position estimate  $\hat{r}$  are both described by a position  $(x, y, z)$  and a class membership mass function  $m$ . In the following, the vector sign is omitted.

The challenge in M2E association lies in initiating and revising a position estimate.<sup>2</sup> If the sparsity of a scenario (i.e., the minimal distance between targets) is smaller than the resolution, accurate position estimates are challenging<sup>3</sup> [6].

The description of the association problem will be twofold. First, we explain the processing of measurements without considering their class membership, represented by the orientation of the shapes of the measurements in Fig. 2. Second, the same case is revisited considering the class membership.

*Association without classification:* The following example is based on Bar-Shalom's terminology [3] and illustrates the complexity of the association problem without classification consideration. The binary validation matrix  $\Omega_{ij}$  (see Eq. 2) expresses all feasible association events between measurements  $r$  and estimates  $\hat{r}$  and is illustrated in Fig. 2. The rows represent the measurements  $(r_1, \dots, r_i, \dots, r_8)$  from top to bottom), the middle column the estimate  $\hat{r}_1$ , and the right column  $\hat{r}_2$ . The left column of the matrix indicates that every measurement may originate from a spurious source. A conjunctive area  $(A \cap B)$  is determined by intersecting validation gates. Measurements that are within this area, such as  $r_5, r_6, r_7$  in Fig. 2, require particular attention because there is more than one possible way of associating them.

A measurement that could be simultaneously associated to multiple estimates is called *unresolved measurement* and is member of set  $C^{ur}$ . A measurement which can be associated to only one estimate  $\hat{r}_j$  is called *resolved measurement* and is member of set  $C_j^{re}$ . Consequently, if there is no associated estimate the measurement is called *unassociated measurement* and is member of set  $C^{ua}$  (see  $r_4$  in Fig. 2). Whether  $r_4$  will initiate a new estimate is determined by its detection probability.

The cases in following expression reflect these three conditions quantitatively. The distinction is based on a sum for a row of the validation matrix over the columns from the first estimated target to the total estimated number of targets  $\hat{N}_t$ .

<sup>2</sup>Tracking of mobile targets i.e., a dynamic state consists of revising the state up to date, since it changes. However, the belief about the static state is **revised** (not updated), because the state does not change.

<sup>3</sup>See measurements  $r_5, r_6, r_7$  in Fig. 2 which could be associated either to estimate  $\hat{r}_1$ , to  $\hat{r}_2$ , to both, or to none.

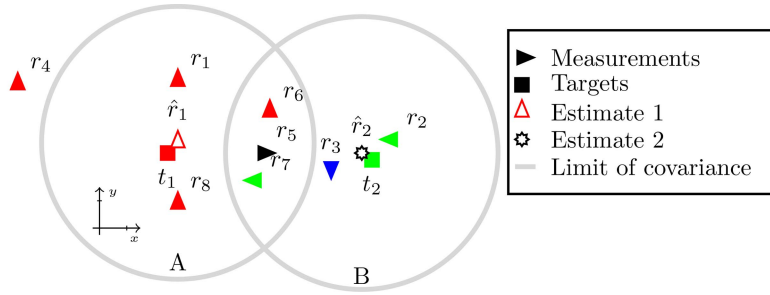


Fig. 2. Measurements (triangles  $r_i$ ) originating from two unknown targets (squares:  $t_1$  from **top** class and  $t_2$  from **left** class). Intersecting validation regions (solid circles) around position estimates  $\hat{r}_1$  (not filled triangle) categorized as **top** class member and uncategorized  $\hat{r}_2$  (not filled star) based on resolved measurements ( $xy$ -plane). The validation regions A, B have the same size because the distribution of measurements is unknown and presumed to be equal for any estimate.

$$r_i \in \begin{cases} C_j^{\text{re}} & \text{if } \sum_{j=2}^{\hat{M}} \Omega_{ij} = 1; \\ C^{\text{ur}} & \text{if } \sum_{j=2}^{\hat{M}} \Omega_{ij} > 1; \\ C^{\text{ua}} & \text{if } \sum_{j=2}^{\hat{M}} \Omega_{ij} = 0. \end{cases} \quad (1)$$

*Association with classification:* This processing reduces the feasible association events as presented in Eq. 2:

$$\Omega_{ij} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \xrightarrow{\text{processing}} \Omega_{ij}^* = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & \mathbf{0} \\ 1 & \mathbf{0} & 1 \\ 1 & 1 & 0 \end{bmatrix}. \quad (2)$$

In the following, three mutually exclusive classes will be examined corresponding to the orientation of the triangles in Fig. 2: **top**, **bottom**, **left**.

