Two formulas of generalized Fermat numbers which seems to generate large primes

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Abstract. There exist few distinct generalizations of Fermat numbers, like for instance numbers of the form $F(k) = a^{(2^k)} + 1$, where $a > 2$, or $F(k) = a^{(2^k)} + b^{(2^k)}$ or Smarandache generalized Fermat numbers, which are the numbers of the form $F(k) = a^{(b^k)} + c$, where $a$, $b$ are integers greater than or equal to 2 and $c$ is integer such that $(a, c) = 1$. In this paper I observe two formulas based on a new type of generalized Fermat numbers, which are the numbers of the form $F(k) = (a^{(b^k)} ± c)/d$, where $a$, $b$ are integers greater than or equal to 2 and $c$, $d$ are positive non-null integers such that $F(k)$ is integer.

Conjecture 1:

There exist an infinity of odd integers $k$ such that the number $p = (2^{(2^k)} + 2)/6$ is a prime of the form $30*n + 13$, where $n$ positive integer.

Examples:

First three such primes $p$, obtained for $k = 3, 5$ and 7:

- $p = 43 = 1*30 + 13$ for $k = 3$;
- $p = 715827883 = 23860929*30 + 13$ for $k = 5$;
- $p = 56713727820156410577229101238628035243 = 1890457594005213685907636707954267841*30 + 13$ for $k = 7$.

Note that for $k = 9$ is obtained a number $p$ with 154 digits!

Conjecture 2:

There exist an infinity of odd integers $k$ such that the number $p = (2^{(2^k)} - 2)/2$ is a prime of the form $30*n + 7$, where $n$ positive integer.

Examples:

First three such primes $p$, obtained for $k = 3, 5$ and 7:
\[ p = 127 = 4 \times 30 + 7 \text{ for } k = 3; \]
\[ p = 2147483647 = 71582788 \times 30 + 7 \text{ for } k = 5; \]
\[ p = 170141183460469231731687303715884105727 = 5671372782015641057722910123862803524 \times 30 + 7 \text{ for } k = 7. \]

Note that for \( k = 9 \) is obtained a number \( p \) with 154 digits!