The relative complement areas of  $A \setminus B$  and of  $B \setminus A$  are predominant for classification of estimates  $\hat{r}_1$  and  $\hat{r}_2$ , respectively. The measurements  $r_1$  and  $r_8$  are located in the predominant area of estimate  $\hat{r}_1$  ( $A \setminus B$ ). Based on its membership to the **top** class, it can be inferred that the estimate  $\hat{r}_1$  most likely belongs to class **top** as well. In contrast, no stringent inference on the class membership of estimate  $\hat{r}_2$  can be achieved, because in its predominant region ( $B \setminus A$ ), conflicting classes are present: **left** and **bottom** of  $r_2$  and  $r_3$ , respectively. However, it can be assumed that with an increasing number of measurements present in the predominance area, the classification errors become negligible.

*Unresolved measurements* for which association is not unique can actually be assessed with respect to the estimates' class that was determined by the *resolved measurements*. **Top**, *unresolved measurement*  $r_6$  can be

associated uniquely to  $\hat{r}_1$  and becomes *resolved*. For measurement  $r_7$  the situation is more complex since  $\hat{r}_2$  has conflicting class membership.  $\hat{r}_2$  could be **left** or **bottom**. However, the **left** class corresponds more to  $\hat{r}_2$  than to  $\hat{r}_1$ . Hence, *unresolved measurement*  $r_7$  can be associated uniquely to  $\hat{r}_2$ , and thus is *resolved*.

The class membership of measurement  $r_5$  is not known. Hence, it can not be associated uniquely to any of the two estimates and remains *unresolved*.

$\Omega$  becomes  $\Omega^*$  (see Eq. 2) when considering class membership. It contains less feasible association events than  $\Omega$ .

These examples convey the complexity of association considering class membership. *Resolved measurements* must allow the determination of the estimate's class. The estimates must be of different classes in order to be able to associate *unresolved measurements* uniquely. If these constraints are fulfilled, *unresolved measurements* in the conjunctive area can be *resolved*.

#### 4. DESCRIPTION OF AC METHOD

Before presenting the two main steps of the AC method, which consist of an initial probabilistic association followed by a post-processing of *unresolved measurements*, the underlying assumptions and simplifications will be explained.

##### 4.1. Assumptions and Simplifications

The presented method can only be applied under the following assumptions:

The localization performed by the sensors may be based on different attributes of the targets. Since these attributes may not be common to all targets, classification becomes feasible depending on the detected attributes of the target. A sensor may be able to detect multiple attributes at once, or heterogeneous sensor technologies can be employed, which offer the opportunity for complementary fusion,<sup>4</sup> and can thus recognize different classes. In this preliminary paper, the classification capability of sensors is assumed to be perfect even if this can not be expected in reality.

<sup>4</sup>Definition provided in [8].

Furthermore, not only do the sensors need to recognize different classes, but targets must be of different classes. It is also presumed that a measurement only originates from one information source, unlike with a JPDAM filter where (to some extent) the model tries to associate a measurement to two targets.

The distribution of measurements around a target is unknown, but is constant in static worlds, and its estimation depends upon the number of measurements available. The measurement noise is usually modeled by a Gaussian distribution ([27], p. 154). An erroneous estimation of the number and position of targets can have two sources. Either there are not enough measurements concerning all targets, or the processing is incorrect. In order to focus on latter, we consider only situations where sufficient measurements for each target are available to determine their number and position.

In situations where not many measurements are to be expected such as during USAR, the initiation can not be based on multiple measurements. Hence, a position estimate is initiated as soon as a new measurement is generated that can not be associated.

#### 4.2. Initial Step of Association Method

Association of measurements is based on the Mahalanobis distance, and consists of finding the correlation between measurements and an information source. This statistical squared distance indicates the probability that measurement  $r_i$  belongs to estimate  $\hat{r}_j$ . In the following, matrices are represented by uppercase letters while vectors are lowercase.

Assuming that a positional error of a measurement  $r$  in two dimensions  $(x, y)$  can be described with a Gaussian distribution, the following covariance matrix can be used, where  $\rho$  is the correlation coefficient for  $x$  and  $y$  [26]:

$$C = \begin{pmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{pmatrix}. \quad (3)$$

Since the distribution of measurements around the target is unknown but constant, it is assumed to be Gaussian. The validation gate is based on an estimate of this distribution. The distance between measurement and estimate is based on the sum of the covariance matrices of the gate  $G$  and of the measurement  $R_i$ . Unlike Kalman filters,  $G$  remains constant. Otherwise it would converge, because fusion leads to a reduction of uncertainty ([3], p. 444), i.e., the fused uncertainty ellipse is strictly contained in the intersection of the two ellipses prior to fusion (see Eq. 6). Hence, for an increasing number of measurements, the probability of association would decrease considerably and hinder the association.

The Mahalanobis distance  $d_{ij}^2$  is defined in Eq. 4. The gate threshold  $\gamma$  gives the maximal distance for the association of measurements to an estimate.

$$d_{ij}^2 = (r_i - \hat{r}_j)^T (R_i + G)^{-1} \underbrace{(r_i - \hat{r}_j)}_{\text{innovation}} \leq \gamma. \quad (4)$$

Following Smith and Cheeseman [26] the revision of an estimate's position  $\hat{r}'_j$  is calculated by:

$$\hat{r}'_j = \hat{r}_j + R_i(R_i + \Sigma_j)^{-1}(r_i - \hat{r}_j) \quad (5)$$

and its corresponding covariance matrix expressing imprecision is:

$$\Sigma'_j = R_i - R_i(R_i + \Sigma_j)^{-1}R_i. \quad (6)$$

The standard application of Kalman filters is to process random signals that are represented "...as the output of a linear dynamic system excited by independent or uncorrelated random signals ("white noise")" [18]. This application differs from the standard application, because it is not a dynamic system.

The processing order of measurements fusion, particularly when targets are spaced close together, influences the association and leads to erroneous associations. This is especially true with *unresolved measurements*. In order to correct these erroneous associations, a post-processing step is performed.

#### 4.3. Post-Processing of Initial Association

The post-processing is based on fusion of class membership information, where the chosen framework is TBM. The advantage of this approach is that inference from contradictory class information is possible. Therefore, the finite set of propositions  $\Theta$  is extended to the set of all subsets  $2^\Theta$  by conjunction and to the empty set. The set of all subsets is also called frame of discernment. With this extension, the exhaustive and the exclusive assumptions on the proposition framework can be disregarded. This disregard is equivalent to the open world assumption. This assumption accepts that none of the propositions may be true [24]. No matter how large the frame of discernment is, it contains three different proposition classes: the empty set  $\emptyset$ , the atomic propositions corresponding to  $\Theta$ , and conjunctive propositions (number:  $2^\Theta - \Theta - 1$ ). Belief in a proposition is quantized by a function called basic belief assignment (bba)  $m(A)$ ,  $A \in \Theta$ . Let  $m : 2^\Theta \rightarrow [0, 1]$ , where following constraints hold:

$$\sum_{A \subseteq \Theta} m(A) = 1, \quad m(\emptyset) = 0. \quad (7)$$

$$(8)$$

It is worth noting that the conjunction of all atomic propositions corresponds to the vacuous function, also referred to as total ignorance. The main difference of the seminal proposition of Dempster and Shafer [22] is that the conjunctive combination of beliefs

$$m_{12}(A) = \sum_{B \cap C = A} m_1(B)m_2(C) \quad (9)$$

in the TBM is not normalized, since:

$$\sum_{B \cap C = \emptyset} m_1(B)m_2(C) > 0. \quad (10)$$

For this reason, TBM is able to express conflict among beliefs. The conflict is given by the mass allocated to the empty set. It is advantageous that the combination is commutative and associative. Conflict, reliability and uncertainty can be expressed quantitatively.

The method is also attractive because of the ease of implementation by matrices, as presented in [25].

#### 4.3.1. Class Membership of Predominant Areas

Class membership is computed by combining all class membership mass vectors  $m_i$  of the *resolved measurements* ( $r_i \in C_j^{\text{re}}$ ) (i.e., measurements in the predominant area of an estimate). The result is again a class membership mass vector for  $\hat{r}_j$ :  $m_j^{\text{p}}$ . If estimates are very close, there are most likely no *resolved measurements* available. This will prove to be a limitation, and will be evaluated in Monte Carlo simulations.

#### 4.3.2. Resolving *Unresolved Measurement*

To determine the most likely association, the mass vector  $m$  of an *unresolved measurement* ( $r \in C^{\text{ur}}$ ) is combined with the mass vector  $m_j^{\text{p}}$  of each  $\hat{r}_j$ , and the minimal conflict is determined. The bijective association is given by the combination with the minimum conflict.

$$m_{ij}^{\text{c}} = m_i \cdot m_j^{\text{p}} \quad \forall r_i \in C^{\text{ur}}. \quad (11)$$

$$\Omega_{ij} = \begin{cases} 1 & \text{if } j = \min m_{ij}^{\text{c}}(\emptyset); \\ 0 & \text{if } j \neq \min m_{ij}^{\text{c}}(\emptyset). \end{cases} \quad (12)$$

ALGORITHM 1 Association with classification (AC)

**Input:** association threshold  $\gamma$ ,  $r_i$

**Output:** position estimates  $\hat{r}_j$

```

1: Evaluate number  $\hat{N}_t$  and  $\hat{r}_j \quad \forall \{r_i \in C_j^{\text{re}}\}$ 
2:  $\Omega_{ij} \leftarrow 1$  if  $d_{ij}^2 \leq \gamma$ 
3: if  $\exists r_i \in C^{\text{ur}}$  then
4:    $m_j^{\text{p}} \leftarrow m_i \cdot m_j^{\text{p}} \quad \forall \{r_i \in C_j^{\text{re}}\}$ 
5:   for  $\forall r_i \in C^{\text{ur}}$  do
6:     for  $j = 1$  to  $\hat{N}_t$  do
7:        $m_{ij}^{\text{c}} \leftarrow m_i \cdot m_j^{\text{p}}$ 
       {Save the association of minimal conflict}
8:       if  $\min m_{ij}^{\text{c}}(\emptyset)$  then
9:          $k \leftarrow j$ 
10:      end if
11:    end for
12:    {Resolve measurement}
13:     $\Omega_{ij}^* \leftarrow 0 \quad \forall j \neq k \wedge j > 1$ 
14:  end for
15: end if
16: return  $\hat{r}_j \leftarrow \bar{r}_i$ 
 $\forall \left\{ r_i \mid \Omega_{ij}^* = 1 \wedge \sum_{j=2}^{\hat{N}_t} \Omega_{ij}^* = 1 \right\}$ 

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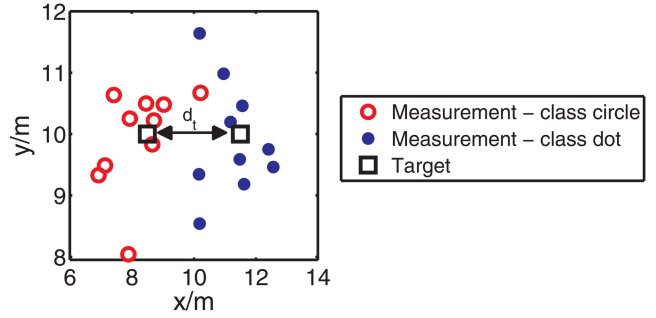


Fig. 3. Measurements ( $N_m = 20$  with normal spatial error  $\sigma_e = 1$  m) around two targets (squares, distance  $d_t = 4$  m). The difference in the class of the measurements (circle or dot) indicates the original target (xy-plane).

The estimates' positions of the post-processed measurements are calculated by taking the average instead of using Eq. 5, as for Kalman filters.

## 5. SIMULATION

The evaluation of the performance of the proposed fusion method AC will be carried out in a Monte Carlo simulation. It is compared with a Kalman filter approach which is called Kalman Gating (KG). It corresponds to the AC method, with the difference that the post-processing step is omitted. The position of the estimates is calculated using the sequential Kalman update Eq. 5.

Two targets will be considered in this simulation, but the methods can cope with multiple targets. The distance  $d_t$  between the two targets is the main evaluation vector, because it simulates various densities which may occur in real scenarios. The scale of the scenario is assumed to be in the order of meters.

Parameters determining AC and KG are the standard deviation  $\sigma = 1$  m of the constant gate covariance matrix  $G$  and the gate threshold  $\gamma = 4$ .

### 5.1. Generation of Simulated Measurements about the World State

Measurements around targets are presumed to be subject to a normal measurement error with standard deviation  $\sigma_e$ . Each target is supported by the same number of measurements. Each measurement's spatial uncertainty is assumed radially symmetric, and is expressed by a covariance matrix  $R_i$ .

There are no outliers and no classification errors. Only two classes are considered. Figure 3 represents such a scenario.

The scenario and the simulation parameters are given in Tables I and II, respectively.

### 5.2. Performance Measures

The quantitative evaluation of fusion methods requires performance measures which are general enough to be applied to all types of scenarios, including those with more than two targets. The determination of the estimates' positions relies on a consecutive order of pro-

TABLE I  
Parameters of the GPL Scenario

$N_t$	$N_m$	$\sigma$ of $R_i/m$	$\sigma_e/m$
2	20	0.4	0.5

TABLE II  
Parameters of the Monte Carlo Simulation

$d_t/m$	Step Size/m	Repetitions
0–5.5	0.1	1500

cessing steps. First, the number of targets has to be estimated. Then the measurements have to be associated to these estimates and finally, the positions of the estimates are calculated based on the associated measurements. The accuracy of the position estimate is expressed with the RMSE, which depends on the correct estimation of the number of targets and the correct association of measurements to the estimates.

It is worth noting that the performance can only be evaluated if all knowledge about the world is available. The so called *ground truth* gives information about the target position and the origin of a measurement.

Even if the RMSE is conditioned by the underlying association performance, the performance of estimating the number of targets and of associating measurements to these position estimates will be evaluated individually.

### 5.2.1. Target Estimation

The fusion methods may generate a number of position estimate  $\hat{N}_t$  that do not correspond to the real number of targets  $N_t$ .

A method's performance in estimating the correct number of targets is given by  $T$ : the quotient of the number of estimated  $\hat{N}_t$  over the number of real targets  $N_t$  as given in Eq. 13. For values  $T < 1$  the method underestimates and for  $T > 1$  it overestimates the number of targets.

$$T = \frac{\hat{N}_t}{N_t}. \quad (13)$$

### 5.2.2. Association

The association performance  $A$  is the quotient of the correctly associated measurements ( $N_m - N_{\text{err}}$ ) over the total measurements  $N_m$  originating from a target (Eq. 14).  $A$  is strictly bound in the interval  $(1/N_t, 1]$ .

$$A = \frac{N_m - N_{\text{err}}}{N_m}. \quad (14)$$

The evaluation of  $N_{\text{err}}$  requires particular attention. When the measurements are created, the index of their original target is saved in an array. The aim of any association method is to reconstruct this array (or "code word"), but there are two complicating factors. First,

there might be  $N_{\text{err}}$  errors that must be detected in the reconstructed code and second, the sequence could possibly be generated with a different character concordance.

For instance, following two code words<sup>5</sup>  $c_1 = 123123$  and  $c_2 = 321321$  contain the same information. However, characters "1" and "3" in  $c_1$  correspond to character "3" and "1" in  $c_2$ , respectively. The code words use different permutations of the character set  $\Psi$ :

$$\Phi_1 = [1, 2, 3] \neq \Phi_2 = [3, 2, 1].$$

Since the concordance is unknown, the generated code word is transcribed with any possible concordances that are all possible permutations of  $\Psi$ . The number of all possible permutation is:  $q!$  (in the example:  $3! = 6$ ).

For each of these possible permutations, the Damerau-Levenshtein distance  $d_{\text{DL}}$  is computed [4, 9]. The Damerau-Levenshtein distance is particularly suited for the evaluation of q-ary codes. The number of association errors corresponds to the smallest  $d_{\text{DL}}$  distance between these transcribed code words  $c_{\text{trans}}^{\Phi_i}$  and the original code word  $c_{\text{orig}}$  (see Eq. 15).

$$N_{\text{err}} = \min_i d_{\text{DL}}(c_{\text{orig}}, c_{\text{trans}}^{\Phi_i}) \quad \forall \Phi_i \in \Psi. \quad (15)$$

### 5.2.3. Root Mean Square Position Error

The position estimate error is given by the mean of the root mean square distance between estimated positions  $\hat{r}_j$  and target positions  $t_k$  given in Eq. 16. Since in GPL practice, the correspondence between estimate and target remains unknown, only the most likely association can be considered. At most, the presumption can be made that the correct correspondence is the one with the minimal Euclidean distance among their positions. The significance of the RMSE is limited, even if the number of estimated targets corresponds to the number of targets ( $\hat{N}_t = N_t$ ) because two estimates can be associated with the same target. It is even more detrimental if the estimation of the number is erroneous ( $\hat{N}_t \neq N_t$ ), which is why previously presented performance indicators about target estimation and association are crucial.

$$\text{RMSE} = \sqrt{\frac{1}{\hat{N}_t - 1} \sum_{j=1}^{\hat{N}_t} \min_k (\|\hat{r}_j - t_k\|)^2}. \quad (16)$$

## 6. RESULTS

The results of the Monte Carlo simulation are presented in Figs. 4, 5, and 6. The analysis of the results represented in Figs. 4 and 5 is performed by comparison with the cumulative intersection distribution function  $P_{\text{int}}$ , i.e., the separation of two Gaussian distributions. Eq. 17 expresses the cumulative intersection probability of two uni-variate normal distributions with equal

<sup>5</sup>Length  $n = 6$ , character set  $\Psi = \{1, 2, 3\}$ , character set cardinality  $q = |\Psi| = 3$ .



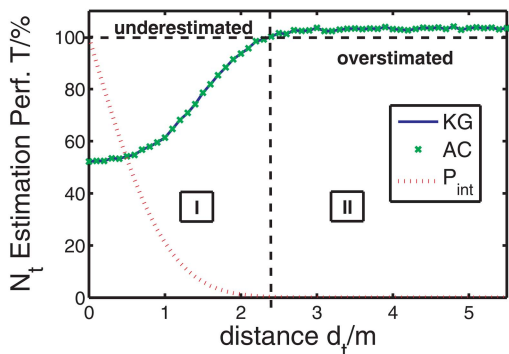


Fig. 4. The target estimation performance  $T$  evaluated through a Monte Carlo simulation is represented over the distance  $d_t$  between two targets for the method  $KG$  and  $AC$  (lines interpolated). The intersection probability  $P_{\text{int}}$  represents the overlap of the measurements' distribution. The methods' performances are identical.

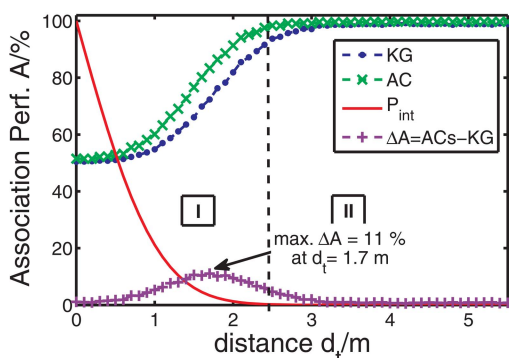


Fig. 5. The association performance  $A$  is represented over the distance  $d_t$  between two targets for the  $KG$  and the  $AC$  method (lines interpolated).  $AC$  outperforms the  $KG$  with respect to  $A$  when their validation intersect, but do not match.

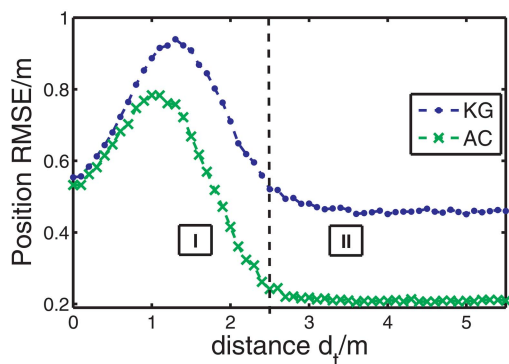


Fig. 6. The RMSE is represented over the distance  $d_t$  between two targets for  $KG$  and  $AC$  (lines interpolated). The characteristic peak in the RMSE is dependent on the methods' capability to detect the two targets.

standard deviation  $\sigma$  as a function of  $d_t$ :

$$P_{\text{int}} = 1 - \operatorname{erf}\left(\frac{d_t}{2\sqrt{2}\sigma^2}\right). \quad (17)$$

As  $AC$  is based on the  $KG$  for target number estimation, their performances with respect to  $T$  are identical

as visible in Fig. 4. The graph is classified into two regions depending on whether the target number  $N_t$  is underestimated (region I) or overestimated (region II).

The  $AC$  method enhances the performance of correct association to estimates whenever validation gates intersect, but do not match. An increase in the distance  $d_t$  in region I (see Fig. 5) improves the association performance  $A$ . The maximal difference in association performance ( $\Delta A$ ) is reached at an intersection probability of  $P_{\text{int}} = 3\%$ .

The positional RMSE for both methods increases in region I with an increasing distance  $d_t$ . However, the maximal error is reached for both methods before the limit of region I.  $AC$  reaches its maximum RMSE around  $d_t = 1.0$  m and  $KG$  around  $d_t = 1.3$  m. After the maximum the RMSE monotonically decreases. The RMSE of  $AC$  is always smaller that of  $KG$ .

## 7. DISCUSSION AND OUTLOOK

The performance of any parametric fusion method depends upon an accurate estimation of parameters. To avoid, for instance, an overestimation of the number of targets ( $T > 1$ ), the standard deviation of the measurements  $\sigma_e$  around the targets has to correspond to the standard deviation of validation gate  $G$ . If  $\sigma$  of  $G$  is larger than  $\sigma_e$ , the association performance  $A$  deteriorates. In contrast, if  $\sigma$  of  $G$  is smaller than  $\sigma_e$ , the number of estimates  $N_t$  is overestimated at a smaller distance of  $d_t$ , enhancing  $A$ .

Comparing Fig. 5 with Fig. 6 allows to reason that the smaller RMSE of  $AC$  in region II—in average 0.25 m better—is not due to better association performance, since in region II,  $A$  converges to the optimum for both. Hence, the smaller position RMSE must be due to the difference on how the estimates' position is evaluated. In region I however, there is an improvement of association performance with  $AC$  which explains why the maximal RMSE is reached at smaller  $d_t$  and why it constantly decreases in comparison to  $KG$ .

The presented method is based on Kalman filtering which intrinsically is based on a motion model for the targets. Despite the focus of this paper on static states, the authors hope to extend the presented framework to dynamic states where situations of limited resolution may also arise. The Kalman filter in the static case behaves like a median or low-pass filter.

The evaluation of the position error (Eq. 16) could be based on an optimal assignment method such as the Hungarian method [19]. However, the Hungarian method may as well produce unsatisfactory results due to its global optimization nature, especially in cases where the estimated target number does not correspond to the true one.

The revisable reasoning characteristic of the TBM method is questionable because the mass of the empty set corresponding to the conflict is a strictly monotonic increasing function. More combinations of conflicting



class information decrease the specificity of information. Robust combination rules such as suggested by Florea or Dezert might be beneficial [7, 10]. What is missing in the TBM combination rule is an expression of the reliability dependent on the total amount of fused information.

## 8. CONCLUSION

This paper demonstrates the potential of basing association not only on spatial aspects, but on classification attributes as well. The post-processing of the AC method enhances the association performance for closely lying targets while diminishing the RMSE. The belief combination framework of the TBM is a valuable tool to quantitatively express conflict, and to infer from class membership even for contradictory class information, which is to be expected for closely lying targets.

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